

## **A fractal theory approach to the initial examination of normal faulting in Central Corinthian Gulf, Greece**

**KONDYLAKIS, J.C. and PAPOULIA, J.E.**

National Center for Marine Research, Institute of Oceanography  
Agios Kosmas, 16604 Athens, Greece  
e-mail: joseph@ncmr.gr

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### **Abstract**

*An application of fractal theory in geological formations in the central Corinthian Gulf, Greece, is presented in an attempt to study the nature of presently active deformation. Fault patterns are approximated under the perspective of fractal theory concept, leading to the conclusion that fractal approach can be considered valid for the region of study. Nevertheless, homogeneity may be expected with the reservation that there are no considerable changes in the viscosities of the ductile layers in the region, so that the characteristic exponent  $b+1-a$  is less than zero.*

**Keywords:** Corinthian Gulf, Greece, Fractals, Normal faults.

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### **Introduction**

A basic investigation of the study of active deformation concerns the preservation of undeformed regions, quite large sometimes, in the midst of otherwise penetrative deformation. To understand the term undeformed in the present case, a representative volume with a length scale LH (length of homogeneity) is introduced. At scales larger than LH the system behaves as if it was homogeneous with affective average rheological laws. As mentioned in DAVY *et al.* (1990), owing to the importance of faults on the deformation field, this approach is certainly valid if no faults of size larger than LH cut the representative volume element in two pieces.

Previous studies considering geological formations under the concept of fractal theory

(VILOTTE *et al.*, 1984; DAVY *et al.*, 1990) lead to the argument that the existence of large underformed regions might arise naturally from a fractal style of faulting.

The meaning of fractal structure was introduced in the study of Poincaré sections of a chaotic attractor in non-linear dynamics (NIKOLIS, 1995). Fractals and chaos are closely intertwined and often occur together. For instance, most chaotic attractors have a fractal texture. Points on such attractors plot as a set of layers that look the same over a wide range of scales. In general, the chaotic attractors of flows or invertible maps are typically fractals; the chaotic attractors of noninvertible maps may or may not be fractals. Other chaos-related geometric objects, such

as the boundary between periodic and chaotic motions in phase space, may also have fractal properties. Because of those close relationships, fractals can help detect chaos (WILLIAMS, 1997).

In the present study, the meaning of fractal structure will be restricted to a pattern repeating the same design and detail or definition over a broad range of scales. Any piece of a fractal appears the same as repeatedly magnified (self-similarity). The usefulness of a fractal approach to geological formation is due to the fact that since deterministic chaos-related geometric objects usually have fractal properties (MOON, 1992), fractals can help in detecting the existence of deterministic chaotic behaviour.

DAVY & COBBOLD (1988) and COBBOLD & DAVY (1988) have successfully explained and simulated the Indian - Asian collision as yielding a predicted distribution of underformed regions that can be easily tested in the field. Furthermore, DAVY *et al.* (1990) have shown that, if faulting is fractal in nature that will limit the validity of homogenization approaches in the modeling of deformation in which the rheology of the lithosphere is assumed to be homogeneous at scales larger than a threshold value.

A similar approach using the concepts of the fractal theory is applied here in the central Corinthian Gulf, Greece, in an attempt to examine the nature of the presently active deformation due to N-S extension. The specific area was chosen because of available accurate data from recent neotectonic surveys, and also its particular interest from the geodynamic point of view, due to the active deformation and high seismic activity (PAPAZACHOS & PAPAZACHOU, 1989).

## Materials and Methods

### *Geotectonic Background*

The Corinthian Gulf is one of the most seismically active areas in Greece and the

