A NON-EXTENSIVE APPROACH IN INVESTIGATING GREEK SEISMICITY

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Abstract

In this work the Greek seismicity is being investigated by means of Non-Extensive Statistical Physics [NESP]. NESP is a generalization of Boltzmann-Gibbs statistical physics and has been successfully used for the analysis of a variety of complex dynamic systems, where fractality and long- range interactions are important. We use a non-extensive model that is derived from the first principles to describe the frequency-magnitude distribution of the Greek seismicity for the period 1976-2009 by using a recent earthquake catalogue for the Hellenic region compiled by Makropoulos et al. (2012). The results indicate that the non-extensive model can describe quite well the observed magnitude distribution for the entire magnitude range. Furthermore, the q parameter of the non-extensive model, along with the b-value of the Gutenberg-Richter relation, are being estimated for different time windows along the time expansion of the catalogue and the variations of these values are being discussed.

Key words: *Frequency – magnitude distribution, non-extensive statistical physics, Gutenberg-Richter relation.*

Περίληψη

Στην παρούσα εργασία εξετάζεται η σεισμικότητα του ελληνικού χώρου χρησιμοποιώντας τη γενικευμένη θεωρία της μη-εκτατικής στατιστικής φυσικής. Η θεωρία αυτή αποτελεί γενίκευση της στατιστικής φυσικής των Boltzmann-Gibbs και έχει επιτυχώς χρησιμοποιηθεί στην ανάλυση πολύπλοκων συστημάτων που παρουσιάζουν κατανομές fractal και συσχέτιση μεταξύ των δεδομένων τους. Χρησιμοποιώντας ένα μοντέλο το οποίο προκύπτει από τις αρχές της θεωρίας αυτής, εξετάζουμε την κατανομή συχνότητας - μεγέθους της ελληνικής σεισμικότητας κατά την περίοδο 1976-2009, όπως αναφέρεται στον πρόσφατο κατάλογο των Makropoulos et al. (2012). Τα αποτελέσματα υποδεικνύουν ότι το μοντέλο αυτό περιγράφει πολύ καλά την κατανομή συχνότητας – μεγέθους για όλο το εύρος τιμών. Επιπλέον, η παράμετρος q του μοντέλου αυτού, καθώς και η παράμετρος b της σχέσης των Gutenberg-Richter, υπολογίζονται σε διαφορετικά χρονικά διαστήματα στη χρονική διάρκεια του καταλόγου και σχολιάζονται οι μεταβολές που εμφανίζουν στο χρόνο.

Λέξεις κλειδιά: *Κατανομή συχνότητας – μεγέθους, μη-εκτατική στατιστική φυσική, σχέση των Gutenberg-Richter.*

1. Introduction

Earthquakesaregenerallyoccurringduetothedeformationandsuddenruptureofpartsoftheearth'sbrittlec rust due to the relative motion of the tectonic plates releasing energy. The energy distribution of earthquakes has a fractal power-law distribution (Turcotte, 1997; Rundle et al., 2003) that in terms of the cumulative magnitude distribution can be expressed through the Gutenberg – Richter (G-R) relation (Gutenberg and Richter, 1944) as:

Equation 1 – Power-law form of the Gutenberg-Richter relation

$$N(>M) \square\ 10^{-bM} ,$$

where $N(>M)$ is the number of earthquakes greater than a threshold magnitude M and b is the slope that describes the size distribution of the earthquake events. This relation is empirical and has not been associated with general physical principles, except a recent attempt (Varotsos et al., 2004), where the G-R relation and the stability of the b-value in the range $0.8 \leq b \leq 1.2$ (Rundle et al., 2003) is simply explained by the maximum entropy principle, if the analysis is performed in the natural time domain (Varotsos et al., 2001).

