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SEISMIC HAZARD ASSESSMENT IN THE NORTHERN AEGEAN SEA (GREECE) THROUGH DISCRETE SEMI-MARKOV MODELING

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Abstract

Semi-Markov chains are used for studying the evolution of seismicity in the Northern Aegean Sea (Greece). Their main difference from the Markov chains is that they allow the sojourn times (i.e. the time between successive earthquakes), to follow any arbitrary distribution. It is assumed that the time series of earthquakes that occurred in Northern Aegean Sea form a discrete semi-Markov chain. The probability law of the sojourn times, is considered to be the geometric distribution or the discrete Weibull distribution. Firstly, the data are classified into two categories that is, state 1: Magnitude 6.5-7 and state 2 Magnitude>7, and secondly into three categories, that is state 1: Magnitude 6.5-6.7, state 2: Magnitude 6.8-7.1 and state 3: Magnitude 7.2-7.4. This methodology is followed in order to obtain more accurate results and find out whether there exists an impact of the different classification on the results. The parameters of the probability laws of the sojourn times are estimated and the semi-Markov kernels are evaluated for all the above cases. The semi-Markov kernels are compared and the conclusions are drawn relatively to future seismic hazard in the area under study.

Key words: *semi-Markov chains, Markov chains, transition probability matrix, sojourn time distribution function*

Περίληψη

Οι ημι-Μαρκοβιανές αλυσίδες χρησιμοποιούνται για τη μελέτη της σεισμικότητας στο Βόρειο Αιγαίο. Η βασική τους διαφορά από τις Μαρκοβιανές αλυσίδες είναι ότι επιτρέπουν μια οποιαδήποτε αυθαίρετη κατανομή για τους χρόνους παραμονής (χρόνοι μεταξύ διαδοχικών σεισμών). Υποθέτουμε ότι η χρονοσειρά των σεισμών που έχουν γίνει στο Βόρειο Αιγαίο αποτελεί μια διακριτή ημι-Μαρκοβιανή αλυσίδα. Θεωρείται ότι οι χρόνοι παραμονής ακολουθούν γεωμετρικές ή διακριτές κατανομές Weibull. Πρώτα ταξινομήθηκαν τα δεδομένα σε δυο κατηγορίες, όπου κατάσταση 1: Μέγεθος 6.5-7 και κατάσταση 2 Μέγεθος>7, και στη συνέχεια σε τρεις κατηγορίες, όπου κατάσταση 1: Μέγεθος 6.5-6.7, κατάσταση 2 : Μέγεθος 6.8-7.1 και κατάσταση 3 : Μέγεθος 7.2-7.4. Εκτιμήθηκαν οι παράμετροι των συναρτήσεων πιθανότητας των χρόνων παραμονής και υπολογίστηκαν οι πίνακες πυρήνες της ημι-Μαρκοβιανής αλυσίδας για όλες τις παραπάνω περιπτώσεις. Έγινε σύγκριση των πινάκων πυρήνων και προέκυψαν συμπεράσματα για τη μελλοντική σεισμική επικινδυνότητα στην υπό μελέτη περιοχή.

Λέξεις κλειδιά: *ημι-Μαρκοβιανές αλυσίδες, Μαρκοβιανές αλυσίδες, πίνακας πιθανοτήτων μετάβασης, συνάρτηση πιθανότητας χρόνων παραμονής*

1. Introduction

Stochastic models are widely used to obtain results concerning the seismic hazard assessment. In Patw-ardhan et al. (1980) a semi-Markov model is developed to estimate the likelihoods of occurrences of great earthquakes (M≥7.8). Fujinawa (1991) studied the earthquake occurrence via a Markov chain and data from China, whereas Al-Hajjar and Blanpain (1997) used a semi-Markov model in a swarm sequence and obtained the optimal value for the total duration of the sequence. Altinok and Kolcak (1999) estimated the earthquake occurrence probabilities by a semi-Markov model and studied the inte-rval transition probabilities. Nava et al. (2005) evaluated the seismic hazard of the Japan area via a Ma-rkov chain and Sadeghian (2010) applied a semi-Markov model to forecast the triad dimensions of ear- thquakes. Votsi et al. (2010a, b, 2012a, b) applied hidden Markov and hidden semi-Markov modeling for the description of seismicity patterns.

