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Abstract

A computational analysis of the periods and structure of surface seiches of Lake Trichonis in Greece and its experimental verification from three simultaneous water gauge recordings, mounted along the shores in Myrtia, Panetolio and Trichonio is given. The first five theoretical modes are calculated with a finite difference code of tidal equations, which yield the eigenperiodes, co-range and co-tidal lines that are graphically displayed and discussed in detail.

Experimental verifications are from recordings taken during spring. Visual observations of the record permit identification of the five lowest order modes, including inter station phase shift. Power spectral analysis of two time series and interstation phase difference and coherence spectra allow the identification of the same five modes. Agreement between the theoretically predicted and the experimentally determined periods was excellent for most of the calculated modes.

Keywords: Surface Seiches, Modelling, Verification, Spectral Analysis.

Introduction

The interest in the study of seiche periods in enclosed seas and lakes started at the beginning of the last century, as a result of the studies of Merian, whose work with lakes of rectangular shape and uniform depth, resulted in the well-known Merian formula. Later, DU BOYS (1891) improved this formula to apply for basins with irregular shapes. His formula was soon replaced by the mathematical theory of CRYSTAL (1904) and later on by DEFANT'S (1961). In 1953, MORTIMER mathematically described the hydrodynamics of lakes. The availability of a computational technique inspired a new start in the study of Lake Hydrodynamics (RAO *et al.,* 1976; HOLLAN *et al.,* 1980). Since then, authors have plotted co-range and co-tidal lines and determined seiche periods for the first few eigen modes. This paper is devoted to the systematical study of the surface seiches of the gravitational oscillations in Lake Trichonis, Greece.

Lake characteristics

Lake Trichonis, the largest and deepest lake in Greece, is situated in the Aitoloakarnania region of western Greece. The lake's surface area is 96.9 Km^2, its length is 20 Km and its greatest width is 6.5 Km (Fig.1). The mean depth is about 40m. The lake is divided into two basins. The

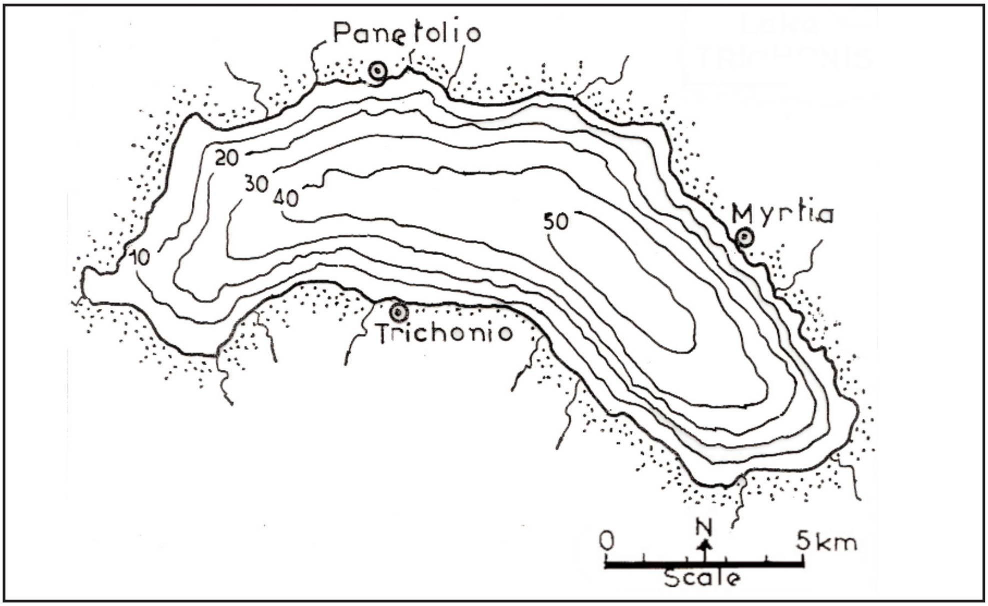

Fig. 1: Depth chart of Trichonis Lake. The basin is divided into a deeper, eastern portion and a shallow, western part. The points marked with the locations Panetolio, Myrtia and Trichonio indicate where the limnigraphs were moored.

western basin is less than 30m deep, while the eastern basin has a maximum depth of 57m. The lake is as reported from KOUSSOURIS (1981): oligotrophic, exhibiting stable thermal stratification, orthograde distribution of dissolved oxygen, high hardness and transparency and dissolved nutrients appearing in low concentrations. Heating and cooling are responsible for the stratification and uniformation of lake Trichonis (ZACHARIAS, 1993). The epilimnion at summer's end attains its maximum average thickness of about 15m when the surface temperature may be as high as 30°C. The thermocline is generally very steep. During the entire year the hypolimnion has a quasi-winter temperature profile at the lake bottom where temperatures of 10°C have been recorded at winter's end when the lake is more or less vertically homogenous (Fig. 2).

