FROM EUCLID TO THE AGE OF THE IMAGE:
FROM THE GEOMETRIC INTERPRETATION OF THE EXPERIENCE OF VISION OF THE CONSTRUCTION OF EXPERIENCE. THE ROLE OF GEOMETRY IN ARCHITECTURAL EDUCATION.

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ABSTRACT

This paper attempts to give an interpretation of the importance of the role of geometry and in particular the courses of perspective in the formation of the perception of space, in the architectural education. A role that seems to focus both on understanding methods of handling space and its objects, as well as on redefining the real-imaginary relationship; physical – virtual, collective - subjective. This inquiry begins with the axioms of Euclid’s Optics, from the period of Girard Desargues to the modern era, placing three different phases in the evolution of the formation of the perception of space, through the sense of vision. In the last part of the work, an application of perspective image remodeling is presented, through mirrors that are placed in an existing space. (fig.1)

Geometry plays a dual role in the educational process of architects. On the one hand it aims to interpret the association between the different elements of space by frequently questioning established stimulus-response relations. The pattern of geometric thinking starts with observation and through analysis and questioning - which is a stage defined by its intense illusory nature; it ends up formulating rules and eventually leads to new interpretations of disciplines and relationships that are already known.

Category: Representation of architecture; perspective; modelling

Keywords: geometry; perspective; Euclidean geometry; representations of architecture; transformations
INTRODUCTION

This paper raises the question of geometry’s significance in shaping the perception of space in architectural education. Initially, an evolutionary creative process is provided, as well as an interpretation of spatial perception, focusing on three main periods: the Euclidean period, the Renaissance and Baroque periods, and finally the modern era of “image.” The work is completed by presenting an example of constructing and capturing an experience at the limits of a physical space with the aid of a camera’s movement in space and the realization of various images. What is supported is the transition from the time of observation and official foundation of the experience of vision to the era of the development of experience, through the manipulation of different images - versions of the existence of a space.

From the moment we are born we start to create a library of stimuli-responses in relation to our surrounding space. This relationship appears to be bidirectional. The same stimulus causes similar responses in such a way that the response recalls the memory of the stimulus. These relationships are stored in our memory in the form of images. However, what happens when this dipole is disrupted in an artificial manner, that is, when the image dictated by the memory is not confirmed, or when other images are inserted in these images? This is when a certain kind of delusion is created, which lasts until the new stimulus-response synapse for the space is restored. It is then that the illusion complements reality, reshaping the perception of it. Perspective appears to be able to provide the geometric tools for the creation of such interventional images.

When we prepare for an action and the topic of the representation is such, and also while carrying out this action, we mainly have three containers, which are automatically set into operation. These centres/containers communicate with each other and their function is to collect stimuli/tools, intangible signs that will prepare us to take action. These include the mind, the body and the spirit. At times our actions are guided by the one and at other times by the other. These automatically pop up, the one in front of the other, constantly, based on our temperament and stimuli, and they dictate our actions according to the goal of the action.

It has been claimed since antiquity that the perception we have of space is determined by our senses. The world, as we perceive it, is in agreement with our senses. However, what does this perception actually mean and to which degree does it require analysis? To which degree are our thoughts dictated by what we already know through our senses and how much room is there for transcendence? Do the objects of the physical world determine their existence on their own? Are they in a position to impose themselves on their observers? Or do they gain importance through a more subjective view of the world and consequently through the creation of relationships of interaction with observers? This paper argues that Geometry taught in schools of architecture plays a secret role: it introduces students to the structural rules applying to such an observation. These are the base rules that will free the spirit in its search for the relationship between the spatial objects and elements, and that which is born from the architect’s imagination, without warning, when entering the process of design action.

The road to interpreting a stimulus starting from a personal observation is defined by an analytical-compositional course, which is organized based on the following pattern:


1. THOUGHTS ON EUCLID’S OPTICS

The perception of space through the phenomenon of vision was first studied by Euclid in his “Optics”, who tried to give a geometric interpretation of visual perception. Based on seven official propositions, the so-called terms, he spoke about the apparent sizes of the shapes and their distances, without relying on graphic methods of their representation, but on how and why they are perceived by the observer. (table!) Employs the imagination, through hypotheses and using intangible optical rays and optical angles, leading the observer to a more active observation of the world.
Table 1 The Seven Terms in Euclid’s Optics

The initial assumption he made was the existence of straight lines connecting the eye with the serrated object. The set of these straight lines, which converge on the eye, was considered to be the surface of a cone, based on the object. In other words, he considered that there is no gap between these straight lines. Therefore the relationship of the points of the object with the number of optical rays is continuous, one to one and on². This is confirmed by the existence of his latest proposition, according to which the more points of view an object has, the more its angles correspond to it and therefore the more accurately it is distinguished.