In this paper a discrete semi-Markov model is proposed for the area under study, which is the Northern Aegean Sea (Greece). This model can be successfully applied in Seismology, considering the earthqua-kes as discrete events of the chain. It allows the interevent times (sojourn times) between two earthqua-kes, to follow any arbitrary distribution, which makes the semi-Markov chains a generalization of Mar-kov chains (Kemeny and Snell, 1976). Using this model, important quantities can be estimated, such as the mean value of the first hitting times (the mean time that an earthquake of state j will occur for the first time given that the previous earthquake was of state i, (Howard, 2007).

The data are obtained by a complete, homogeneous and accurate catalogue from the Geophysics Depar-tment of the Aristotle University of Thessaloniki and cover the period 1845-2008.

In this paper the quantity that is studied, is the discrete semi-Markov kernel, which gives the probabili- ty that an earthquake of state j will occur after k time units, given that the previous earthquake was of state i. It is assumed that the probability law of the sojourn times is either the geometric or the discrete Weibull distribution and the results are compared.

2. Semi-Markov Kernel for the two Dimensional State Space

The state space is firstly assumed to be two dimensional by classifying the data into two categories, ac-cording to the range of magnitudes (smaller and larger earthquakes). The sojourn times are supposed to follow geometric or discrete Weibull distributions, in order to examine the differences of the probabili- ties related to the aforementioned distributions.

2.1 Geometric Sojourn Times

In this section, it is assumed that the sojourn time distribution law is the geometric which is a common distribution law and it can be well adapted in the area under study (Pertsinidou, 2012). The probability mass function of the geometric distribution is the following:

Definition 1-Geometric distribution

$P(X = k) = (1-p)^{k-1}p, \ \ k = 1,2,\ldots$

In the sequel we give some definitions concerning the semi-Markov chains which are necessary for what follows (Barbu and Limnios, 2008).

Let E={1,…,s} be a finite state space, whose evolution in time is governed by a stochastic process $Z = (Z_k)_{k\in N}$. Let us also denote by $S = (S_n)_{n\in N}$ the successive time points when state changes in $(Z_n)_{n\in N}$ occur and by $J = (J_n)_{n\in N}$ the chain which records the visited states at these time points. Let $X = (X_n)_{n\in N}$ be the successive sojourn times in the visited states. Thus, $X_n = S_n - S_{n-1}, \ n \in N^*$, and, by convention, we set X_0=S_0=0. If $P(J_{n+1} = j, S_{n+1} - S_n = k \,|\, J_0,\ldots,J_n; S_0,\ldots,S_n) = P(J_{n+1} = j, \ S_{n+1} - S_n = k \,|\, J_n)$, then $Z = (Z_n)_{n\in N}$ is called a semi-Markov chain and the couple (J_n, S_n) is call- ed a Markov renewal chain. The visited-state chain $(J_n)_{n\in N}$ is called the embedded Markov chain. We denote by p_{ij} the transition probabilities, that is:

Definition 2 – Transition probabilities

$p_{ij} = P(J_{n+1} = j | J_n = i), i, j \in E, n \in N.$

The matrix $P = (p_{ij})$ is called the transition probability matrix. The distribution function of the sojourn times is defined as follows:

Definition 3 – Sojourn time distribution function

$f_{ij}(k) = P(X_{n+1} = k | J_n = i, J_{n+1} = j).$

The semi-Markov kernel probabilities that we study throughout this paper are defined as follows:

Definition 4 – Discrete-time semi-Markov kernel probabilities $q_{ij}(k)$

$q_{ij}(k) \coloneqq P(J_{n+1} = j, X_{n+1} = k | J_n = i) = p_{ij} f_{ij}(k).$

Then the semi-Markov kernel is the matrix $Q(k) = (q_{ij}(k))$ and constitutes the essential quantity which defines a semi-Markov chain.

