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MOMENT TENSOR DETERMINATION USING A NEW WAVEFORM INVERSION TECHNIQUE

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Abstract

In this study a new waveform inversion methodology was developed to determine the source parameters of an earthquake. This technique is based on analyzing data recorded both at teleseismic and regional distances. To apply the inversion three different methods, which are the normal equations, the QR-decomposition and the singular value decomposition (SVD), were successfully tested, similar results were obtained and the SVD method was selected. The proposed inversion methodology was applied to large, as well as to earthquakes of moderate magnitude. Analysis of moderate events is crucial for seismogenic volumes, where an important number of such earthquakes occur which allow the calculation of their source parameters. Thus, the seismotectonic characteristics of the study area can be determined. The proposed methodology is successfully applied to events located in Greece and its surrounding regions in near real time.

Key words: *focal mechanism, moment tensor inversion, body waves.*

1. Introduction

Moment tensors represent the key information for seismotectonic studies. The moment tensor as a mathematical description of equivalent forces and moments is used to study the source processes. The propagation and the source effects characterize variations of the observed seismograms. Mathematically, each one of these effects can be calculated in order to generate synthetic seismograms that can be compared directly to the corresponding observed ones. The best solution is obtained by the minimization of the difference between the observed and the synthetic seismograms (Aki and Richards, 1980).

A seismic source is represented by a moment tensor M with six independent elements. However in most tectonic earthquakes, there is no change of volume at the source (Kikuchi and Kanamori, 1991) and thus we have five independent elements of the moment tensor. Then the synthetic seismogram can be calculated by the linear combination of the five terms which are elements of the matrix M and the corresponding Green's function. A powerful tool for the moment tensor calculation is the waveform inversion. In the past different methodologies were developed both in time (Das and Kostrov, 1990; Hartzell and Heaton, 1983; Kanamori, 1972; Kikuchi and Kanamori, 1982; Langston and Helmberger, 1975; Langston, 1976; Madariaga and Papadimitriou, 1985; Papadimitriou, 1988) and frequency domain (Brune, 1970; Cotton and Campillo, 1995). Langston and Barker (1981, 1982) developed a method using a generalized inversion technique to determine the source parameters of an earthquake.

In the present study we propose a new technique of waveform inversion to rapidly estimate both source time function and the seismic moment tensor of an earthquake. We assume a pure double – couple source mechanism to stabilize the solution when data from a small number of seismic stations are used. This method is based on the eigenvalues and eigenvectors analysis of the moment tensor in order to calculate its components, (Jost and Hermann, 1989). In the case where the sum of the eigenvalues vanishes the moment tensor has only deviatoric components. In general a seismic moment tensor can be decomposed into a dipole of forces, the best double couple (DC), which indicates the slip movement along the fault surface (Lay and Wallace, 1995) and in a compensated linear dipole (CLVD) which describes seismic sources with no volume changes.

An important number of moderate earthquakes were analysed in near real time and the results are published on the website of the Department of Geophysics of the University of Athens: www.geophysics.geol.uoa.gr. In the present study two earthquakes, one large recorded at teleseismic distances and one moderate at regional, are presented.

2. Methodology

2.1 Seismic source representation

Seismic sources can be represented by equivalent forces, producing displacements on the earth's surface, which are identical to those created by the physical process of the source. The displacement field $u_n(x, t)$ using the representation theorem for body – waves (Aki & Richards, 1980) can be calculated at a position x and time t in the following equation:

(1)

where $u_i(\xi, \tau)$ is the dislocation function, $G_{nk,1}$ is the partial derivative of Green's function and expresses the nth component of the displacement at position x and time t. Note that x is a vector that denotes the position of a station and ξ denotes the position vector of a point source on the rupture surface. Σ is the fault plane surface, η_j is the jth component of the n which is the vector normal to Σ. Furthermore, c_{ijkl} is the elastic constant tensor of Hooke law while $u(\xi, \tau)\cdot c_{ijkl}$ denotes the moment density tensor m_{kl} which can be define as:

(2)

The representation theorem for displacement at x due to general discontinuity $u(\xi,\tau)$ across Σ is

(3)

Thus, for an effective point source

(4)

where the moment tensor components are

(5)

The dimension of the symmetric matrix M is 3x3 and depends on the type of faulting. The three diagonal elements represent vector dipoles, while the six off-diagonal elements represent force couples. Considering no volume change, the trace of the matrix is equal to zero. Otherwise there an isotropic part exists, where positive values indicate explosion and negatives implosion. In the case where the determinant is equal to zero, the deviatoric moment tensor represents a pure double couple (DC). In general, the moment tensor can be decomposed in an isotropic part, in a pure double couple and a compensated linear vector dipole (CLVD).