In 2004, Sotolongo-Costa and Posadas starting from first principles developed a general physical model for the earthquake generation mechanism that contains the G-R relation as a particular case. In this model, the local breakage and the displacement of the asperities and the fragments between the fault planes are the cause of the earthquake energy release. Then, the released energy can be considered proportional to the volume of the fragments and the energy distribution function can be obtained in terms of the fragment size distribution (Sotolongo-Costa and Posadas, 2004).Sotolongo-Costa and Posadas have considered that interactions between the fragments exist and derived the model in the frame of non-extensive statistical physics (NESP). NESP has been proposed by Tsallis (1988) as a possible generalization of Boltzmann-Gibbs (BG) statistical physics and provides a consistent theoretical framework for the analysis of complex dynamical systems that exhibit fractal structures and long-range correlations (Tsallis, 2009). The NESP concept has been successfully applied to various fields of geophysics (see Vallianatos and Telesca, 2012 and references therein) including earthquakes (e.g. Vallianatos et al., 2012; 2013; Vallianatos and Sammonds, 2013).

The non-extensive model for the earthquake energy distribution, as was later revised by Silva et al. (2006), has been successfully applied to regional seismicity (Silva et al., 2006; Telesca, 2010a; 2010b) covering diverse tectonic regions and volcano related seismicity (Vallianatos et al., 2013). The question of whether this model can also describe the earthquake activity at the Hellenic region is addressed in this work. We use this model, along with the G-R relation, to study the earthquake activity during the period of 1976-2009, as it is referred in the recent earthquake catalogue for the area of Greece by Makropoulos et al. (2012). We perform the analysis for the entire time period as well as in different time intervals, in order to recognize patterns that are related to the evolution of the earthquake activity and the results are being discussed.

2. Dataset

In this work we use a recent catalogue for the area of Greece, compiled by Makropoulos et al. (2012) that expands from 1901 to 2009. In our analysis we consider the moment magnitudes (M_w) of the shallow seismicity (depth $\leq$ 40 km) that occurred in the period 1976 – 2009, as for this period the catalogue can be considered complete for magnitudes (M_w) greater than 4.1 (Makropoulos et al., 2012). We decluster the catalogue in order to remove the aftershocks from the dataset and to perform the analysis directly to the main earthquake events. For this purpose, we use the window method by Gardner and Knopoff (1974), as it was later on modified by Uhrhammer (1986). After the declustering procedure, a dataset of 2153 earthquakes emerges that is used further on in the analysis.

3. Non-extensive Model for Earthquake Energies

The non-extensive model for the earthquake energies distribution is derived in the frame of NESP. This concept refers to the non-additive Tsallis entropy S_q(Tsallis, 1988) that incorporates the parameter q, which is a measure of the non-extensivity of the system. In the limit of $q\rightarrow1$, S_q reduces to the ordinary BG entropy S_{BG}. The main difference among them is that S_{BG} is additive while S_q is non-additive. According to this property S_{BG} includes only short-range correlations, while S_q allows all length-scale correlations between the elements of the system (Tsallis, 2009). In terms of the probability $p(\sigma)$ of finding a fragment of surface σ, S_q is expressed as:

Equation 2 – Integral formulation of Tsallis entropy

$$S_q = k_B \frac{1 - \int p^q (\sigma) d\sigma}{q - 1} \, ,$$

where k_B is Boltzmann's constant and q is the non-extensive index. For the sake of simplicity we set k_B=1. To find the probability $p(\sigma)$ the maximum entropy principle is applied, under the constraint of the normalization of $p(\sigma)$:
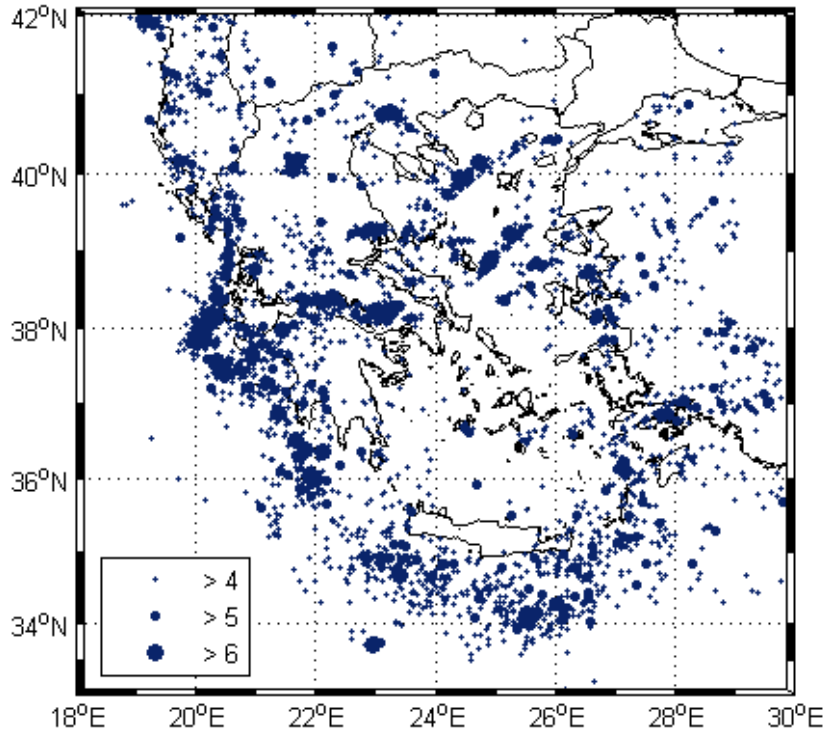