The data concerning earthquakes that occurred in Northern Aegean Sea from 1845-2008 are classified, according to their magnitude, into two categories which are state 1: Magnitude 6.5–7 and state 2: Magnitude>7. In order to study the semi-Markov kernel probabilities, we need first to estimate the transition probabilities. The estimators of the transition probabilities are (Barbu and Limnios, 2008):

Definition 5 – Estimators of the transition probabilities

$\hat{p}_{ij}(M) = N_{ij}(M)/N_i(M)$, if $N_i(M) \neq 0$,

where $N_i(M)$ is the number of transitions of the embedded Markov chain to state *i*, until time *M*, and $N_{ij}(M)$, is the number of transitions of the embedded Markov chain from state *i* to state *j*, until time M. If $N_i(M) = 0$ we set $\hat{p}_{ij}(M) = 0$ for all M and if $N_{ij}(M) = 0$ we set $f_{ij}(M) = 0$ for all *M*. The time unit is considered to be the year and the transition matrix for our data is found to be

$P=\begin{pmatrix} 0.71 & 0.29 \\ 0.8421 & 0.1579 \end{pmatrix}.$

If we assume that the sojourn times follow geometric distributions, the maximum likelihood estimators of these geometric distributions are found to be (Pertsinidou, 2012):

$f_{11}(n)=\frac{39}{109}\left(\frac{70}{109}\right)^{n-1}$, $f_{12}(n)=\frac{2}{3}\left(\frac{1}{3}\right)^{n-1}$, $f_{21}(n)=\frac{8}{17}\left(\frac{9}{17}\right)^{n-1}$, $f_{22}(n)=\frac{3}{4}\left(\frac{1}{4}\right)^{n-1}$.

Then the kernel of the semi-Markov chain becomes

$Q(n)=\begin{pmatrix} 27.69 * 70^{-1+n} * 109^{-n} & 0.58 * 3^{-n} \\ 6.74 * 9^{-1+n} * 17^{-n} & 0.47 * 4^{-n} \end{pmatrix}$, n=1,2,3,…

The corresponding graphs, in which the decay of the kernel probabilities as time passes can be observed, are the following:

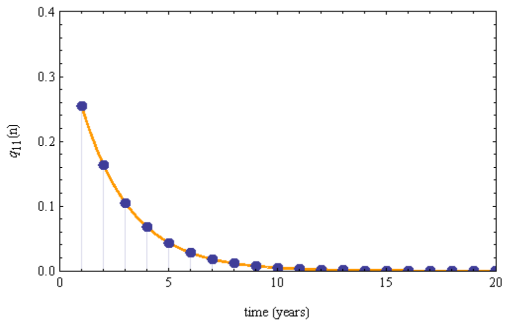

Figure 1 - Kernel probabilities $q_{11}(n)$, of the geometric distribution.

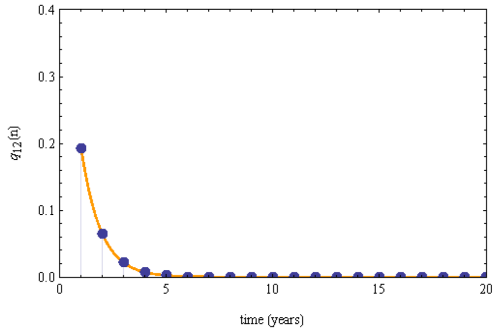

Figure 2 - Kernel probabilities $q_{12}(n)$ of the geometric distribution.

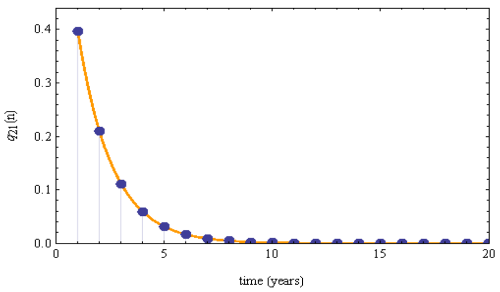

Figure 3 - Kernel probabilities $q_{21}(n)$, of the geometric distribution.

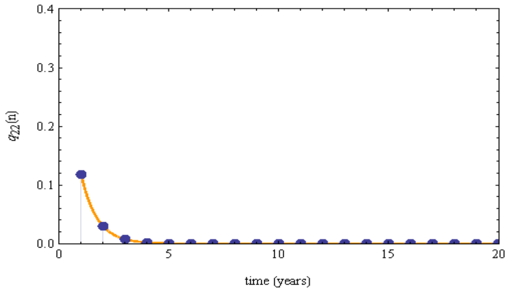

Figure 4 - Kernel probabilities $q_{22}(n)$, of the geometric distribution .