Methods of study

For the study of seiches in the lake, five-water gauges were installed. The Merian formula was first used to determine the periods. The full two dimensional tidal theory was used for the study of seiches since the simple one dimensional theory model was not considered accurate due to the bathymetry and the crencentic shape of the lake. Furthermore, knowing that a higher order mode structure is bound to have transverse features, at least locally, the two dimensional

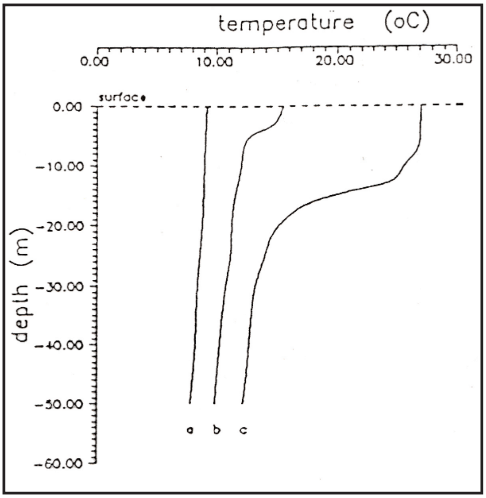

Fig. 2: The thermocline in the summer (c) and in the winter (a).

Table 1
Theoretically predicted and observed periods in minutes of the lowest five gravitational seiches in Lake Trichonis.

Mode	Type	Merian period (min)	Theoretical period (min)	Relative error (%)	Observed period (min)
1	Longit	33.67	34.57	3	34.5
2	Longit	16.83	16.13	6	16
3	Longit	11.22	12.03	9	12.5
4	Longit	8.41	9.26	11	9.5
5	Longit	6.73	7.52	15	6.5

tidal theory would indicate the optimal sites for future, more detailed, limnographical studies. There are several methods to study measured time series to be analysed by spectral analysis. The aim is to obtain, in a certain frequency range, spectral densities and phases of the measured data. For this, the continuously recorded time series of surface elevations were digitised with a temporal step of about 2.5 minutes to guarantee that periods of five or more minutes were realistically covered. The power spectra and the spectra of interstation phase differences and coherence were calculated using the autocorrelation function, followed by a smoothing of the power spectral distribution function. The confidence limits for the correlation distance were chosen to be 1/4 of the full twelve-hour interval. The time length of the correlated seiche episodes were over twelve hours; allowing the spectra of phase differences and coherence to be estimated with high accuracy.

Surface seiches of lake Trichonis

A detailed analysis of the water level recordings yielded seven characteristic oscillation periods (Table 1) using the Merian formula:

$$Tn = 2\,l/(n*SQR(gh))$$

(Where: g= acceleration due to gravity, l= length of the lake, h= depth of the lake and n= number of nodes).

The following seiche periods were calculated: a) A period of 33.67 min. using h=40m and l=20km. This marked the first mode longitudinal osci1lation of the entire lake. b) A period of 16.83 min. constituted the second mode oscillation and was observed at Trichonio, Panaitolio and Myrtia. c) A period of 11.32 min. the third mode oscillation, was detected at almost all stations. d) Periods of 8.41 min. and 6.73 min. were observed mainly in Trichonio. e) Periods of 5.61 min. and 4.83 min. were not observed at any of these stations. Table 1 shows that the shape of lake Trichonis is such that the periods, which were calculated using the Merian formula, give reasonable values that are in close agreement with the observations.