The last equation indicates that the displacement in a position x and time t is defined as a convolution between the moment tensor and the partial derivative of Green's tensor. Furthermore, a synthetic seismogram, in teleseismic distances, can be calculated at a specific geographical position using the following equation:

(6)

where $g(\Delta, h)$ is the geometric spreading, r_0 is the radius of the earth, ϱ is the density at the source, R denotes the radiation pattern, c denotes the v_P (P waves) or v_S (S waves) velocity and is the moment rate.

The discrete wavenumber method of Bouchon (1981, 2003) to compute the Green's function was used to calculate the source parameters for regional waveforms. In this study the method by Zahradnik et al. (2005) was modified and then adapted for the analysis using regional – local data.

The moment tensor is represented by a 3x3 symmetric matrix. Using a set of data which consists of synthetics and the corresponding observed recordings, it is possible to construct nx6 matrix and, by applying the Generalised Inverse Method, to calculate the source parameters of the earthquake.

2.2 Inverse Problem

Following, the data, denoted by a vector D, can be represented by a linear combination of a non square matrix G representing elementary Green's functions and a vector M representing the model parameters:

$$D = G \cdot M \tag{7}$$

The dimension of the matrix G is nxm where n is the number of observations and m the number of fundamental Green functions. The inverse problem consists of inverting the non symmetric matrix G in order to determine the parameters of the model. Several methods exist to calculate the inverse of the matrix G like the normal equations, QR-decomposition, and the singular value decomposition (SVD). All three methods are used and the obtained results were similar. In the present study the singular value decomposition method is chosen and can be used either in overdetermined or underdetermined systems. First, two symmetrical matrices the $U= GG^T$ and the $V= G^TG$ that have the same eigenvalues are defined. Following, the diagonal matrix Λ is defined by calculating the positive square root of the non zero eigenvalues, named singular values (Lanczos, 1950). Thus, the model parameter M can be calculated by the relation:

$$M = G^{-g} \cdot D \tag{8}$$

where is the generalised inverse matrix. The matrix G is decomposed into:

$$G = U \cdot \Lambda \cdot V^T \tag{9}$$

The obtained eigenvectors are parallel to the principal stress axes and the norm of the matrix is equal to the seismic moment which is symmetric and has 6 independent elements. In case of an earthquake both the trace and the determinant of the matrix must be equal to zero. Considering the two constraints the obtained moment tensor consists of 5 independent elements.

In summary, by creating five elementary Green functions it is possible to calculate a synthetic seismogram that can directly be compared to the observed one. The model parameters are estimated by minimizing the difference between observed and synthetics.

3. Source parameters determination

For large earthquakes synthetic seismograms are calculated at teleseismic distances between 30°<Δ<90° while for regional – local events for distances less than Δ<6°. Green's functions are generated and combined in order to produce the P, SV, SH movements. Broad band data are tested and each component of good quality is selected and used for the inversion. For large events waveforms at teleseismic distances (epicentral distances between 30° and 90°) were selected from stations belonging to the Global Seismograph Network (GSN). For moderate events regional data from the Hellenic Unified Seismological Network (HUSN) were used. We applied deconvolution of instrument response from all the waveforms and integration of the data to produce displacement waveforms. For the teleseimsic events the synthetic seismograms were generated for a point source in a half space. The attenuation effect is taken into account using Futterman's (1962) operator with t^*=1 sec for P – waves and t^*=4 sec for S – waves. The regional – local data were generally band passed between 0.02 and 0.1 Hz and then converted into displacement with a sampling rate of 0.2 sec.