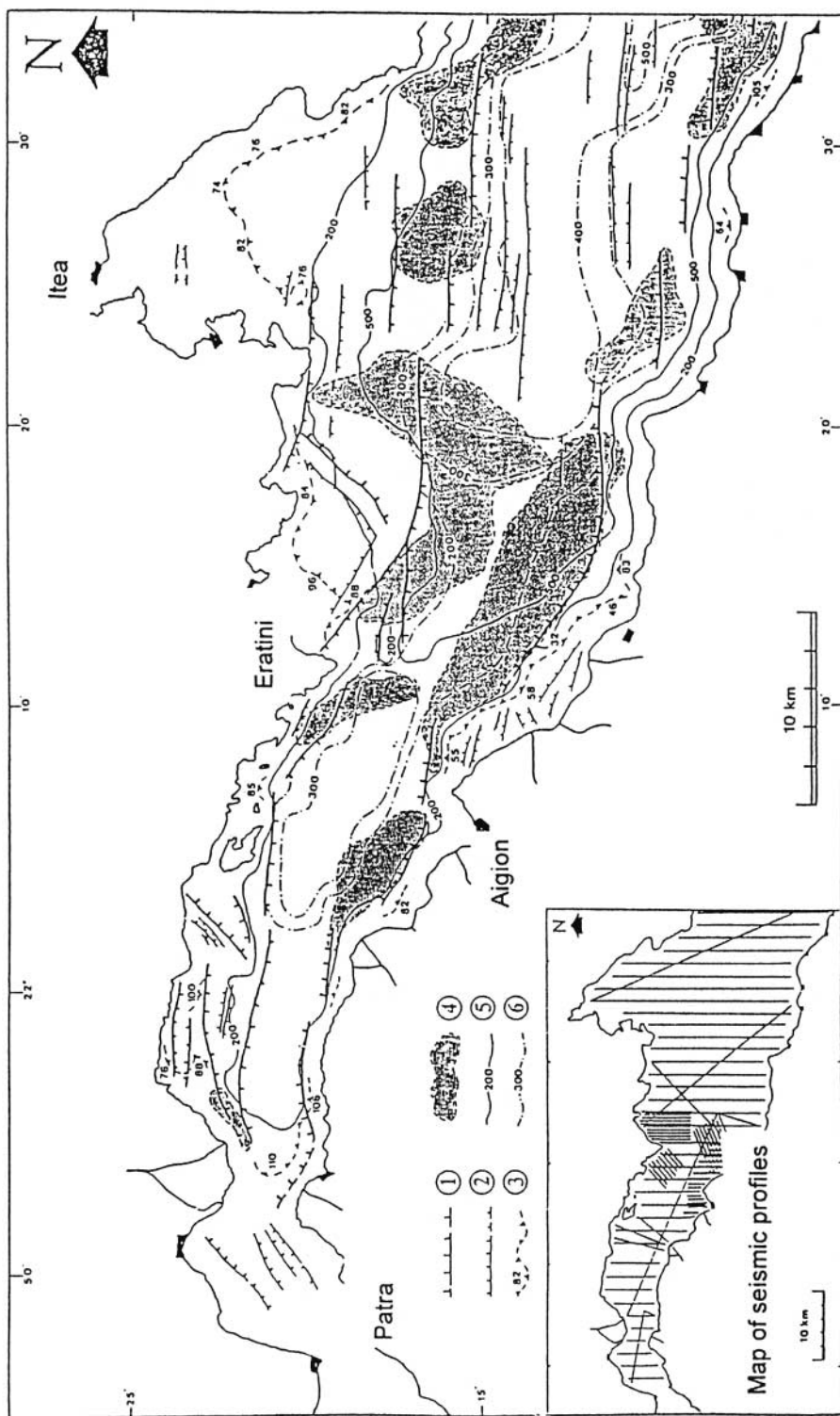
southeastern Mediterranean (DELIBASIS, 1968, 1981, LEYDECKER, 1975; MAKROPOULOS, 1978). The Gulf is a post-alpine tectonic basin, WNW-ESE oriented, almost perpendicular to the alpine geotectonic units of continental Greece, known as the Hellenides. The geodynamic evolution of the Gulf dates back to the upper Miocene, and is controlled by normal faults. These have caused an almost 1,000m uplift of the Pleistocene sediments of the northern Peloponnese (PHILIPPSON, 1892, KELLETAT *et al.*, 1976, DOUTSOS *et al.*, 1988, 1992), and intense geodynamic phenomena (extended landslides, sediment accumulation) in the broader region.

The broader area of the Corinthian Gulf is characterized by high seismic activity (PAPAZACHOS & PAPAZACHOU, 1989). From fault plane solutions it is found that the dominant stress field is extensional, N-S oriented (McKENZIE, 1972), as also verified by geodetic measurements showing a rate of extension of the order of 1cm/yr (BILLIRIS *et al.*, 1991), and dipping to the south (TSELENTIS & MAKROPOULOS, 1986).

The central and western parts of the Gulf have been the subject of a detailed investigation of active faulting in relation to seismic hazard (PAPANIKOLAOU *et al.*, 1997). The above survey involved a total of more than 2,500 km length of seismic profiling, using different seismic sources (AIR GUN 1,5,10,40in3, 3,5KHz). The central part of the Gulf, where the recent Ms6.1 Aigio-earthquake of 15/6/1995 occurred, was particularly investigated. The neotectonic map obtained from the above study is the basis of the present application (see Figure 1a,b). All of the faults in this area are active since they all displace the recent Holocene sediments, creating minor or major morphological discontinuities on the sea bottom.