The mathematical model

The equations governing the dynamics of surface seiches are shallow water equations for a homogenous fluid on the steady rotating earth. Suggesting the vertically averaged velocity has the components, u and v (in the east and north direction). The fluid has an equilibrium depth of h, and the acceleration of gravity be denoted by g and the local Coriolis parameter by f, the resulting mathematical description of seiches referred to a horizontal Cartesian system (x, y) may be written as:

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial n}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} - fu + g\frac{\partial n}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0$$

In the above equation (1), frictional effects and non-linear terms were not taken into consideration. Boundary conditions were zero mass flux through the boundary therefore:

$$n_x u + n_y v = O \qquad (2)$$

Where (n_x, n_y) is a horizontal unit vector defined a long the boundary perpendicular to the shoreline. The boundary condition (2) does not take into consideration any in and out flux tributaries. However, this neglect is insignificant for lakes of such size and residence time as Trichonis (MORTIMER, 1974). In seeking solutions to the boundary value problem, (1,2), which are periodic in time, it is convenient to eliminate time by assuming that all fields have exponential time dependence with frequency N. The emerging boundary value problem then becomes an eigenvalue problem for the discreet spectrum of eigenfrequences and eigenfunctions, which consequently determine the seiche periods and their mode structure.

Analytical solutions of (1,2) are only known for ideal 1akes with very simple geometrical shapes. Several numerical procedures based on the finite difference (RAO & SCHWAB, 1976) the finite element (HAMBLIN, 1978) and the Kantorovich technique (HUTTER & RAGGIO, 1982) have been developed to find the surface seiche structure in natural lakes. Here, a finite difference technique was used to subdivide the lake into a finite number of grids there by reducing the problem to an algebraic eigenvalue equation. In the explicit finite-difference model described here in the discreet values of the variables are described on a space-staggered grid. The water level of Trichonis was computed at integer values of i and j, while the values of h were given at half-integer values of i and j. The following short notations were used to avoid the lengthy finite difference formula. The position and the time coordinates (x,y,t) were represented on the finite grid by (i,Dx,Dy,nDt) and i,j,n=0,1/2,1,3/2,... Lake Trichonis was partitioned into 210 space-staggered grids. The elongated shape of the lake would require a great number of squares if a sufficiently accurate transverse resolution of the mode structure of high order mode was sought. This, however, would be computationaly time consuming. The depth variable, h, was taken from the bathymetric chart for Lake Trichonis with a scale of 1:5000 and an equidistance of 1ines of equal depth of 5m. (ZACHARIAS *et al.*, 1998).

Computational results

The mathematically determined periods of the eigenmode are listed in Table 1 with those observed in the field and those obtained by the use of the Merian formula. The mathematically calculated periods of the lowest five modes range from 34.57 min to 7.52 min. These periods are similar to

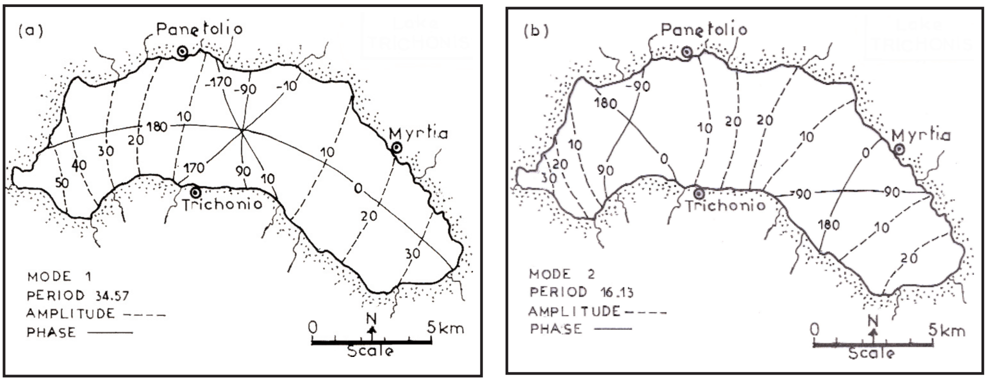

Fig. 3: Co-range (dashed) and co-tidal (solid) lines for mode one (a) and mode two (b) of the Lake Trichonis. The period are 34.57 minutes and 16.13 respectively.