The proposed methodology was successfully applied to the largest earthquakes that happened recently in Greece as well as to a large number of moderate events. Following, two examples will be presented. The first application concerns the most recent large event which is the 2009 Crete earthquake. The second example is a moderate event that occurred in the area of the Corinth Gulf.

3.1 The M_w=6.2 2009 Crete earthquake

On 1 July 2009 (09:30 GMT) an earthquake of magnitude M_w=6.2 occurred south of Crete island. Using waveforms from HUSN the epicenter was manually located at 33.92°N, 25.47°E. Broadband teleseismic recordings from GSN network were collected and those at epicentral distances between 30° and 90° degrees were selected. For the inversion method, 11 teleseismic stations were used with satisfactory azimuthal coverage for the generation of P waves, 3 for SH and 2 for SV waves. The moment tensor components and the focal depth of the mainshock were determined using a single trapezoidal source time function with a duration of 5 sec. Reverse type faulting was revealed after applying body wave inversion. The obtained focal mechanism is φ=101°, δ=62°and λ=87°. The seismic moment is equal to $3.2 \cdot 10^{25}$ dyn·cm, for a focal depth equal to 22 km. The inversion resulted a DC equal to 90%, while the compensated linear vector dipole was equal to 10%.

It is worth noticing that after several applications the percentage of these quantities depends on the quality of the data. For this reason a second methodology was developed and applied to better estimate the final solution, as well as the variance of the parameters, based on grid search.

The second methodology was developed mainly for moderate events recorded at regional distances in cases of low DC percentage, noisy waveforms etc. This technique is based on the use of an iter-

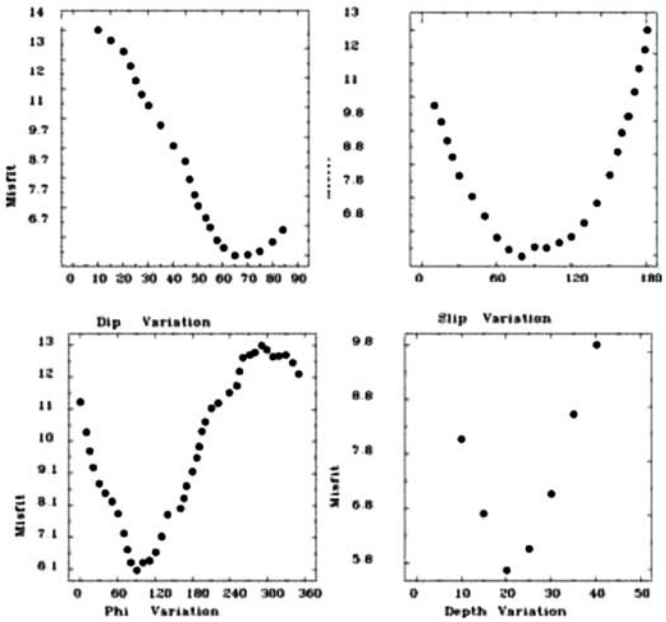

Fig. 1: Misfit of the source parameters of the 2009/07/01 Crete Earthquake.

ation procedure. As initial parameters the solution resulted by the inversion procedure is used. The source parameters determined using this technique is more accurate, while the range of each parameter can be individually estimated.

Figure 1 presents the results of this technique. In the first stage the dip, the rake and the depth were fixed and the azimuth varied, as described in the low left subfigure. Initially the azimuth varies between 0º and 359º with a step of 10º. A clear minimum was observed around 90º. Following, the same procedure is applied for the range 90º±20º with a step of 1º in order to determine a more accurate final minimum value which was found equal to 92º±5 º. Using the same procedure the dip was determined. Initially, in the upper left subfigure, the dip varies between 10º and 80º with a step equal to 5º and indicates a minimum around of 70˚. Following, for the range 60º - 80º (step 1º) the dip was 67º±3˚. In the upper right subfigure the variation of rake indicates a minimum around 80º, while after the second iteration 80º±5º. Finally, following the same procedure (low right subfigure) the depth was initially found around 20 km and finally 22±3 km.

