LABORATORY INVESTIGATION ON THE CORRELATION BETWEEN THE FRICTION ANGLE OF ROCK JOINTS AND THE CONSTANT M$_I$ OF THE HOEK AND BROWN CRITERION

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Abstract

A correlation between the non-dilational friction angle (φm) of rock discontinuities and the constant mi of the Hoek and Brown criterion for intact rock is investigated, using the results of a focus oriented laboratory program. The program consisted of two types of laboratory tests: a series of triaxial compression tests on intact rock samples for the determination of the constant mi and an independent series of direct shear tests on tensile fractures of the same rock types for the determination of the rock joint friction angle φm. Four typical rock types from Northern Greece were used: a granite, a sandstone, a limestone and a marble, covering a range of mi between 8 and 34, and an unconfined compressive strength between 60 and 120 MPa. Apart from the certain range of parameters that is presented for this specific rocks, the experimental results show that the non-dilational friction angle of the rock fracture determined by direct shear testing (φm) decreases logarithmically with the value of the constant mi.
Keywords: *mi constant, direct shear tests, triaxial compressive strength, rock joints, laboratory testing, Hoek and Brown criterion.*

Περίληψη

Στην παρούσα εργασία διερευνάται η συσχέτιση μεταξύ της μη - διαστολικής γωνίας τριβής (φm) ασυνεχειών και της σταθεράς mi του κριτηρίου Hoek and Brown για τον άρρηκτο βράχο, με τη χρήση των αποτελεσμάτων ενός προσανατολισμένου προγράμματος εργαστηριακών δοκιμών. Το πρόγραμμα περιλαμβάνει δύο τύπους εργαστηριακών δοκιμών: μια σειρά από δοκιμές τριαξονικής θλίψης σε ακέραια δείγματα πετρωμάτων για τον προσδιορισμό του mi και μια ανεξάρτητη σειρά δοκιμών άμεσης διάτμησης ασυνεχειών σε επιφάνειες ασυνεχειών, που προέκυψαν από εφελκυστική αστοχία από τα ίδια πετρώματα για τον προσδιορισμό του φm. Χρησιμοποιήθηκαν τέσσερις αντιπροσωπευτικοί τύποι πετρωμάτων από τη Βόρεια Ελλάδα: ένας γρανίτης, ένας ψαμμίτης, ένας ασβεστόλιθος και ένα μάρμαρο, καλύπτοντας ένα εύρος τιμών mi μεταξύ 8 και 34, και αντοχής σε ανεμπόδιστη θλίψη μεταξύ 60 και 120 MPa. Εκτός από το εύρος των παραμέτρων που παρουσιάζεται για τα 4 αυτά συγκεκριμένα πετρώματα, τα πειραματικά αποτελέσματα δείχνουν ότι η χωρίς διαστολή γωνία τριβής της ασυνέχειας βράχου που προσδιορίζεται από τη δοκιμή άμεσης διάτμησης (φm) μειώνεται λογαριθμικά με την τιμή της σταθεράς mi.

1. Introduction

The Hoek-Brown (H-B) failure criterion is used extensively in practice to describe the mechanical behavior of an intact rock and a rock mass (Hoek and Brown, 1980, 1970) (Hoek *et al*., 1992, 2002). In this criterion, the m_i constant is an important factor which defines the slope of the initial portion of σ_1-σ_3 curve. This parameter is very approximately analogous to the angle of friction, φ, of the conventional Mohr-Coulomb failure criterion (Hoek, 1983). The estimation of m_i is based on triaxial tests under confining pressures in the range 0-0.5σ_c, where σ_{ci} is the unconfined compressive strength. In the absence of such tests, its value is estimated from a table of values provided by Hoek-Brown depending on rock type (Hoek and Marinos, 2000). From this table it can be concluded that the value of m_i is higher for silicate rocks (granites and sandstones) than for carbonate ones (limestones and marbles).