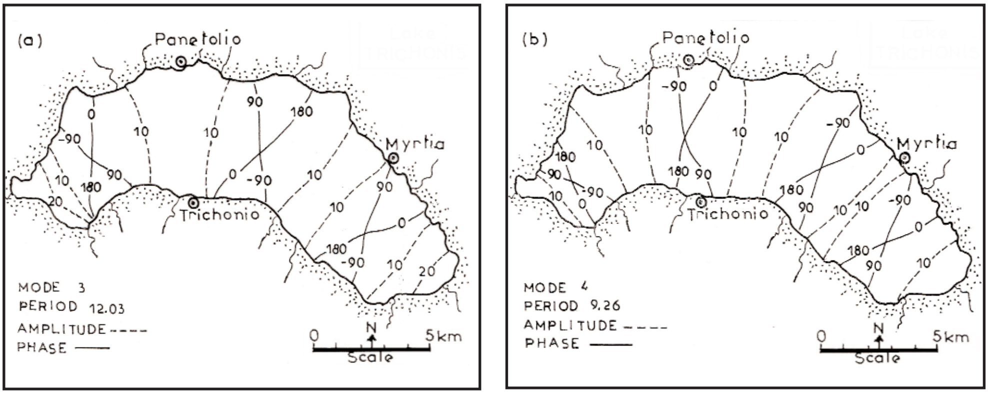

Fig. 4: Co-range (dashed) and co-tidal (solid) lines for mode three (a) and four (b) of the Lake Trichonis. The periods are 12.03 and 9.26 minutes respectively.

those calculated using the Merian formula and those observed during the measuring period with the exception of the fifth mode which shows a considerable difference. This difference may result from a topographic effect of the lake's floor. The mode structure of the co-range and co-tidal lines are shown on figures 3 and 5. The lowest mode (Fig. 3a) has a single positive amphidromic point north of Trichonio near the middle of the lake and is longitudinal. The distributions of the water level fluctuations are unevenly throughout the lake. Their maxima occur at the lake's ends, with 30 units at the western end and 50 at the eastern one. The difference in the height between the two ends may exist because the eastern part of the lake is deeper than the western part. The second mode (Fig. 3b) has two positive amphidromic points near Myrtia and Panaitolio. The mode is mainly longitudinal and the water level fluctuations are larger on the far ends of the lake than in the area between the two-amphidromic points. The gradient of the water level is steeper in the western end of the lake, which may result from a greater water depth in the eastern end. The propagation of the phase angle runs clockwise in agreement with the longitudinal structure of the mode.

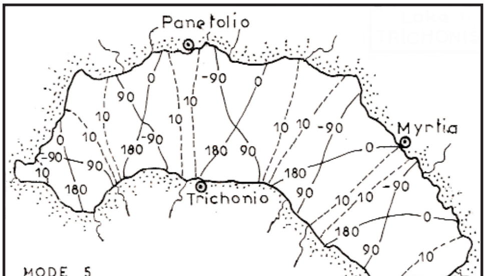

Fig. 5: Co-range (dashed) and co-tidal (solid) lines for mode five of the Lake Trichonis. The period is 7.52 minutes.

The third mode (Fig. 4a) is also longitudinal and has three amphidromic points. Maximum surface elevations appear at the far ends of the lake as a result of the topographic effect. The maximum water level was found in a small enbayment, west of Trichonio. This level may have occurred from a strong excitation of the third mode in this enbayment or, more simply, from a peculiarity of the mathematical model due to the coarse finite difference grid.

The fourth mode (Fig. 4b) has four amphidromic points which were nearly equally distributed along the lake. The water level was evenly distributed and the mode was mainly longitudinal. The maximum amplitude, of about 14 units, arose at the eastern and western ends. The phase angle was clockwise. The fifth mode (Fig. 5) portrays the phase angle moving clockwise in four of the five cells. The water level distribution was as in the fourth mode.

Verification

Direct analysis of data

The water gauge stations were operated from 13 April to 12 May 1989. Daily synoptically inspection of all data sets disclosed the following: (a) Episodes of wind induced surface oscillations, lasting for several hours or even days, were never fully harmonic and the first five seiche periods - could be determined by visual determination. (b) The episodes, lasting twelve to twenty-four hours, with the water level undulating considerably did not permit visual inspection of a typical period between 34.5 and 6.5 min. For these, time series spectral analysis methods are used to identify the modes. (c) Periods, lasting several days, when the lake was at rest. A synoptically view of the time series for all stations and different episodes is given on figure 6. This figure is marked by lines which depict the distance periods of 34, 16, 12, 9 and 6 min. Figure 6a. shows the