Comparing the results of both methodologies, the obtained values for all parameters are similar. Nevertheless, the results obtained by the second method are preferred, since the misfit between the recorded waveforms and the synthetics is more satisfactory. Furthermore, a better estimation of the errors of the source parameters is achieved.

The results of the applied modeling are presented in Fig. 2. For this event the selected stations presented a good azimuthal coverage. The complexity presented in the observed waveforms is probably due to the seismic source. In this study a simple trapezoidal source time function is applied therefore no high frequencies are observed in the synthetics. However the obtained source parameters were precisely determined.

The main shock was followed by the occurrence of numerous aftershocks. The source parameters of the five largest moderate aftershocks (M>4) were computed. Their focal mechanisms revealed reverse type of faulting, as the main shock, while their depths vary between 15 and 25 km.

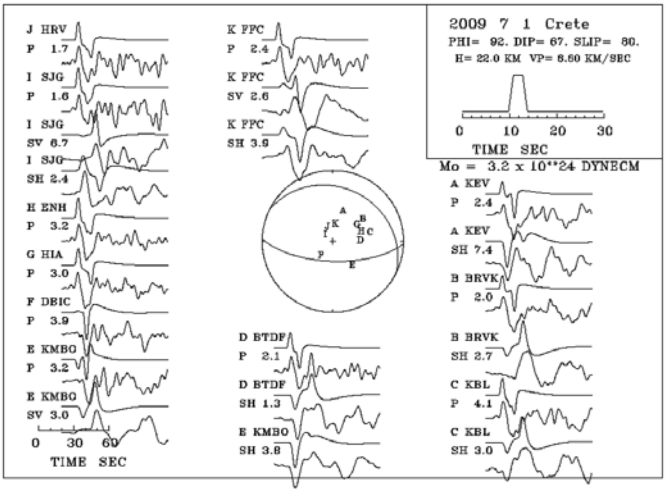

Fig. 2: Body wave modelling of the 2009/07/01 Crete earthquake.

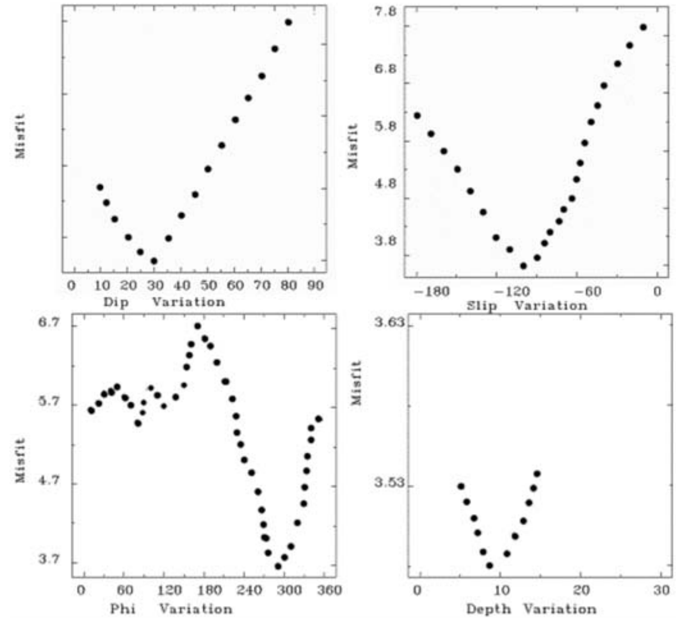

Fig. 3: Misfit of the source parameters of the 2009/06/07 Aigion earthquake.

3.2 The M_w=4.2 earthquake in the Corinth Gulf (07/06/2009)

On 7 June 2009 (08:57 GMT) an earthquake of magnitude M_w =4.2 occurred in the Corinth Gulf, near to Aigio town. The event was manually located at 38.14° N, 22.68° E, using recordings of the HUS Network. To determine the source parameters of this moderate event 7 station in local and regional distances were used.

The source parameters were first calculated applying the moment tensor inversion method. The best fit solution is: ϕ=265°, δ=25°, λ= -78° and seismic moment $1.1 \cdot 10^{22}$ dyn · cm for a depth of 7 km. Furthermore, normal faulting was indicated. The calculated double couple was found equal to 68%, while the compensated linear vector dipole (CLVD) to 32%.