Figure 1 – Spatial distribution of the shallow earthquake activity (depth ≤ 40 km) in the area of Greece and the adjacent areas during 1976 – 2009.

Equation 3 – Normalization condition

$$\int\limits_{0}^{\infty} p(\sigma)d\sigma = 1$$

and the condition about the q-expectation value (Tsallis, 2009):

Equation 4 – Definition of q-expectation value

$$\sigma_q = \left\langle \sigma \right\rangle_q = \frac{\int_0^\infty \sigma p^q(\sigma)d\sigma}{\int_0^\infty p^q(\sigma)d\sigma} \, .$$

After the maximization procedure, the following expression for the fragment size distribution function is derived (Silva et al., 2006):

Equation 5 – Probability distribution function for the size of the fragments

$$p(\sigma) = \left[1 - \frac{(1-q)}{(2-q)}(\sigma - \sigma_q) \right]^{1/(1-q)} \, .$$

Assuming that the energy release E is proportional to the volume of the fragments $E \sim r^3$ (Silva et al., 2006), in accordance to the standard definition of seismic moment scaling with rupture length (Lay and Wallace, 1995), this proportionality becomes:

Equation 6 – Relation between the size of the fragments and the earthquake energy

$$\sigma - \sigma_q = \left(\frac{E}{\alpha} \right)^{2/3} \, .$$

In the last equation, σ scales with r^2 and α is the proportionality constant between E and r^3. By using the latter deformation, the energy distribution function becomes:

Equation 7 – Probability distribution function for earthquake energies

$$p(E) = \frac{C_1 E^{-\frac{1}{3}}}{\left[1 + C_2 E^{\frac{2}{3}} \right]^{1/(q-1)}} \, ,$$

with $C_1 = \dfrac{2}{3\alpha^{\frac{2}{3}}}$ and $C_2 = -\dfrac{(1-q)}{(2-q)a^{\frac{2}{3}}} \, .$

In the last equation, the probability of the energy is $p(E)=n(E)/N$, where $n(E)$ is the number of earthquakes with energy E and N is the total number of earthquakes. The cumulative number of earthquakes with energy E can be now estimated by integrating Eq. (7):

Equation 8 –Integration of the probability density function

$$\frac{N(E > E_{th})}{N} = \int_{E_{th}}^\infty p(E)dE \, ,$$

where $N(E>E_{th})$ is the number of earthquakes with energy E greater than the threshold energy E_{th} and N the total number of earthquakes. The cumulative distribution in terms of the earthquake

magnitude M can now be obtained, if we consider that the magnitude M is related to the energy E as $M = \frac{2}{3}\log(E)$ (Kanamori, 1978). Thus:

Equation 9 – Normalized cumulative distribution of earthquake magnitudes

$$\frac{N(>M)}{N} = \left[1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^M}{\alpha^{2/3}}\right)\right]^{\frac{2-q}{1-q}}.$$