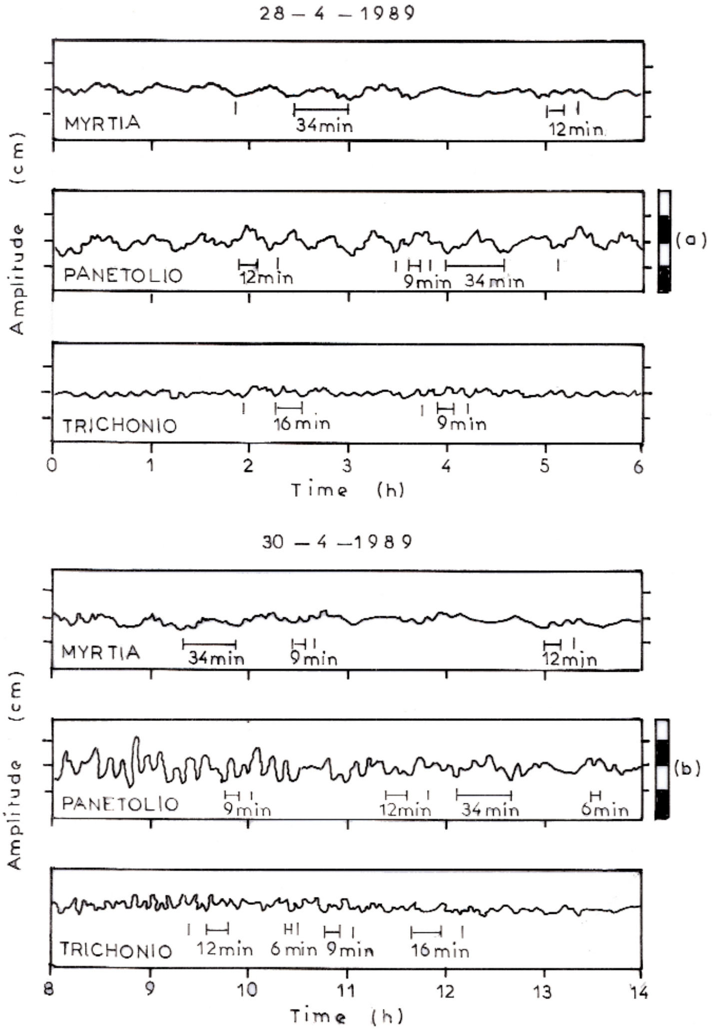

Fig. 6: (a). Six-hour episode of the time series for the Panetolio, Myrtia and Trichonio gauges on 28 of April from 00.00 to 6.00 h. (b) Six-hour episode of the time series for the Panetolio, Myrtia and Trichonio gauges on 30 of April from 8.00 to 14.00 h.

00.00h to 06.00h event on 28 April which is part of an episode lasting twenty-four hours. The Myrtia recording is noteworthy for it consists primarily of two harmonics with periods of 34.4 and 12.5 min. This behaviour lasted for several days and was a combination of first and third seiche. The Panetolio recordings are also of interest since they possess primarily three harmonics with periods of 34.5, 12.5 and 9.5 minutes. This behaviour was a combination of the first, third and fourth seiche. The Trichonio recordings are less regular and seem to contain components of the second and fourth seiche, which have periods of 16 and 9.5 minutes. Because the position of the gauge at Trichonio was close to the amphidromic point of the fundamental mode, it is clear why this mode cannot be seen in the Trichonio recordings. Figure 6.b illustrates a more complicated six-hour episode on 30 April. Here the Myrtia recordings consisted primarily of three harmonic periods of 34.5, 12.5 and 9.5 minutes which are combinations of the first, third, and fourth seiche. Panetolio recordings are also challenging and consist of four harmonic periods of 34.5, 12.5, 9.5 and 6.5 minutes. Finally, Trichonio recordings are the most complicated ones disclosing a combination of the first, second, third, fourth and maybe the fifth seiche, but possessing very small amplitudes. The recordings in this figure (6b) are complex. However, they allow the examination of higher modes such as the fifth seiche. Further modes cannot be detected by direct inspection of the water level recordings.

Spectral analysis

Simultaneous excitation of the limnigraphs, at all three stations, suitable for spectral analysis occurred on 28 and 30 April. Twelve-hour episodes were selected on these days for a careful spectral analysis permitting identification of five eigenperiodes. In the following figure, results for station pairs will be illustrated.