### *Methodology*

In the following, fault patterns are assumed to be self-similar, with certain fractal



**Fig. 1a:** Simplified Neotectonic Map of the western Corinthiakos gulf.

1: Main Active Faults, 2: Secondary Active Faults, 3: Shelf Break, 4: Submarine Landslides, 5: Contour of Depth, 6: Isopach of Quaternary Sediments.

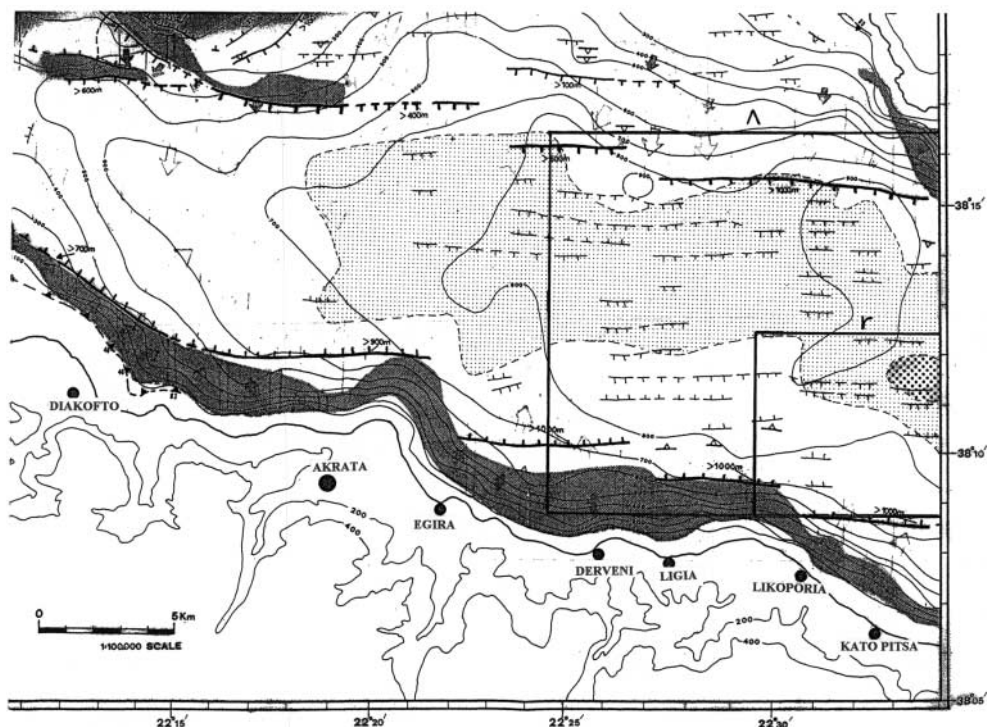


Fig. 1b: b. Area of application (Squares  $\Lambda$  and  $r$ , enlarged from figure 1a).

dimensions,  $\Delta f$ . The fractal dimension is the similarity fractal dimension as defined in the theory of non-linear and chaotic dynamics.

Let  $N(l, r)$  be the density of faults per unit length around faults of length  $l$  in a box of size  $r$ . Then,  $N(l, r)$  may be expressed by the equation.

$$N(l, r) = dN_f / dl \quad (1)$$

so that  $N(l, r) \cdot \Delta l$  represents the number of faults of length between  $l$  and  $\Delta l$  in a box of size  $r$  which is found of the form (SORNETTE *et al.*, 1990).

$$N(l, r) = C r^b l^{-\alpha} \quad (2)$$

where  $b$  affects the fractal (similarity) dimension of the fault barycenters,  $\alpha$  is an exponent constant measured by fitting to experimental data, and  $C$  is a normalization constant.

After normalization by the condition  $\int_{l_{min}}^{\Lambda} N(l, r = \Lambda) dl = N_{tot}$ , where  $N_{tot}$  is the total number of faults larger than  $l_{min}$  in a system of size  $\Lambda$ , it is found that,

$$N_{tot} = \{ C / (\alpha - 1) \} \Lambda^b l_{min}^{-(\alpha-1)} \quad (3)$$

and

$$C = N_{tot} \cdot (\alpha - 1) \Lambda^{-b} l_{min}^{(\alpha-1)} \quad (4)$$

Consequently,  $N(l, r)$  can be rewritten

$$N(l, r) = (\alpha - 1) l_{min}^{-1} N_{tot} (r/\Lambda)^b (l_{min}/l)^{\alpha} \quad (5)$$

The above equation expresses the fractal dimension in a box of size  $r$ . Thus the fractal (similarity type) dimension  $\Delta f$  defined by the scaling law  $L(r) \cong r \Delta f$ , should be equal (if  $\alpha \geq 2$ ) to the barycenter exponent  $b$ , whereas if  $\alpha < 2$  then  $\Delta f = b + 2 - \alpha$ , and should be independent from the precise length distribution (DAVY *et al.*, 1990).