It is evident from Figure 1, that there is a higher probability for an earthquake of state 1 to be followed by an earthquake of state 1 during the next year (0.254). There is also a still high probability that such an earthquake will occur after two or three years, while these probabilities decay quickly from three years on. In Figure 2, given that the previous earthquake was of state 1, there is a high probability that the next earthquake of state 2 will occur in the next year. The probabilities $q_{12}(n)$ decay very quickly and, as we can also observe by the values given in Table 1 below, they become nearly 0 for n>5. Figure 3 shows that if the previous earthquake was of state 2, then it is very probable that the next earthquake of state 1 will occur in the next four years and for n>4 the probabilities become considerably smaller. Figure 4 shows that if the previous earthquake was of state 2 then there is a small probability that the next earthquake will also be of state 2, but if so, this is to be expected in the next four years. For n>4 the probabilities become zero. The aforementioned probabilities are given analytically below (for n∈[1,20]).

Thus, the probability that an earthquake of state 1, will be followed within three years by an earthquake of state 1, is high and from the third year on the probabilities decay quickly. If the next earthquake is of state 2, given that the last earthquake was of state 1, then this is expected to occur in the first five years. An earthquake of state 2, is more probable to be followed by an earthquake of state 1 in the next three years. Finally, if we assume that the an earthquake of state 2, will be followed by an earthquake of state 2, then this is more likely to happen within the next two years. As already mentioned, the probabilities $q_{12}(n)$ and $q_{22}(n)$ decay very quickly, which means that visiting state 2 (M>7) is less probable as the sojourn time increases.

2.2 Discrete Weibull Distributions for the Sojourn Times

It is now assumed that the transition probability matrix is the same as previously, but the sojourn time distribution function is the discrete Weibull of equation 2 that follows. This distribution allows the sojourn times to obtain greater values than the geometric, thus the time between two

Table 1 - Semi-Markov kernel probabilities of the geometric distribution (two dimensional case).

n	$q_{11}(n)$	$q_{12}(n)$	$q_{21}(n)$	$q_{22}(n)$
1	0.254	0.193	0.396	0.118
2	0.163	0.064	0.209	0.029
3	0.104	0.021	0.111	0.007
4	0.067	0.007	0.059	0.002
5	0.043	0.002	0.031	0.000
6	0.027	0.000	0.016	…
7	0.018	…	0.009	
8	0.011		0.005	
9	0.007		0.002	
10	0.005		0.001	
11	0.003		0.000	
12	0.002		…	
13	0.001			
14	0.000			
…	…	…	…	…
20	0.000	0.000	0.000	0.000

successive earthquakes can now be greater. In the sequel the probability mass function of the discrete Weibull distribution is cited, where x stands for the sojourn time and q and b are positive parameters.

Equation 2 – Discrete Weibull

$f(n) = q^{(n-1)^b} - q^{n^b}$, $n=1,2,\ldots$, $0<q<1$ and $b>0$.

The parameters of the discrete Weibull distribution can not be estimated via the maximum likelihood method. There exists an empirical estimation effort (Kulasekera, 1994) which can not be used in our dataset, because of the small sample size of the sojourn times. Therefore, the parameters are estimated numerically and the distribution functions derived are (Pertsinidou, 2012):

$f_{11}(n) = 0.59^{(n-1)^{0.76}} - 0.59^{n^{0.76}}$, $f_{12}(n) = 0.53^{(n-1)^{0.94}} - 0.53^{n^{0.94}}$

$f_{21}(n) = 0.37^{(n-1)^{0.56}} - 0.37^{n^{0.56}}$, $f_{22}(n) = 0.4^{(n-1)^{0.62}} - 0.4^{n^{0.62}}$

Then the kernel matrix turns out to be

$$Q(n) = \begin{pmatrix} 0.71 * (0.6^{(-1+n)^{0.87}} - 0.6^{n^{0.87}}) & 0.29 * (0.25^{(-1+n)^{0.58}} - 0.25^{n^{0.58}}) \\ 0.8421 * (0.5^{(-1+n)^{0.89}} - 0.5^{n^{0.89}}) & 0.1579 * (0.34^{(-1+n)^{1.4}} - 0.34^{n^{1.4}}) \end{pmatrix}$$

The corresponding graphs of the discrete semi-Markov kernel functions are:

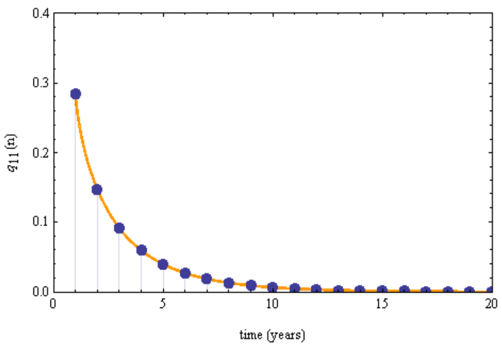

Figure 5 - Kernel probabilities $q_{11}(n)$, of the discrete Weibull distribution.