Angles are a measure of the quantification of vision set by Euclid, which is more accurate in shaping perception than the optical rays later adopted by the classical perspective, since the human eye is spherical and not level, such as the Renaissance Perspective table.

Specifically, he mentions (Proposition 8) that sizes that are equal and parallel and that are different distances from the eye, do not appear depending on their distances, but depending on their viewing angles. In contrast to the linear perspective there is equality of reasons. He also stated (Suggestions 35 and 36) that equal circles belonging to the frontal plane do not appear as circles, but appear so “distorted”, depending on their angle of view.

Actually in a system of spherical perspective, a system in which the table of perspective would not be flat, but spherical, such as the human eye, the resulting images would be closer to the experience of human vision. Figure 1 shows the construction of a perspective on a spherical table and then its transfer to the level, with the system of stereoscopic projection. The horizontal straight lines of space are projected in arcs of circles passing through the edges of the horizontal diameter of the view of the sphere. Respectively, the vertical lines of space are projected in arcs of circles passing through the two poles of the sphere. Consequently straight lines are represented as parts of circular arcs.

But the question that arises is: While these axioms and Euclid’s propositions were based on a geometric interpretation of the phenomenon of vision, they were essentially invented to interpret the experience of vision, for the most part, when one observes images resulting from the application of the perspective rules of this approach, the object itself is not recalled in memory, to the extent that it is recalled by observing a flat perspective image - a photograph? Does it have to do only with the perceptual mechanism that has been developed and which connects

1 With the mathematical significance of continuity
2 There was no concept of scarcity then
the physical object with its flat image? Or is it related to the evolution of collective attitudes towards the way we process visual stimuli?

2. BAROQUE QUESTS
- THE QUEST FOR A REASONED CONNECTION WITH THE UNKNOWN

The Baroque period is the period that strongly expresses the need for humans to connect with an idea of the world. Physical forms imitate – they refer to some other distanced forms, a perfect model that resides in the world of ideas. The geometry of “being” gradually gives way to the geometry of “appearance” and this happens with the simultaneous shift from the Euclidean formulation of space to another, projective view. Euclid’s optical rays and angles acquire literal meaning, since the infinite point of the line remains at infinity, only now it is considered fully defined and known, at infinite distance. The intersection in thinking about space is introduced by Desargues’ Projective Geometry. This point - present but inaccessible - functions as the center of projection, as the apex of the Euclidean optical cone, as the eye of God. The objects of the physical world are considered distorted versions of perfect patterns of another world, and the transition to thought from the earthly world to the celestial implies a turning point; a kind of discontinuity. The discontinuity implied by death.

The art and architecture of this period often express the agony of the mental connection of humans with the world of ideas. The mirror or water element justifies the relationship between the perfect model and its distorted version. The tool of constructed illusions in architecture dictates the asymmetry between the experience of space and the perception of the natural, through the sense of vision. The asymmetry between image and pattern, between knowledge and experience seems to prevail.

Desargues’ geometry supports this need by setting the laws for the study of space as a consequence of an act of projection. The man himself is a product of this projection. Thought and perception work together towards a new view of the world, a little more complex than Euclid. A consideration that includes what he cannot control - the unattainable - the remote.

For the baroque society the here and there consists of a stable system. The present on Earth is the image of a distant model. The whole society believes in the existence of an external - general - judge (of God), who controls the figurative correspondence of the standard world with the earthly. (fig.26) There is the belief in the figurative correspondence of the two worlds and the expression of this correspondence with similarity in terms of art and architecture, according to the development of terms by Deleuze.1

Distinctive was the attempt of René Descartes 2 (1595-1650) to prove the existence of this other distant divine world. If he had achieved this, faith would have secured its meaning - they would believe in something that existed - and the only thing that would separate the human element from the divine it would be a very long distance: infinity. Leonhard Euler had made a similar effort and came to the conclusion that there is a God.3

Descartes4 began his reasoning with the concept of imagination, which he connected with the projection of a primary image of a concept from one’s mind. So he argued that this thing may not exist, but everyone can have their own idea of it. This is the aura of things, unchanging and internal. The object imposes its aura on the human mind and he accepts it without a second judgment. As such an object we can imagine a triangle.5 A complex of three