## Results

In the present application equation (5) is used to predict the distribution of stable undeformed regions within the central Corinthian Gulf, Greece. The consequence of fractal fault geometry on homogenization approaches to the modeling of deformation is furthermore tested.

Two squares of dimension  $\Lambda$  and  $r$ , within the coordinates presented in figure (1a), are chosen for the application.

All individual faults, having length greater than  $l_{min}=1\text{ km}$  with step of length increase  $\Delta l=1\text{ km}$ , associated with these square areas, are measured, as presented in Table 1. The accuracy of measurements is of the order of  $\pm 0.1\text{ km}$ .

After computer fitting of equation (2) with the experimental data (Table 1), using graphical/observation trial the parameters  $\alpha$ ,  $b$  and  $c$  are found equal to 2.8, 1.2 and 1.593, respectively (Figs 2,3). Since the question of if fractal faults structure exist in the investigated region is of the main interest in the present study, rather than the detailed measured results, the results of the fitting procedure may be considered reasonably adequate for our initial degree of approximation study.

Thus equation (2) yields

$$N(l,r) = (1.593) \cdot l^{1.2} \cdot r^{-2.8} \quad (6)$$

In our initial approach the fitting results were relatively sensitive to variations of  $\alpha$  and less sensitive to variations of  $b$ . Nevertheless, our initial approximation conclusions of the article may be regarded as valid to a

satisfactory degree of approximation, because of the approximate regularity of fault patterns, even in greater spatial scales which include the region under study.

To test the hypothesis of the existence of large undeformed areas, as the result of fractal fault pattern with no need of macroscopic variations of strength, the scaling law of equation (5) is used to predict the distribution of stable areas  $A$ , and their maximum size  $A_{max}(\Lambda, N_{tot})$ .

For a domain  $D$  of size  $L$ , and area  $a=L^2$ , the average number of faults of all lengths, as follows from equation (5), is  $N_{tot} (L/\Lambda)^b$ . Assuming that the fractals are randomly distributed according to the fractal distribution given by equation (5), the probability  $P(A)$  that the domain  $D$  contains no faults is

$$P(A=L^2) \cong [N_{tot} (L/\Lambda)^b]^{-1/2} \exp \left\{ -N_{tot} (L/\Lambda)^b \right\} \quad (7)$$

The maximum stable domain size  $A_{max}(\Lambda, N_t)$  is obtained from the condition

$$P(A_{max}) \left( \Lambda^2/A_{max} \right) \cong 1 \quad (8)$$

From (7) and (8) it is obtained

$$A_{max}(\Lambda, N_{tot})/\Lambda^2 \cong \left[ \left\{ (b+2)/2b \right\} \left( \log N_{tot} / N_{tot} \right) \right]^{2/b} \quad (9)$$

In the present application  $\alpha=2.8$  ( $>2.0$ ), therefore  $b \cong D_f = 1.2$ .

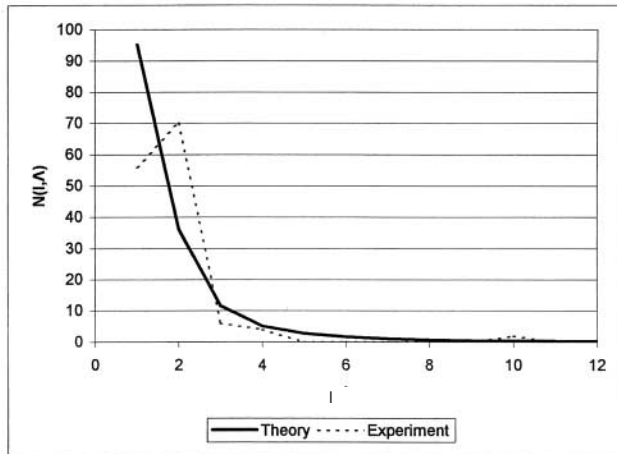
From  $L_{max} = \sqrt{A_{max}(\Lambda, N_{tot})}$ , for  $N_{tot}=70$ , it is estimated  $L_{max}=1.663$ . This value is comparable with the experimental one obtained from the analysis (Table 1).