The last expression describes from the first principles and in NESP formalism, the cumulative distribution of the number of earthquakes N greater than the threshold magnitude M in a seismic region, normalized by the total number of earthquakes. Above a certain threshold, the G-R relation can be deduced as a particular case with $b = (2-q)/(q-1)$ (Telesca, 2012). Taking in account the minimum magnitude M_0 of an earthquake catalogue, Eq. (9) should be slightly changed to (Telesca, 2012):

Equation 10 – Normalized cumulative distribution of earthquake magnitudes taking in account the minimum magnitude M_0

$$\frac{N(>M)}{N} = \left[\frac{1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^M}{\alpha^{2/3}}\right)}{1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{M_0}}{\alpha^{2/3}}\right)}\right]^{\frac{2-q}{1-q}}.$$

4. Data analysis

The non-extensive model that has been described in the previous section is now applied to the normalized cumulative magnitude distribution for our dataset. We estimate the values of q and α by fitting the observed distribution to the non-extensive model of Eq.(10) by applying a non-linear least squares algorithm. The results of this analysis are presented in Figure 2. The non-extensive model describes quite well the data for the values of q=1.46 ±0.018 and α=3.25·10^5±1.7·10^5. For comparison, a power-law fit that corresponds to the G-R relation (Eq. (1)) is also plotted in Figure 2 for the value of b=1.076 ±0.027. The b-value is estimated according to the maximum likelihood technique (Aki, 1965), as was later revised by Utsu (1978).The mean square error estimation for the non-extensive model and the power-law fit indicates that the former describes better the observed magnitude distribution, as it has been also observed in previous studies (e.g. Vallianatos et al., 2013).

An interesting feature is whether the q and b parameters vary with time and how these variations are related to the evolution of the earthquake activity. This kind of analysis can provide useful insights into the physical mechanism of seismogenesis. We perform such an analysis by estimating q and b in different time intervals along the time spanning of the earthquake catalogue. We define these intervals by a sliding window, which is characterized by the length l and the sliding factor w. These factors should be defined in a way that optimal estimations, concerning the statistical significance of the results and the resolution, can be obtained. In our case we set l=200 and w=20, illustrating that the estimation of q and b is performed for time intervals that contain 200 events and slide every 20 events, resulting on 90% overlapping between the successive windows. Though the selection we made may produce large deviations, it is preferred here in order to gain better
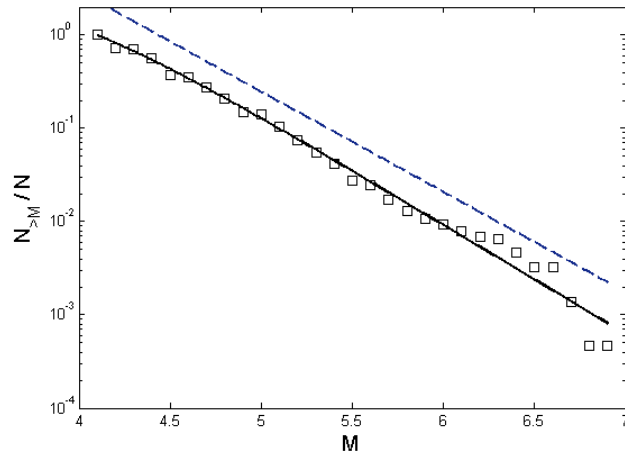
Figure 2 – Normalized cumulative magnitude distribution (squares) and the model of Eq. (10) (solid line) for the values of q=1.46 ±0.018 and α=3.25·10⁵±1.7·10⁵. The dashed line represents the G-R relation for b=1.076 ±0.027 (transposed vertically).

resolution in the variations with time. The estimated values are then associated to the time of the last event in each window.