On the other hand, there is adequate evidence that the friction angle (φ_m) of flat rock surfaces resulted from sand-blasted, rough-sawn and residual surfaces is higher for carbonate rocks than for silicate rocks (for example see Barton, 1976). Based on this remark, an investigation of the possible correlation between the friction angle of rock joints and the m_i constant is hereby presented.

The determination of the parameter mi Hoek (1983) requires a data acquisition process from triaxial tests in the range 0≤σ3≤0.5σci, as well as, data from tensile tests to anchor the envelope. The correct adjustment of the envelope and the determination of mi constant requires at least five equally spaced triaxial data points covering a confining pressure range from 0 to 0.5σci. The nonlinear failure criterion by Hoek *et al*. (2002) in the generalized format is given by equation 1.

Equation 1: Hoek and Brown criterion for intact rock

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_i \frac{\sigma_3}{\sigma_{c_i}} + 1 \right)^{\alpha} \tag{1}$$

where

σ1   is the major principal stress

σ3   is the minor principal stress,

σci   is the uniaxial compressive strength of the intact rock material

mi   is constant which depends on the texture and internal structure of the rock

a   is a constant which controls the curvature of the envelope and depends on the properties o f the rock. This constant is often assumed to be equal to 0.5

The determination of the friction angle of the rock wall material (φ_m) was based on the Papaliangas' criterion (Papaliangas, 1995). For each shear test the measured peak shear strength is analyzed in two components of shear strength: a) The dilational (geometrical) component, which arises from overriding of asperities at an angle determined by the slope of the asperities. b) The non-dilational component, which arises from the shearing resistance rock contacts. The peak shear strength criterion used for the analysis of the experimental results is given by the following expression:

Equation 2: Peak shear strength of rock joints

$$\tau_p = \sigma_n \tan(\varphi_m + \psi) \tag{2}$$

Equation 3: Dilation angle of rock joints

$$tan\,\psi = tan\,\psi_o\,log_{10}\frac{\sigma_{nT}}{\sigma_n}\,/\,log_{10}\frac{\sigma_{nT}}{\sigma_{no}} \qquad (3)$$

Where:

τ_p is the peak shear stress

σ_n is the normal stress

φ_p the peak friction angle of the rock joint

φ_m the friction angle of the rock wall material under high normal stress and

ψ the instantaneous dilation angle corresponding to the peak shear strength.

ψ_0 the maximum dilation angle which is approximately equal to maximum asperity slope angle

σ_{n0} is the normal stress under no-damage normal stress

σ_{nT} is the normal stress where the dilation is fully suppressed

The friction angle φm is different from the "basic friction angle" (Barton, 1971), and its relevance to the field shear strength of rock surfaces has been demonstrated elsewhere (Papaliangas et al., 1996, 1997). The non-dilational component of shear strength is for an effectively planar yet naturally textured surface and, for design, it can be used with a low shear strength factor of safety as a lower bound (Hencer, 1995).