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5 Dionysis Anapeltanos (1985), ibid, pp.80
points in space that are connected to each other by the condition called "triangle". This is accepted by all. Once accepted, the mind enters the process of explaining and proving in order to understand its properties. But the strong and completely understandable is always the original triangle, which exists and therefore lives, in everyone's mind. It is something that everyone can talk about and meaning everyone has their own triangle, everyone can communicate with each other. It is for everyone distinct and completely personal and at the same time common for everyone. The triangle as an idea works unifyingly. This is exactly the different approach through the texts of Rene Descartes, who thus seeks the link - the balance between truth and existence. It tries to strike a balance between the concepts "what I clearly understand with my imagination is true" and "what is true exists". Imagination is connected with instinct. According to Martin Briggs, Baroque art and architecture reflect the power of the Church and are largely aimed at stimulating the imagination and unleashing religious instinct. It is in this context Descartes distinguishes the objective reality of things from conventional reality. The first represents the idea behind everything. It is the image that is recalled in the mind, or even better the sensation that is evoked as a stimulus, when hearing the name of this concept. The second reality - the conventional one - is the formalized concept. When the concept is overshadowed by the face of the object.

4. THE EXAMPLE OF THE CONSTRUCTION OF AN ILLUSION, INTERFERING WITH THE PHYSICAL BOUNDARIES OF A SPACE.

Nowadays, the relationship between physical and virtual reality is very often explored. This search is usually done in a digital environment. In the framework of an example of the work presented, a condition of alteration of the physical boundaries of a space is designed, which depends on the tour of an observer inside


While Euclid gave a geometric interpretation of the experience of vision, meaning that he gave an interpretation of the relation of the perception we form of space and the mechanism of vision; Desargues set rules of perspective, in an attempt to set up a closed system between the natural world and the world of ideas, while Descartes attempted to prove the existence of this distant model, from the projection of which every earthly experience derives, the modern age shows a tendency to construct experience, through the manipulation of the image. Through the management of fragmentary impressions of space (for the world), setting up a synthetic mechanism, which takes the different images - versions of the existence of space, in order to establish a reality beyond the physical limits and under the condition of the observer's involvement. The observer in this view is not a passive recipient of a static image, but attempts to participate in the shaping of his experience, shaping both his action and the image of space.

With this term, the perception of the space is not static, but changes and is related to the experience of the visitor. The following is a description of an example of a construction in real space, which aims to create the impression of removing the physical boundaries of space.

Fig 2. The model of the room with the interior walls.

4.1. THE CASE OF A ROOM

The example on which this paper focuses assumes that there is a real space with certain indoor walls (fig. 2). With the help of mirrors, the rules of perspective and the laws of reflection, an image is created there where there is none, thus eliminating the physical boundaries of space. The process is inverse to perspective. What is examined is how one can start from that which they wish to observe from a point of vision through the planes of curved mirrors placed in different points in space, and from there to go on to the construction that will have the desired image.

Suppose a room, which has inside it some walls placed vertically. (fig. 2) It is possible, with the help of mirrors of suitable shape and size, which will be placed on the walls, to project an image in the empty space of the room, in a desired position, thus removing the boundaries of the walls. This image can be such to complement the image of the rest of the space from a point of view, even from the front door to the room. This results in an image of the room, which is composed of the projection of both the parts of the physical elements of the room and the parts of the image. In other words, a hybrid, alternative aspect of space emerges, which calls into question the physical boundaries of space and therefore their image.

4.2. THE GEOMETRIC DESIGN APPROACH

The floor plan of the space is given. (fig. 3) The point where the observer will stand is taken in the middle of the entrance opening and at a certain height (the height of the observer). On the walls that are directly visible with the entrance of the observer, three mirrors k1, k2, and k3 are placed, with the necessary condition that there is visual continuity, i.e., there are no gaps or overlaps in the optical rays projected by the mirrors. (fig. 3)

We first consider the successive optical rays OA, Oβ, Oλ, and OZ. Of these, Oβ and Oα have a defined position from the beginning, so that they pass through...
wall edges, while the other two result with the criterion that reflections are not prevented. This results in the positions of the mirrors $k_1$, $k_2$, $k_3$ on the walls.

The three parts of the image are chosen to be placed on three walls of the given space, with the criterion that they are not immediately perceptible by the observer, as well as that the reflections are not obstructed by some walls. For this purpose, we consider the symmetries of the point of view $O^*$ with respect to the planes of the mirrors, where the points $O_1^*$, $O_2$ and $O_3$ respectively result. If we project from $O_1^*$ to $AB$, it appears on the wall where the width of the image $E_1$ has been selected, as the intersection of the resulting pyramid with the level of the wall. Similarly, projecting from $O_2$ to $BI$, the image $E_2$ emerges and projecting seen by $O$, through the three mirrors, on the other hand, the parts of the mirrors that are required to show the image.