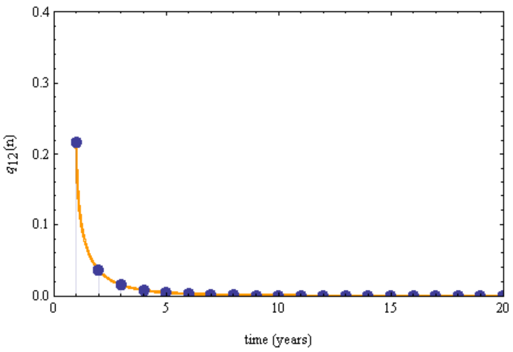

Figure 6 - Kernel probabilities $q_{12}(n)$, of the discrete Weibull.

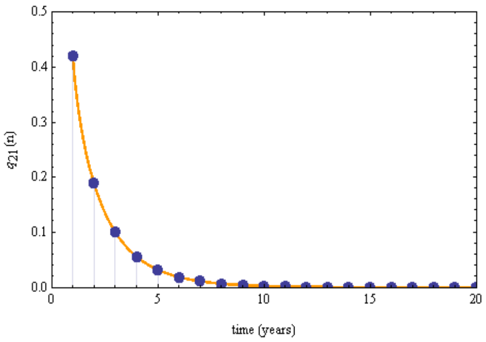

Figure 7 - Kernel probabilities $q_{21}(n)$, of the discrete Weibull distribution.

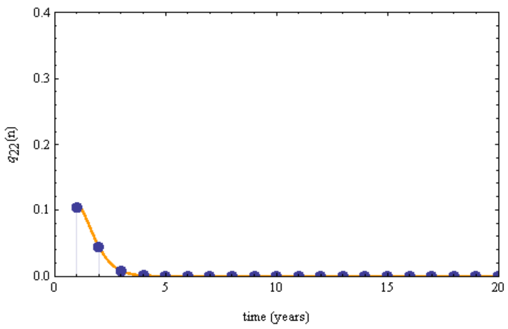

Figure 8 - Kernel probabilities $q_{22}(n)$, of the discrete Weibull distribution.

Comparing the above figures and also the values of the semi-Markov kernel probabilities given below, with the corresponding figures and values of the geometric distribution presented in the previous section, it can be seen that the results are similar, though we would expect the discrete Weibull kernel probabilities to decay much slower, than they do. This reinforces the previous conclusions concerning the expected seismicity. The values are given analytically (for comparison reasons) in the following Table 2.

3. Semi-Markov Kernel for the three Dimensional Transition Matrix

It is useful to classify the data into more than two categories, in order to observe if there are any differ- rences in the results. Now the data will be classified into three categories (we notice that more than three categories would lead to estimation problems due to the already small size of the dataset.) It is again firstly assumed that the times between two successive earthquakes follow the geometric distribu- tion and secondly the discrete Weibull distribution.

3.1 Geometric Sojourn Times

The data, concerning earthquakes that occurred in Northern Aegean Sea, are now classified into three categories, i.e. state 1: 6.5-6.7, state 2 : Magnitude 6.8-7.1, state 3 : Magnitude 7.2-7.4. The number $N_i(M)$ of visits in each state i and the transitions $N_{ij}(M)$ from state i to state j, until time M are found to be