The results of this analysis are presented in Figure 3a and Figure 3b for b and q respectively, along with their standard deviations. Both parameters exhibit variations during the different time periods. The b-value varies between 0.9 and 1.37, while q between 1.26 and 1.54. In Figure 3c the cumulative earthquake energy in each time interval, according to $E \sim 10^{1.5M}$ (Kanamori, 1978), is also presented. The time and the magnitude of the earthquakes with $M_w \geq 6$ are also presented in Figure 3c.The b-value seems to increase during more quiescent periods and reduce to a value close to 1 during periods where higher magnitude earthquakes occur, though this is not evident for the period 1982-1984 where b suddenly increases and the released earthquake energy obtains higher values as well. In the other hand, there is a clear correlation between the q-value variations and the cumulative energy in each time interval. Such a result can be interpreted in terms of the physical meaning of q that measures the degree of non-extensivity. For q approaching 1, equilibrium and the transition to Boltzmann-Gibbs statistical physics is obtained. In the other hand, as q increases, the system is getting away from equilibrium and larger earthquakes occur. Thus, q may be considered as a characteristic parameter for the seismic history of a particular area. A closer inspection of Figure 3b and Figure 3c also implies a periodicity in the q-values and the released earthquake energy during 1980 – 2009, where 5 – 7 years periods of increased activity and higher q s are followed by more quiescent periods that last 7 – 10 years.

5. Conclusions

In the present work, the earthquake activity at the Hellenic region during 1976 – 2009 is studied by means of non-extensive statistical physics. We use a physical model, derived in a NESP formalism, to investigate the frequency - magnitude distribution of the shallow earthquake activity that occurred during 1976 – 2009. In terms of this model, the cumulative magnitude distribution can be well reproduced for the values of q=1.46 ±0.018 and α=3.25·10⁵±1.7·10⁵. For the sameperiod, the b-value of the G-R relation is b=1.076 ±0.027. By performing the analysis in different time intervals along the evolution of the earthquake activity, variations in the values of b and q emerge that for the latter correlate well with the relative cumulative earthquake energy in each time interval. Thus, the non-extensive analysis and particularly the q-value can provide valuableinformation on the state of the seismogenic process in a particular area and should be considered in future seismological studies.
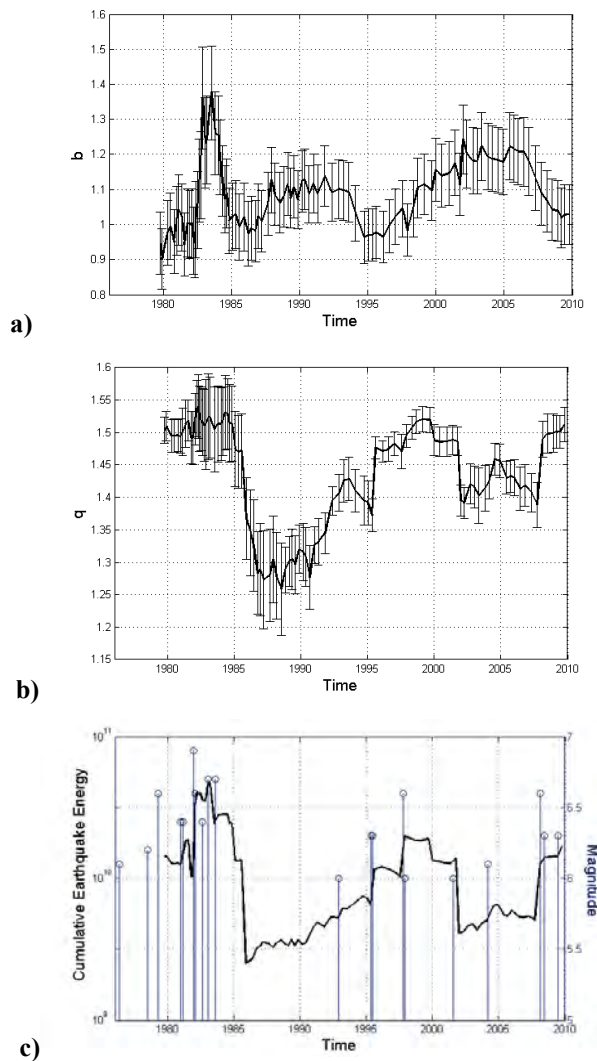
Figure 3 – a) Variations of the _b_-value with time, estimated in successive time intervals with 90% overlapping and the associated standard deviations, plotted as error bars, b)_q_-value variations with time, estimated as previously and theassociated standard deviations, plotted as error bars, c) cumulative earthquake energy in each time interval (solid line) and the magnitude of earthquakes with _M_ ≥ 6 with time.

6. Acknowledgements

7. References

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