The middle portions of the figures (Fig. 7 to 10) show the power spectral distribution functions plotted against frequencies or "period of time" series in which the linear trend was subtracted prior to the Fourier transform. The scale is the logarithm of the power density. However, because different time series do not have the same optimal

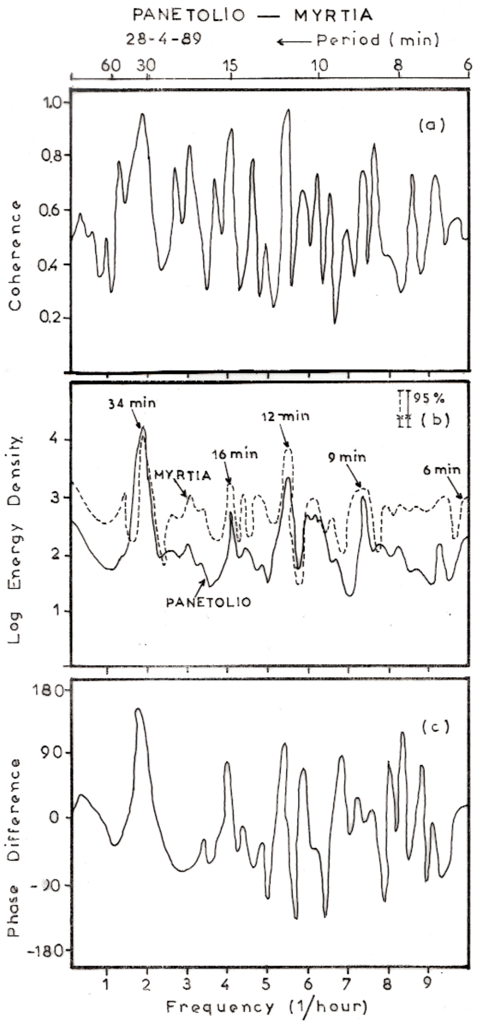

Fig. 7: Power spectra of water level fluctuations at stations Panetolio and Myrtia (b) for a twelve-hour event on 28 of April1 and spectra of corresponding phase difference (c) and coheence (a). Here and in later figures, energy density is plotted in a logarithmic scale, but dimensions are not shown for reasons explained in the text. The 95% confidence limit is also shown.

linear fit, the energy scales of the distribution functions of different stations could not be easily related. This explains why the dimension in the logarithmic energy density in the subsequent figures could not be shown. In the lower portions of the figures the spectra of the interstation phase difference is shown and in the upper portion lies the coherence spectra. The coherence spectra suggest whether the two time series, at a certain period, are statistically correlated; a value between 0.6 and 1 means strong to perfect correlation (BLACKMAN *et al.,* 1958).

Episode 28-4 00.00-12.00

Figures 7a,b,c show, respectively for this episode, the coherence spectra, the power spectra and the phase difference spectra of Panetolio and Myrtia stations. The first and third modes appeared highly excited at both stations and possessed relatively high energies. The second and fourth modes were also well defined, but they possessed less energy due to the locations of the water gauges in relation to the position of the nodal points as figured by the mathematical model (Fig. 3, 4). The fifth mode could not be determined in the power spectra for this episode. Interstation coherence (Fig. 7a) is nearly unity for the first and third mode and over 0.6 for the other two modes. For the fifth mode the coherence is very low about 0.5. The phase difference (Fig.7c) between Panetolio and Myrtia for the first, second, third, fourth and fifth modes is approximately 170°, 90°, 120°, 30°, 5° respectively. The first three modes are in close agreement with the model results, while the last two ones are in lesser agreement.

Figures 8 a,b,c show respectively, for this episode, the coherence spectra, the power spectra and the phase difference spectra of Panetolio and Trichonio stations. On figure 8.b it is clear that the lowest mode, with a period of 34. 5 minutes, is dominantly excited in Panetolio station but not in Trichonio. This is due to the position of the station in relation to the nodal point of the first mode. The same occurred with the third mode with a period of approximately 12.5 minutes. The second and fourth modes, with periods of 16 and 9.5 minutes respectively, are excited at both stations and posses relatively high energies. The fifth mode, with a period 6.5 minutes, is much less energetic than the lowest modes and cannot be identified in this episode. The phase difference (Fig. 8c) for the three modes is near 20°, 90°, and 60° for