### 4.3. Determining the shape of the mirrors

Initially, the shapes and dimensions of the mirrors are determined, depending on the positions in which they will be placed. The $k_1$ mirror will look promisingly in the $ABC\Delta$ shape of the blue wall. The $k_2$ will be the $BEZ\Gamma$ of the green wall and $k_3$, the $EH0Z$ of the yellow wall. (fig.5) The altitudes of these points can be calculated in perspective, with inverse perspective lines. To calculate the altitude $h$ of any point on the straight line $\alpha$, given perspective $(\alpha)$, it is sufficient to have point $M$, where $\alpha$ meets the plane of the table, and point $1$, where the projection of $\alpha$, or $\alpha^*$ meets the same level. (fig.6)

According to this, the actual heights of the points belonging to the blue wall are measured perpendicular to the table at point 1. (fig.6) For the green wall, the heights are measured perpendicular to the table at point 2 and for the yellow at point 3. Thus, to calculate the altitude of points $A$ and $D$, for example, which belong to the blue wall, we project them from the escape point $\Phi_1$ of this wall and at the intersection of these lines vertical to the table from point 1, the heights of these points are obtained.

Similarly, to calculate the altitude of point $G$, if we consider it to belong to the green

![Fig.5 The graphic determination of the altitudes of the mirrors corners.](image)

![Fig.6 Explanation of the determination of the altitude of the points of a horizontal line $\alpha$ from its perspective image.](image)
wall, we project it from \( \Phi_2 \), which is the escape point of this wall, and vertical to the table at point 2. Its height is obtained. If we consider \( Z \) to belong to the yellow wall, then we project it from \( \Phi \), and its height is vertical to the table at point 3. Respectively we calculate the altitudes of all the points that define the vertices of the parts of the mirrors required for to show the whole picture.

The true shape of the parts of these mirrors is obtained if we project them in parallel to them. Thus, projecting the mirrors \( k_1 \) and \( k_3 \), in a vertical plane \( E_3 \) with trace \( Y_13 \) and placing the altitudes of the points as measured from the perspective, the true size of the mirrors will be obtained as well as their distance from the ground level. (fig.7) Respectively the mirror \( k_2 \) results if we project it in a vertical plane \( E_4 \), with trace \( Y_14 \).

### 4.4. Determining the Shape of the Parts of the Image

The next stage is to calculate the format that the parts of the image will take. (fig.8) The images will appear as sections of the pyramids projected by the mirrors from the symmetry of the point of view to the plane of the mirrors, with the surfaces of the walls where the images are placed.

For example, the image \( E_3 \) appears as a section of the pyramid projecting \( k_3 \) from \( \Phi_3 \). In the projection parallel to the plane of the image \( E_3 \), its real shape will emerge, but also its position on the wall, i.e., the altitudes of its points.
from the ground level. Respectively, the actual sizes of the images $E_1$ and $E_2$ are obtained.

In the final phase, we divide the image into the corresponding sections, using an image editing software (Photoshop, Corel Draw, etc.). (fig.9) The resulting three parts will be rectangular. In the same software we deform the three parts, so that they take the form that resulted from the constructions, simultaneously inverting the left with the right border of each piece. The final images are placed, taking into account their orientation.

4.5. THE CONTROL OF THE SPACE EXPERIENCE

For the purpose of controlling the impression created while navigating the room, a scale model of the space was constructed and inside a camera was set in motion, which gave the image of the space from different observation positions inside. (fig.10) The first feeling, from the point of entry into space, is created by the impression of the existence of a rectangular opening, which removes the structural continuity of the walls of the space and from which the selected work of Picasso is seen frontally. (fig.11) During the movement of the camera, the four-sided shapes of the mirrors are gradually revealed, as well as the hidden parts of the image. (fig.12)

Fig.9 Further processing in Photoshop environment.
Fig. 10 Different shots inside the room via camera.

Fig. 11 The observer's impression from the point of entry into space O.

Fig. 12 Mirror and image parts.
CONCLUSION

From Euclid until today, the way one perceives the natural space and projects one’s thought or imagination on it has changed. From the official foundation of the visual experience and through an ever-expanding perception of the limits and the form of the natural space, the modern era highlights the tendency towards the practical formation of impressions - experiences, with the image of the natural as a canvas.

The role of geometry in the education of architects is to expose them on an analytical compositional pattern and to enable them to reset the rules in order to handle the objects as is necessary in each circumstance; to question what is already known; to set rules or adopt existing rules (seeing as there are already so many) in order to transcend the boundaries of interpretation.

Geometry is essentially a compositional process of thinking that fosters design action and shapes perception. Geometry offers the knowledge that helps us manage and redefine the rules, driven by the action of thinking in each instance. Rules exist in order to ensure that there are no barriers and everything is open.

REFERENCES


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