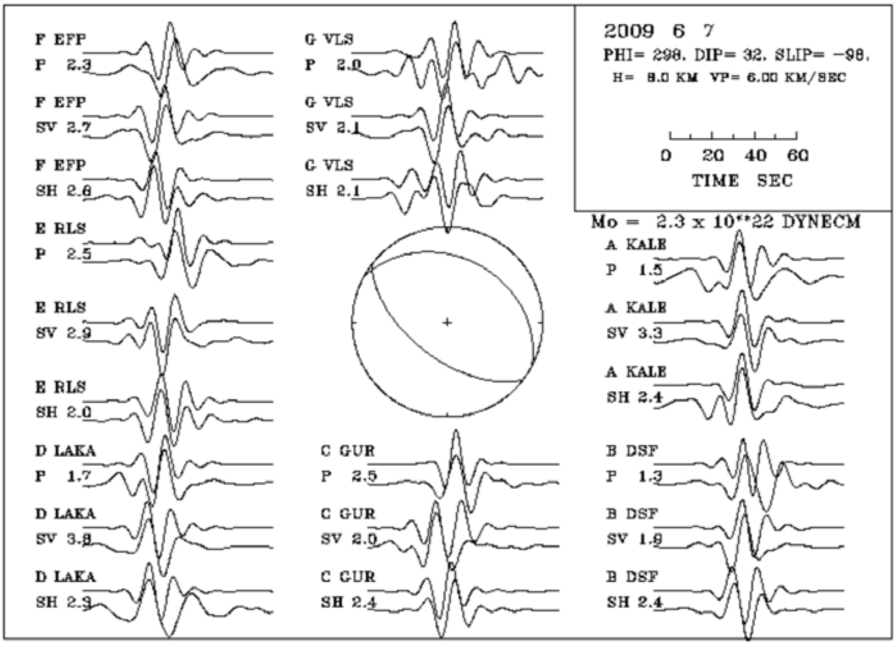

Fig. 4: Body wave modelling of the 2009/06/07 earthquake in the Corinth Gulf

The inversion procedure did not provide a high double couple component. For this reason, the second method is applied, using as starting solution the one obtained by the moment tensor inversion and the results are present in Fig. 3. In the first stage the dip, the rake and the depth were fixed and the azimuth was left to vary as described in the low left subfigure. The minimum was close to 300° while the obtained solution was 298°±10°. The upper left subfigure indicates the dip variation with a minimum close to 30°, while the obtained final solution was 32°±5°. Following the same procedure the rake (upper right subfigure) was estimated close to -100° while the final solution was 98±5°. In the down left subfigure the variation of azimuth is described. Finally, the depth (down right subfigure) was initially found around 8 km, which was also the final solution.

The results of the applied procedure are presented in Fig. 4. For this event 7 three components stations of HUS Network were used. Waveforms with a good signal to noise ratio were selected in regional distances. As we can see the fit between the synthetics and the corresponding observed is in good agreement and as a result the calculated source parameters are well determined.

4. Discussion

In this study, source parameters were calculated for the strongest events that occurred during the period 1995 – 2009 using teleseismic body wave modelling. The obtained source parameters of those earthquakes present in Table 1.

In most cases, the inversion procedure revealed simple source time function, as in the cases of the Aigio, Kozani and Athens earthquakes. In addition, cases with complicated source time functions were also considered, as the 2003 Lefkada earthquake. Between 14 February 2008 and 20 February 2008 three strong (M_w>6.0) earthquakes occurred in the sea, close to the town of Methoni. For two

Table 1. Earthquakes source parameters determinate in the present study.