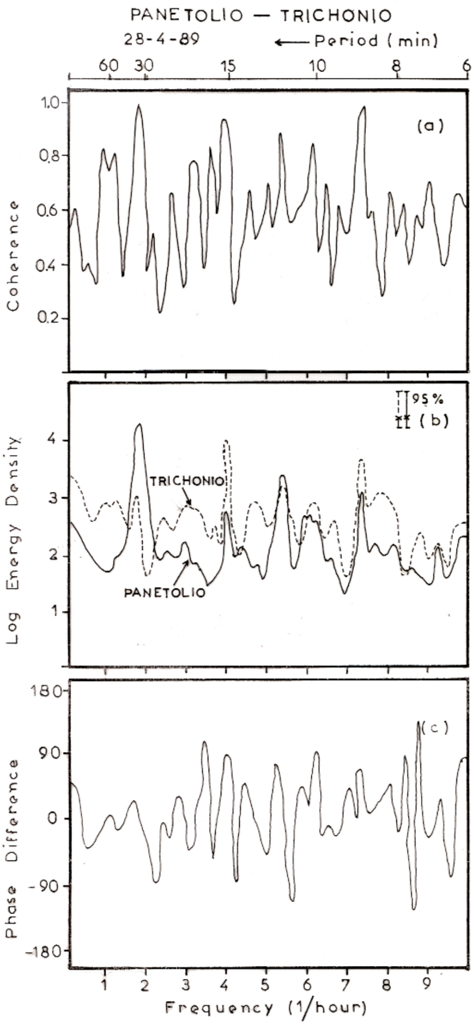

Fig. 8: Power spectra of water level fluctuations at stations Panetolio and Trichonio (b) for a twelve-hour event on 28 April and spectra of corresponding phase difference (c) and coherence (a).

the first, second and third respectively. This agrees well with the findings of the mathematical model. For the higher two modes the phase difference is 50° and 80° which is in less agreement with the model results. The coherence spectra (Fig. 8a) shows the relative maximum: almost 1, for the second and fourth modes, over 0.8 for the first and third modes and almost 0.5 for the fifth mode.

Episode 30-4 8.00-20.00

The time spectra analysis of this episode shows that the fifth mode is excited and possesses large energies. Figures 9, 10 show, from top to bottom, the coherence function for the Panetolio-Myrtia and Panetolio-Trichonio gauges, power spectra distribution functions and the spectra for the phase difference for the same stations. On figure 9.b, the power spectra indicate a dominant excitation of the fifth mode in Panetolio and Trichonio gauges, which is much smaller in Myrtia (Fig. 10).

Similarly, in the same figure it can be seen that the first, third and fourth modes have relatively high energies, whilst the second mode is characterised by very little energy.

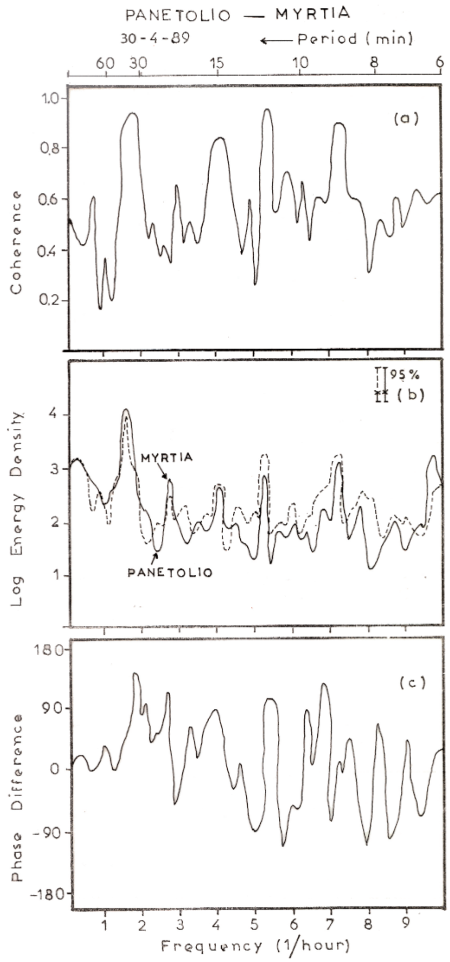

Fig. 9: Power spectra of water level fluctuations at stations Panetolio and Myrtia (b) for a welve-hour event on 30 April and spectra of corrsponding phase difference (c) and coherence (a).