2. Laboratory test results

The testing program consisted of a series of laboratory direct shear tests on artificially generated joints and a series of triaxial compression tests of intact core rock specimen in marble, sandstone, limestone and granite. Each rock type was from a different site of Northern Greece. The Kavala marble and Demati (Ioannina) sandstone were supplied by a local stone supplier, limestone was sampled from the quarry of Lafarge Beton at Messaio village, Kilkis and granite was sampled from the area of Arnaia, Chalkidiki. It is important to mention that, for each rock type, the specimens used for the direct shear tests and the triaxial compression tests were prepared from the same block. All laboratory tests were conducted at the Laboratory of Geomechanics of the Department of Civil Engineering TE of the Alexander Technological Educational Institute of Thessaloniki.
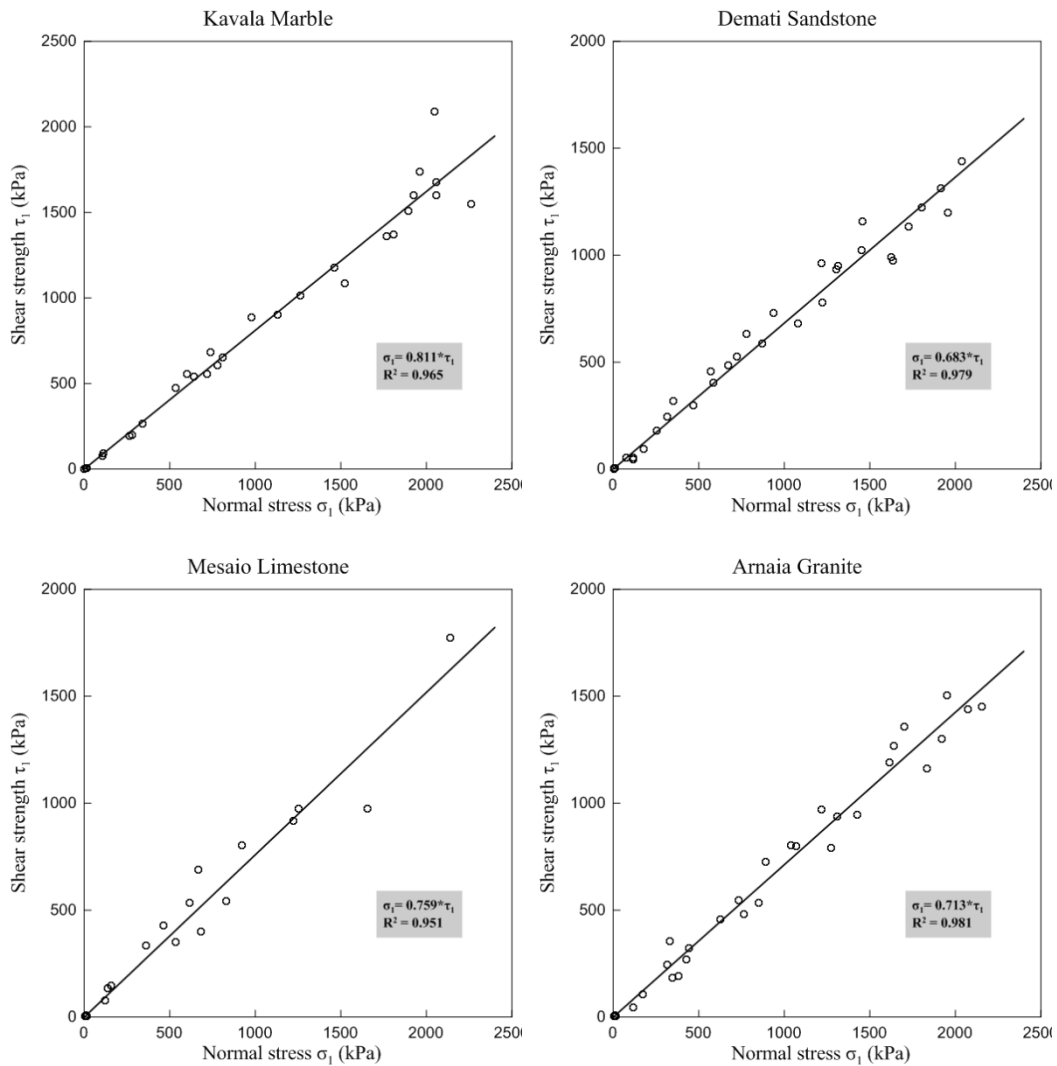
2.1 Direct shear tests

Four sets of samples were used. One set of each rock type (marble, sandstone, limestone, granite) comprises 6 samples of artificial surfaces generated by fracturing of larger samples subjected to bending tests till failure. The resulted surfaces represent the highest degree of surface roughness since they resulted from tensile failure. Each sample was subjected to multistage direct shear testing (6 different normal stresses, including one under their self weight which represents a normal stress of approximately 5 kPa). At the end of each stage, the sample was unloaded, the surfaces were examined and photographed and the loose debris was removed. The sample was then reset at its original position prior to changing the normal load for the next stage. The resulting total number of direct shear tests was 138.