Table 2 -. Semi-Markov kernel probabilities of the discrete Weibull distribution (two dimensional case)

n	$q_{11}(n)$	$q_{12}(n)$	$q_{21}(n)$	$q_{22}(n)$
1	0.284	0.217	0.421	0.104
2	0.147	0.035	0.188	0.045
3	0.091	0.015	0.099	0.008
4	0.059	0.008	0.055	0.000
5	0.039	0.005	0.032	…
6	0.027	0.003	0.018	
7	0.018	0.002	0.011	
8	0.019	0.001	0.006	
9	0.009	0.000	0.004	
10	0.006	…	0.002	
11	0.004		0.001	
12	0.003		0.000	
13	0.002		…	
14	0.001			
15	0.001			
16	0.000			
…	…	…	…	…
20	0.000	0.000	0.000	0.000

$N_1(M)$=29, $N_2(M)$=33, $N_3(M)$=12, $N_{11}(M)$=11, $N_{12}(M)$=14, $N_{13}(M)$=5, $N_{21}(M)$=15, $N_{22}(M)$=12, $N_{23}(M)$=5, $N_{31}(M)$=3, $N_{32}(M)$=7, $N_{33}(M)$=2.

The transition matrix is

$$P=\begin{pmatrix} 0.37 & 0.48 & 0.15 \\ 0.45 & 0.36 & 0.19 \\ 0.25 & 0.58 & 0.17 \end{pmatrix}.$$

Using the maximum likelihood function (Pertsinidou, 2012) we obtain the sojourn time distributions:

$$f_{11}(n)=\frac{11}{25}\left(\frac{14}{25}\right)^{n-1},\ f_{12}(n)=\frac{2}{5}\left(\frac{3}{5}\right)^{n-1},\ f_{13}(n)=\frac{5}{7}\left(\frac{2}{7}\right)^{n-1}$$

$$f_{21}(n)=\frac{5}{12}\left(\frac{7}{12}\right)^{n-1},\ f_{22}(n)=\frac{1}{2}\left(\frac{1}{2}\right)^{n-1},\ f_{23}(n)=\frac{5}{7}\left(\frac{2}{7}\right)^{n-1}$$

$$f_{31}(n)=\frac{3}{13}\left(\frac{10}{13}\right)^{n-1},\ f_{32}(n)=\frac{1}{2}\left(\frac{1}{2}\right)^{n-1},\ f_{33}(1)=1, \text{ and } f_{33}(n)=0 \text{ for } n>1.$$

The transitions from state 3 to state 3 found in the data are only two, which explains the fact that $f_{33}(1) = 1$, and $f_{33}(n) = 0$ for $n>1$.

The kernel matrix is

$$Q(n)=\begin{pmatrix} 4.07*14^{-1+n}*25^{-n} & 0.96*3^{-1+n}*5^{-n} & 0.75*2^{-1+n}*7^{-n} \\ 2.25*7^{-1+n}*12^{-n} & 0.36*2^{-n} & 0.95*2^{-1+n}*7^{-n} \\ 0.75*10^{-1+n}*13^{-n} & 0.58*2^{-n} & 0 \end{pmatrix}, n=1,2,3,\ldots$$

The only difference is that the probabilities now decay faster, compared with the two dimensional case, in most of the cases. We cite as an example only the first graph.

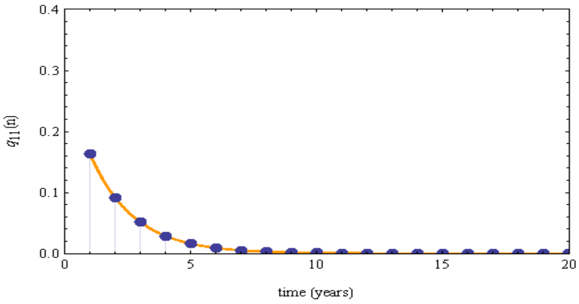

Figure 9 - Kernel probabilities $q_{11}(n)$, of the geometric distribution.

The kernel probabilities are the following (n∈[1,20]):

Table 3 - Semi-Markov kernel probabilities of the geometric distribution (three dimensional case).