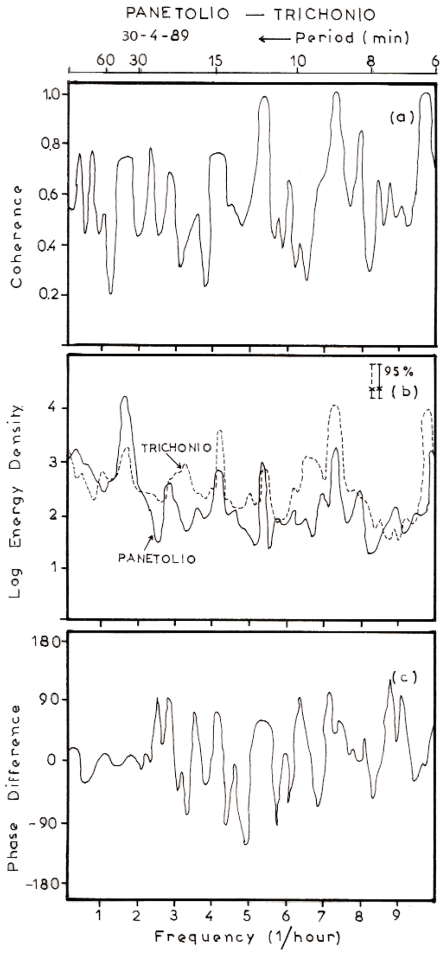

Fig. 10: Power spectra of water level fluctuations at stations Panetolio and Trichonio (b) for a twelve-hour event on 30 April and spectra of corresponding phase difference (c) and coherence (a).

The fifth mode has high energy in Panetolio but not in Myrtia. The coherence (Fig. 9.a) is almost one for the first and the third modes, over 0.8 for the second and fourth modes and 0.6 for the fifth mode. This occurs since Myrtia has low energy in the fifth mode unlike Panetolio. The phase difference (Fig. 9b) between these stations for these five modes is 160°, 100°, 110°, 10°, 20°, respectively. For the first three there is good agreement with the mathematical model results, but we have lesser agreement with the fourth and fifth modes.

On figure 10b it may be seen that all the modes, with the exception of the second mode in Panetolio, have high energies. The fifth mode has a very high energy, which did not happen in the previous episode. It is dominantly excited in both Panetolio and Trichonio stations with a period of 6.39 minutes. The periods of the lowest four modes are 34.48, 15.87, 12.33 and 9.32 minutes. The phase difference (Fig. l0c) for the three modes is 15°, 80°, 75°, respectively. For the higher two modes it is 50° and 45° which is not in close agreement with the theoretical results. The coherence (Fig. 10.a) spectra is nearly unity for the third, fourth and fifth modes which means very good correlation between those stations at these periods. It is almost 0.7 for the lowest two modes because the Panetolio has high energy in the first mode and the Trichonio has it in the second.

Discussion

In this paper, a detailed attempt was made to determine the structures of the surface seiche and the eigenmode periods of Lake Trichonis and to correlate and verify these findings with measured water level fluctuations at various stations in the lake.

In Table 1 the periods of the lowest five modes are listed. They range from almost 3 to 6 minutes. It can be seen that the relative error between the theoretical results and the Merian formula are much smaller than from Zurich and Lugano Lakes in the work of HUTTER, *et al.,* 1982. This is chiefly because the shape of Lake Trichonis is more rectangular than the above mentioned lakes. Because the seiches of Lake Trichonis occur in highly regular trains of oscillations, the periods may be estimated very accurately. Consistent underestimation of the observed periods by a few percent is due to the neglect of frictional and non-linear effects in the mathematical model of seiche.

Power spectra, in the first episode in Trichonio, indicate high energies in the first four modes and low energy in the fifth mode. This is an indication of the fact that the geometry of the basin hampers excitation of the fifth mode and favours the lower ones. In the second episode, power spectra indicate high energy in all five modes. This is probably due to the meteorological forcing at this date. The wind on the 28 of April was from west to east with a speed of 4 Beauforts and on the 30 of April it was the same direction with a speed of 6 Beauforts. Confirmation of this can also be found in the coherence spectra which show low values in this frequency range.

It may be worth noting that in the entire observation period, the water level undulations in Lake Trichonis were composed of a series of harmonics. This accounts for the reason why the fundamental mode in the first episode appears as the first mode and as the fifth mode in the second episode.

After analysing the recordings gathered between 13 April to 12 May, it is evident that the fundamental mode and the first three higher order modes are relatively easily excited. A fifth mode (period of 6.5 minutes) is only excited about as half as often. Higher order modes could not often be identified. If this is compared with wind measurements, it can be seen that when strong winds from west to east exist, a high excitation of higher order modes results.

Based on measurements of different lakes, the following mechanisms are regarded as main causes for the generation of seiches (MORTIMER, 1976). a) Temporal and local fluctuations of the air pressure during passage of fronts and thunderstorms. b) Horizontal winds, which cause a stress on the water surface. c) Rainfall onto one part of the lake. d) Vertical air currents. From the present data it appears that the wind forcing on the whole is more influential than the pressure gradient and the other mechanisms.

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