All samples having a length 8-12 cm, were first tested under their self weight (negligible surface damage), so that the maximum dilation angle ψo could be determined. This is considered to represent the maximum asperity angle of the sample. Following this "zero normal load" test, the samples were subjected to shearing under higher normal stresses up to a maximum of 2.0 MPa for the joints. All tests

were performed using a purpose-built direct shear box, capable of accommodating samples as long as 1.20 m and applying a maximum shear load of 300 kN and a maximum normal load of 250 kN.

For each data pair of shear (τ) and normal (σ) stress the non-dilational shear and normal stress ($\tau 1$ and $\sigma 1$) was determined using the instantaneous dilation angle (ψ). These stresses represent shearing along a plane surface with natural texture, i.e. shearing without dilation (in other words shearing under constant volume conditions). This method is presented by Hencher and Richards (1989) and is based on the assumption that shearing of a dilating joint is equivalent to a shear movement along an inclined plane, having a slope angle equal to the instantaneous dilation angle. Analysis of stresses along the inclined plane ($\tau 1$) and its perpendicular direction ($\sigma 1$) results in the following relations:

Figure 1 - Results of direct shear tests on rock joints showing the non-dilational friction line for the investigated rocks.

Equation 4: Shear stress acting on the inclined plane

$$\tau_1 = (\tau \cos \psi - \sigma \sin \psi) \cos \psi \qquad (4)$$

990

Equation 5: Normal stress acting on the inclined plane

$$\sigma_1 = (\sigma \cos\psi + \tau \sin\psi)\cos\psi \qquad (5)$$

The results presented as shear strength τ_1 vs. normal stress σ_1 diagrams for all the investigated rock types are presented in Figure 1.

1.2. Triaxial compression tests

The triaxial compression testing program included at least eight specimens per rock type. More specifically, ten triaxial and one uniaxial for Arnaia granite, ten triaxial and two uniaxial, for Kavala marble nine triaxial and two uniaxial for Demati sandstone and eleven triaxial and one uniaxial compressive test for Messaio limestone. The total number of triaxial compression tests was 42 including 4 uniaxial compression tests. The specimens were cylindrical, with a diameter of 54cm and a ratio diameter/height of 1:2. The confining pressure used for all rock types were in the range 0 to 70 MPa, however only the range between 0 and $0.5\sigma_{ci}$ was used to determine the value of mi. All tests were carried out according to E103 - 84(6) and ASTM D7012 - 14A.

The results presented as axial strength σ1 vs. confining pressure σ3 diagrams for all rock types are presented in Figure 2, with the Hoek-Brown fit curve and parameters σci and mi shown on each diagram. The values of mi were all within the ranges suggested by Marinos and Hoek (2000) except for the Mesaio limestone that was found to be marginally above the upper limit (15.9 vs. 15).

3. Correlation between friction angle and constant m_i

The values of the non-dilational friction angle and the constant m_i of the Hoek-Brown criterion for intact rock are given Table 1.

Table 1 - Friction angle of rock joints φ_m and constant m_i.

| Rock type	Kavala marble	Messaio limestone	Demati sandstone	Arnaia granite
φ_m	39.0°	37.2°	34.3°	35.5°
m_i	8.6	15.9	18.9	34.0

The correlation between the two parameters in graphical form is presented in Fig.3. A logarithmic curve described by Equation 6 has been found to fit the data satisfactorily.

**Equation 6: Relation φ_m -m_i**

$$\varphi_m = -2.79 \ln m_i + 44.44 \qquad (6)$$

The friction angle φ_m appears to be independent of the normal stress and decreases from about 39° for the carbonates Kavala marble and Mesaio limestone (m_i=8.9 and 15.9 respectively), to about 3 5° for the silicates Arnaia granite and Demati sandstone (m_i=34.0 and 18.9). The higher values of φ_m observed for the carbonates of this study as compared to those of silicates are in line with the va lues reported by Barton (1976) for the same rock types and are attributed to the difference in the fri ction angle of the corresponding rock-forming minerals. From the values of mineral friction report ed by Horn and Deere (1962), it is clear that the friction angle of calcite, the mineral constituent of marble and limestone, is clearly higher than that of quartz and feldspar, the main constituents of gr anite and sandstone.