n	$q_{11}(n)$	$q_{12}(n)$	$q_{13}(n)$	$q_{21}(n)$	$q_{22}(n)$	$q_{23}(n)$	$q_{31}(n)$	$q_{32}(n)$
1	0.163	0.192	0.107	0.187	0.18	0.138	0.057	0.29
2	0.091	0.115	0.031	0.109	0.09	0.039	0.044	0.145
3	0.051	0.069	0.009	0.064	0.045	0.011	0.034	0.073
4	0.028	0.041	0.002	0.037	0.022	0.003	0.026	0.036
5	0.016	0.025	0.000	0.022	0.011	0.000	0.02	0.018
6	0.009	0.015	…	0.013	0.005	…	0.015	0.009
7	0.005	0.009		0.007	0.003		0.012	0.005
8	0.003	0.005		0.004	0.001		0.009	0.002
9	0.002	0.003		0.002	0.000		0.007	0.001
10	0.000	0.002		0.001	…		0.005	0.000
11	…	0.001		0.000			0.004	…
12		0.000					0.003	
13							0.002	
14							0.002	
15							0.001	
…	…	…	…	…	…	…	0.001	…
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

We recall that $q_{33}(1) = p_{33}$ and $q_{33}(n)$=0 for n>1. We remind that in the two dimensional case it was $q_{11}(1)$=0.254. The corresponding probabilities appear to be smaller due to the different classification. We now expect fewer events of state 1, (6.5-6.7) given that the previous state was 1, which is reasonable since the class is smaller. From the values of Table 3, we realize that if the previous earthquake was of state 1, and the next one of state 2, then this is more probable to occur in the next 5 years. From the fifth year on, the probabilities decay and they become nearly zero for n>11 years. Furthermore, it is more likely for an earthquake of state 3 to be followed by an earthquake of state 1 in the next year and the probability that this transition will occur for n>4 years is almost zero. However, if the previous earthquake was of state 1, it is difficult to determine which one earthquake of the three classes is more likely to happen in the following year, because the related probabilities are found to be very close. We can also realize that if the last earthquake was of state 2, an earthquake of state 1 is more likely to happen after one or two years. For n>2 the probabilities are smaller and for n>10 they tend to zero. Also, given that the last earthquake was of state 2, an earthquake of state 2 is more likely to occur in the next five years, and for n>8 these probabilities become almost zero. If the previous earthquake was of state 2 and we assume that next one will be of state 3, then this is more likely to happen after one year, and the probability decays very quickly since for n>4 it is almost zero. Finally, comparing the values of $q_{31}(n)$ with $q_{32}(n)$ we find out that if the previous earthquake was of state 3, then an earthquake of state 2 is more likely to happen than an earthquake of state 1, in the next four years.

3.2 Discrete Weibull Distributions for the Sojourn Times

It is now assumed that the transition matrix is three dimensional, as estimated in the previous section, while the sojourn times follow discrete Weibull distributions. The parameters are estimated numerical- ly (Pertsinidou, 2012):

$f_{11}(n)= 0.54^{(n-1)^{0.98}} - 0.54^{n^{0.98}}$, $f_{12}(n)= 0.51^{(n-1)^{0.52}} - 0.51^{n^{0.52}}$

$f_{13}(n)= 0.4^{(n-1)^{1.4}} - 0.4^{n^{1.4}}$, $f_{21}(n)= 0.47^{(n-1)^{0.79}} - 0.47^{n^{0.79}}$

$f_{22}(n)= 0.51^{(n-1)^{0.83}} - 0.51^{n^{0.83}}$, $f_{23}(n)= 0.2^{(n-1)^{0.3}} - 0.2^{n^{0.3}}$

$f_{31}(n)= 0.89^{(n-1)^{1.4}} - 0.89^{n^{1.4}}$, $f_{32}(n)= 0.52^{(n-1)^{1.4}} - 0.52^{n^{1.4}}$

Then the kernel functions are found to be:

$q_{11}(n)= 0.37 * (0.54^{(-1+n)^{0.98}} - 0.54^{n^{0.98}})$, $q_{12}(n)= 0.48 * (0.51^{(-1+n)^{0.52}} - 0.51^{n^{0.52}})$

$q_{13}(n)= 0.15 * (0.4^{(-1+n)^{1.4}} - 0.4^{n^{1.4}})$, $q_{21}(n)= 0.45 * (0.47^{(-1+n)^{0.79}} - 0.47^{n^{0.79}})$

$q_{22}(n)= 0.36 * (0.51^{(-1+n)^{0.83}} - 0.51^{n^{0.83}})$, $q_{23}(n)= 0.19 * (0.2^{(-1+n)^{0.3}} - 0.2^{n^{0.3}})$

$q_{31}(n)= 0.25 * (0.89^{(-1+n)^{1.4}} - 0.89^{n^{1.4}})$, $q_{32}(n)= 0.58 * (0.52^{(-1+n)^{1.4}} - 0.52^{n^{1.4}})$

$q_{33}(n)=0$

We will cite, indicatively, only the graph of $q_{31}(n)$ which seems to differ from the others, which decay in a similar way to the already presented graphs in page 5.