Date	Time	Event	Lon (E)	Lat (N)	M_w	Depth km	Strike o	Dip o	Slip o
1995-05-13	08:47:13	Kozani	40.15	23.65	6.5	15	260	34	-90
1995-06-15	00:15:23	Aigio	38.40	22.48	6.2	12	277	28	-30
1999-09-07	11:56:34	Athens	38.10	23.56	6.0	8	105	56	-82
2001-07-26	00:21:46	Skyros	39.03	24.27	6.5	14	160	65	7
2002-01-22	04:35:00	Karpathos	35.56	26.73	6.2	90	95	89	50
2003-08-14	05:14:54	Lefkada	38.82	20.60	6.3	15	80	170	15
2006-01-08	13:34:54	Kythira	36.23	23.39	6.6	65	205	45	55
2008-01-06	05:14:18	Leonidio	37.09	22.65	6.0	85	114	75	120
2008-02-14	10:09:22	Methoni	36.30	21.69	6.7	35	290	16	69
2008-02-14	12:08:54	Methoni	36.34	21.92	6.1	38	337	11	123
2008-02-20	18:27:05	Methoni	36.21	21.71	6.0	16	249	67	-9
2008-06-08	12:25:28	Andravida	37.95	21.53	6.4	23	218	89	176
2008-07-15	03:26:32	Rodos	35.92	27.84	6.3	56	280	82	-40
2009-07-01	09:30:09	Crete	33.92	25.47	6.2	20	92	67	80

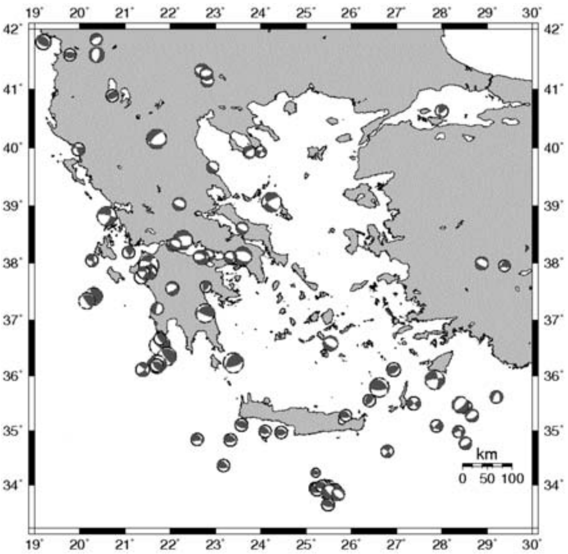

Fig. 5: Focal mechanisms solutions determined in this study between 1995 – 2009.

of these events complicated source time functions was also calculated. In total, source parameters for 14 large earthquakes were successfully determined in different seismotectonic settings.

The knowledge of the source parameters for moderate earthquakes is very important for seismically active regions, especially in the case where no large events occur. In general, it gives the opportunity for analytical studies, reveal the tectonics and the seismogenic characteristics of a specific region as in the case of the May 2009 seismic sequence that occurred at the lake Doirani (north of Thessaloniki). The area was activated for the first time during the last decades. The modelling of the strongest earthquakes, using regional data of this sequence were studied and revealed the tectonics of the region which is characterized by normal faulting. Thrust type faulting appears south of Zakynthos island, of the town of Methoni and close to the Kythira island, while in the eastern Aegean region different types of faulting appeared. Common events were compared with other studies (Agalos et al., 2007) and they were in very good agreement. The focal mechanisms solutions for the events determined in this study are appear in the Fig. 5.

5. Conclusions

In the present study, the results of a new developed method were presented. In the proposed methodology data both from teleseismic and from local-regional distances were used. This methodology is useful for regions were important moderate seismicity is observed. For the inversion P, SH and SV waves are used. Mathematically, the moment tensor inversion is an overdetermined problem and the general solution is separated in a DC and a CLVD part. In this case the components of the seismic moment tensor can be calculated using the generalized inverse technique by applying different methodologies. The most commonly used is the singular value decomposition method and this is the one also used in this study.

For teleseismic events this method gives, in general, satisfactory results. On the contrary, this is not always the case for moderate events due to poor quality of the large period component. For that reason a second methodology, based on a grid search was developed to overcome this difficulty. The proposed methodologies were successfully applied to a large number of moderate earthquakes that occurred in Greece the recent years. It is worth noticing that in some seismic crises like Doirani (north of Thessaloniki), Evia and Ionio islands, no large event recently occurred. The obtained results can contribute to the study of the seismotectonic characteristics of these regions.

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