The value of m_i can be determined if φ_m is known by Equation 7.
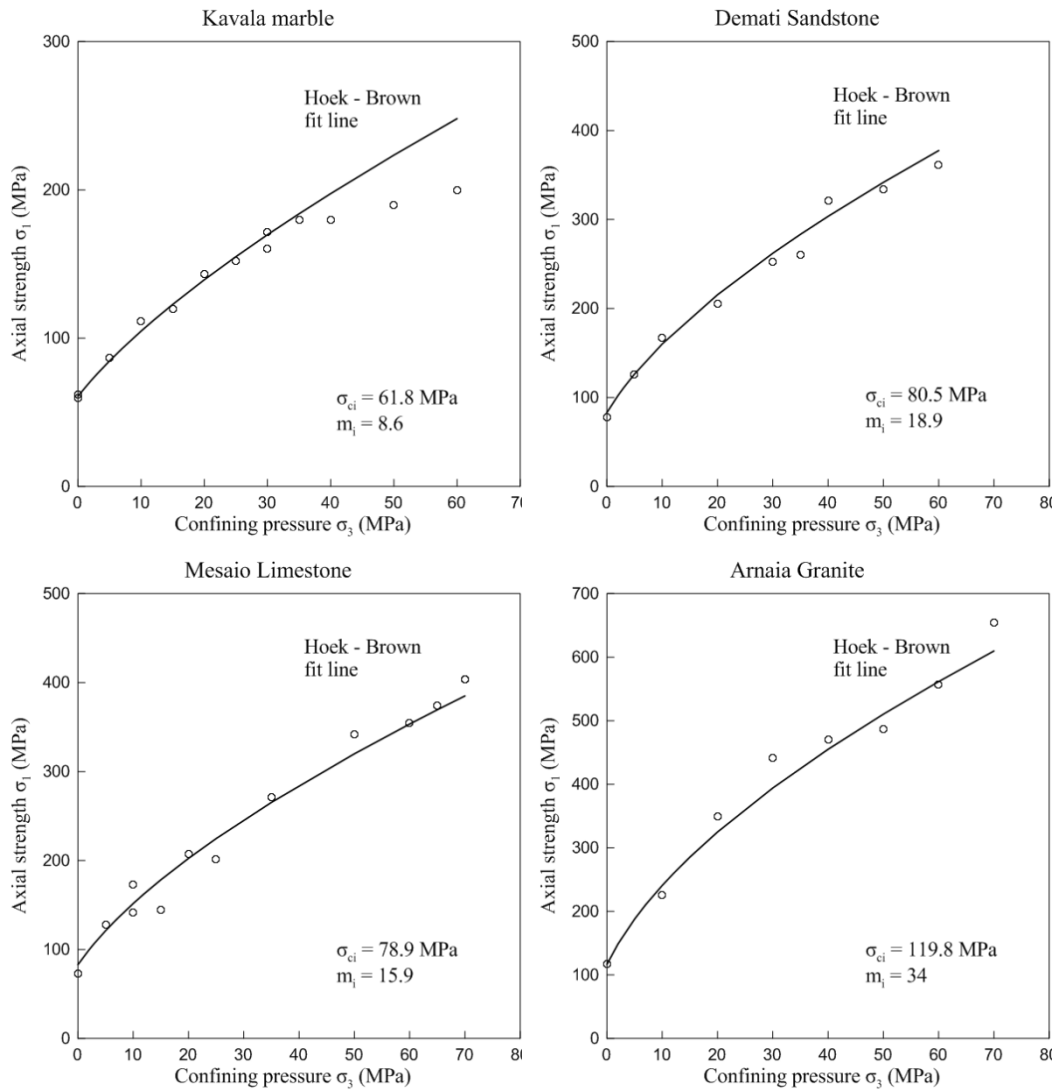
Figure 2 - Axial strength vs. confining pressure diagrams for the four investigated rocks.
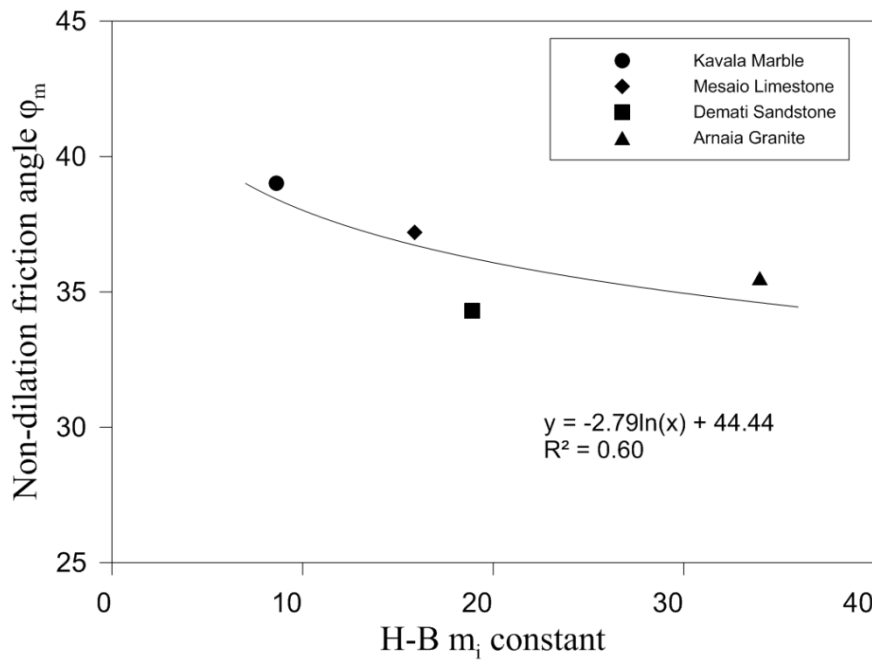
Figure 3 - Correlation between the friction angle of rock joints and the constant m_i of the Hoek-Brown criterion.