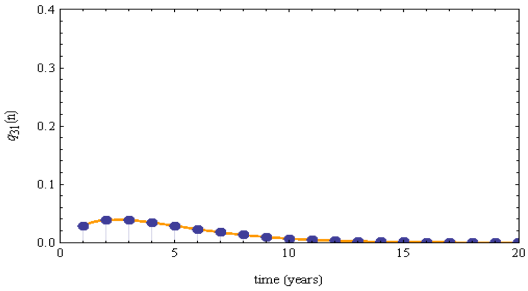

Figure 10 - Kernel probabilities $q_{31}(n)$, of the discrete Weibull distribution.

From the above graph we observe that the probabilities exhibit an increase for n=2,3. The values are shown in the following table.

Table 4 - Semi-Markov kernel probabilities of the discrete Weibull distribution (three dimensional case).

n	$q_{11}(n)$	$q_{12}(n)$	$q_{13}(n)$	$q_{21}(n)$	$q_{22}(n)$	$q_{23}(n)$	$q_{31}(n)$	$q_{32}(n)$
1	0.170	0.235	0.09	0.238	0.176	0.152	0.027	0.278
2	0.090	0.062	0.047	0.089	0.075	0.011	0.039	0.198
3	0.049	0.037	0.011	0.047	0.041	0.005	0.038	0.075
4	0.027	0.025	0.002	0.027	0.024	0.003	0.028	0.021
5	0.015	0.018	0.000	0.016	0.015	0.002	0.023	0.005
6	0.008	0.014	…	0.010	0.009	0.002	0.017	0.000
7	0.005	0.011		0.006	0.006	0.001	0.009	…
8	0.003	0.009		0.004	0.004	0.001	0.006	
9	0.001	0.008		0.003	0.007	0.000	0.005	
10	0.000	0.006		0.002	0.002	…	0.003	
11	…	0.005		0.001	0.001		0.002	
12		0.005		0.000	0.000		0.001	
13		0.004		…	…		0.000	
14		0.004					…	
15		0.003						
16		0.003						
17		0.002						
18		0.001						
19	…	0.001	…	…	…	…	…	…
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

From the above table we realize, that it is more probable for an earthquake of state 1, to be followed by an earthquake of state 2 or of state 1, and less probable of state 3, if the earthquake occurs during the next year. The probability of having a transition from state 2 to state 1 is more probable to happen after one year. The same holds for the probabilities $q_{22}(n)$ and $q_{23}(n)$. Also, given the fact that an earthquake was of state 3, the probability that the next will be of state 1 is more likely to happen between 2-4 years, as we mentioned before in the graph of $q_{31}(n)$. Finally if an earthquake of state 3, will be followed by an earthquake of state 2, we expect this to happen in the next five years, since for n>5 $q_{32}(n) \approx 0$.

4. Conclusions

The use of semi-Markov chains is a useful tool that provides the probabilities that the chain will visit a state after a certain time given the previous state. In our case this means, that knowing the previous ear-thquake we can evaluate the probability that the next earthquake will occur after n time units and will be of state j. Classifying the states to earthquake clusters, allows us to obtain results concerning the sei-smic hazard. The discrete semi-Markov kernel, is studied in the Northern Aegean Sea. The kernel pro- babilities derived under the assumption that the sojourn times follow geometric or discrete Weibull di- stributions, in the two dimensional case, are very similar in most of the cases. Concerning the three di-mensional case we observe a mixed behavior for small number of steps, but as time increases the geo-metric probabilities decay faster than the discrete Weibull probabilities. A remarkable observation is that the $q_{31}(n)$ probabilities of the discrete Weibull distribution, are higher for n=2,3,4. This means that the occurrence of an earthquake of state 1, given that the previous earthquake was of state 3, is more li-kely to happen for n=2,3,4. We notice here that the mean hitting times of the various states for the mo-dels we studied in the present paper are also of main interest concerning seismic hazard assessment and have already being studied in Pertsinidou and Tsaklidis (2012).

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