Thus, one may conclude in the degree of approximation we are working that the test in

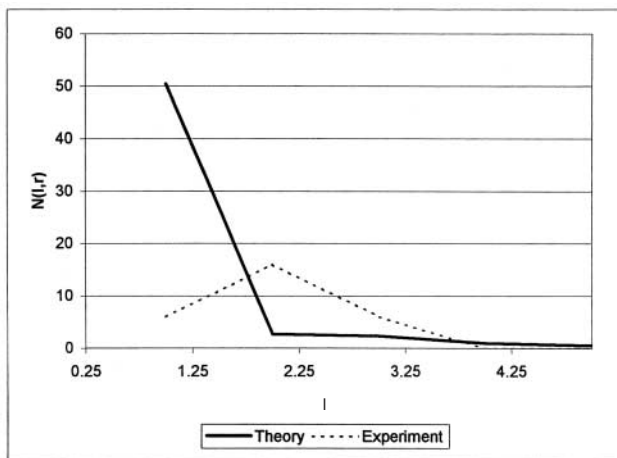
**Table 1**  
Faults with length greater than  $l_{min}=1\text{ km}$ , associated with square areas of size  $\Lambda$  and  $r$ .

1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	$N_{tot}$
$\Delta l \bullet N(l, \Lambda)$ ( $\Delta l=1$ )	28	35	3	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	70
$\Delta l \bullet N(l, r)$ ( $\Delta l=1$ )	3	8	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14





**Fig. 2:** Density of number of faults per unit length as a function of faults of length  $l$ , for square of size  $\Lambda$ . ( $\alpha=2.8$ ,  $b=1.2$ ,  $c=1.593$ )



**Fig. 3:** Density of number of faults per unit length as a function of faults of length  $l$ , for square of size  $r$ . ( $\alpha=2.8$ ,  $b=1.2$ ,  $c=1.593$ )

the present hypothesis is positive, suggesting that fractal structure of fault patterns may be valid in the region of study. To further explain, one may recall the fact that fractals in nature are approximate and statistical (SAUPE, 1988; WILLIAMS, 1977). These can be composed according to the same rules as deterministic (derived from mathematical formulas) fractals but with the additional element of randomness or noise included. The random element is critical for reproducing natural features, such

as landscapes. Such features, when scaled down or up, never look completely alike; all differences between the original and scaled versions are attributable to chance. In the present case one may consider the fault patterns as approximately self-similar, at least in the range of 1 to 20 km in the region of study.

Furthermore, the scaling equation (5) allows the analysis of the consequence of fractal fault geometry on homogenization procedures (DAVY *et.al.*, 1990).

Assuming a representative volume with homogeneous length  $L_H$ , the system is considered to behave as homogeneous with effective average rheological laws, if no faults of size larger than  $L$  divide the volume in two pieces. Using equation (5) the average number of faults  $N(l \geq L_H)$  is estimated and compared with the number one, for the chosen representative homogenization length  $L_H$ . For the recent case, from equation (5) is obtained.

$$N(l \geq L_H) = 70 \cdot (r/\Lambda)^{\alpha-1} \cdot (\Lambda_H/\Lambda)^{b+1-\alpha} \quad (10)$$

Since the exponent  $b+1-\alpha$  is equal to -0.6, less than 0, it is noted that  $N(l \geq L_H)$  decreases at larger and larger lengths  $L_H$ . Therefore, it is concluded that, in the present case, homogenization may be considered valid, at some sufficiently large lengths  $L_H$ . Because the exponent  $\alpha$  may depend on the viscosities of the ductile layers (DAVY *et al.*, 1990), great care should be recommended before using the homogenization approach to study systems that could present fractal fault patterns.

As for the origin of fractal structures, one may speculate this to be a result of no passive deformation and self-organization of the lithosphere. Tectonosphere is a hierarchical dissipative structure, which resulted in the process of cooperative behaviour of its microelements. It may be considered that the perlocation system of the Earth is the main structural motive, which provided the structural arrangement of tectonosphere and its self-organization. The data here do not contradict such an idea: the fractal dimension of a fault is 1.2, as for perlocation clusters.

Future studies may involve a higher degree of approximation approaches to our theme and also other geological formations and comparison of the results with the present approach will help to better conceptualize the application of fractal theory to the particular region, with the aim of a better understanding of the mechanics of geological structure formation.

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