Equation 7: Relation m_i-φ_m

$$m_i = e^{\frac{44.44 - \varphi m}{2.79}} \tag{7}$$

4. Conclusions

The aim of this paper was to investigate experimentally the relation between the friction angle of rock surfaces determined by direct shear testing and the constant m_i of the Hoek and Brown criterion for intact rock. Four different rock types were used in two types of laboratory tests: a series of triaxial compression tests on intact rock samples for the determination of the constant m_i and an independent series of direct shear tests on tensile fractures of the same rock types for the determination of the rock joint friction angle φ_m.

The laboratory tests suggest that the constant m_i of the Hoek and Brown criterion increases logarithmically with decreasing friction angle of the rock wall material (φ_m). The relation can be used to estimate the constant m_i from the angle φ_m, which is determined from direct shear testing of discontinuities and vice-versa. However, the main utility of the method is the indirect determination of the constant m_i using a series of direct shear tests on discontinuities from the same rock, when suitable triaxial compression test results are not available. It should be noted that the correct determination of m_i constant requires at least five cylindrical specimens for the production of five equally spaced triaxial data points covering a confining pressure range from 0 to $0.5\sigma_{ci}$. On the other hand, for the determination of φ_m a multi-stage direct shear test on a single rock joint sample of any shape tested under three different normal stresses is acceptable (ISRM, 2014).

Another important contribution of this method is the estimation of m_i for weathered rocks. Direct shear test on weathered rock joints can be carried out without any experimental difficulty, whereas cylindrical specimens for triaxial testing require coring from weathered rock pieces, which is quite difficult. Therefore, the decrease in the value of m_i for a weathered intact rock sample can be estimated by this method.

The relation between the two parameters described earlier resulted from only four different rocks t ypes which covered the range of constant m_i between 8 and 34. Consequently, the equation gives o nly a trend and not an accurate estimate of the constant m_i from φ_m. To establish a more accurate e xpression, a number of additional different rocks types needs to be used. Moreover, for further deta iled investigation of the correlation, specimens subjected to triaxial compressive tests can be used a fterwards for direct shear testing.

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