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Seasonal and interannual variability of the water exchange in the Turkish Straits System estimated by modelling

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Supplementary Data

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Appendix

A1.The equations of strait model

The strait is a shallow and narrow channel connecting two deep basins. A definition sketch for the strait model is given in Fig. A1. The coordinate x is taken along the strait and it is directed from the basin with light water to the basin with dense water. For sake of simplicity, the strait is rectangular with length L, depth $H=H_1+H_2$ and width $A=A_0+A_s$. The instantaneous thicknesses of upper and bottom layers are $H_1=D_1-\eta+\varsigma$, $H_2=D_2+\eta-H_s$. The averaged along strait depths of upper layer and bottom layer are D_1 and D_2, respectively, and $D=D_1+D_2$. Here ς is the free surface elevation, η is the interface deviation and H_s, A_s are the deviations of the bottom level and width from the averaged values D and A_0, respectively. Invoking the hydrostatic and Boussinesq approximations, the stationary governing equations of the two-layer model of the strait (Maderich & Konstantinov, 2002) can be written as

$$\frac{d}{dx}AH_1u_1^2+AH_1\frac{d}{dx}g\zeta=-c_iA|u_1-u_2|(u_1-u_2), \qquad (A1)$$

$$\frac{d}{dx}AH_1u_2^2+AH_2\frac{d}{dx}(g\zeta+g'\eta)=c_iA|u_1-u_2|(u_1-u_2) -c_bA|u_2|u_2, \qquad (A2)$$

where x is coordinate along the strait, g is the gravity, $g'=g(\varrho_2-\varrho_1)/\varrho_0$, ϱ_0 is undisturbed density, ϱ_1 and ϱ_2 are densities in the upper and bottom layers, respectively; c_i and c_b are interface and bottom drag coefficients, respectively. The volumetric flow rates in the upper (Q_1) and bottom (Q_2) layers relate to the velocity u_1 and u_2 by $Q_1=AH_1u_1$ and $Q_2=AH_2u_2$. At a small density difference $((\rho_2-\rho_1)/\rho_0<<1)$ the control condition (Farmer & Armi, 1986) is

$$G^2=F_1^2+F_2^2=1, \qquad (A3)$$

where

$$F_1^2=\frac{Q_1^2}{g'H_1^3A^2}, F_1^2=\frac{Q_2^2}{g'H_2^3A^2}, \qquad (A4)$$

Equations (A1)-(A2) are complemented by the control conditions (A3) at the critical sections in the strait. For sake of simplicity the strait geometry is chosen such that the control points are placed at the ends of the strait. Following Özsoy *et al*. (1998) and Maderich & Konstantinov (2002), the volumetric flow rates in the upper and bottom layers are related to the level difference along the strait using the Bernoulli equations for upper and lower layers. We obtained the Bernoulli equation in the upper layer between the sea, that is in the rest, and the left end of the strait (x=0). In a similar manner, we obtained the Bernoulli equation in the lower layer between the right end of the strait ($x=L$) and another sea, that is in the rest. The resulting equation that relates Q_1, Q_2, H_1(L), $H_2(L)$ to the level difference $\Delta\varsigma$ between adjacent seas is

$$\frac{1}{2}\left(\frac{Q_1^2}{A^2(L)\ H_1^2(L)}-\frac{Q_2^2}{A^2(L)\ H_2^2(L)}\right) +g'H_2(L)=g\Delta\varsigma-c_i\int_0^L\frac{|u_1-u_2|(u_1-u_2)}{H_1}dx. \qquad (A5)$$

The details of derivation of (A5) are given by Maderich & Konstantinov (2002). An assumption $\varsigma<<H_1$ allows reducing the system of equations (A1)-(A2) to the single equation for H_2

$$\frac{1}{2}\left[\left(\frac{D-H_s}{H_2}\right)^{1/2}F_1-\left(\frac{D-H_s}{D-H_s-H_2}\right)^{1/2}F_1\right]^2+c_bF_2^2. \qquad (A6)$$

This differential equation is of the first order and it contains the unknown variable $H_2(x)$ and unknown values of Q_1 and Q_2. It is completed by equation (A5) and the control condition (A3) acting at the two control points at x $=0;L$. The nonlinear system of equations (A3)-(A6) is solved by method of successive approximations.

A2. The equations of model of the Marmara Sea

The evolution of the vertical thermohaline structure in the Marmara Sea is described by the equations for the horizontally averaged temperature and salinity in the layers including the surface mixed layer (SML), internal and bottom layers. The equations for the temperature and salinity in the SML are

$$V_u\frac{dT_u}{dt}=\sigma_u(T_u-T_u)(w_e-w_u)-q_T\sigma_u+(T_1^B(M)-T_u)Q_1^B(M), \qquad (A7)$$

$$V_u\frac{dS_u}{dt}=\sigma_u(S_u-S_u)(w_e-w_u)-S_TQ_f^M+(S_1^B(M)-S_u)Q_1^B(M), \qquad (A8)$$

where $Q_1^B(M)$, $T_1^B(M)$ and $S_1^B(M)$ are the volumetric flow rate, temperature and salinity of water flowing from the upper layer of the entrainment box in the Bosphorus

Strait into the Marmara Sea, $V_u = \sigma_u(\varsigma_M + h_u)$ is the volume of the SML, h_u is the height of SML from the undisturbed sea surface. The equation for the evolution of SML height is

$$\sigma_u \frac{dh_u}{dt} = \sigma_u w_e - Q_2^D(M), \qquad (9)$$

where w_e is the entrainment velocity at the lower boundary of SML, $Q_2^D(M)$ is the volumetric flow rate in the bottom layer of the Dardanelles Strait at the Marmara Sea exit. The equations for temperature and salinity in the internal layers are

$$V_i \frac{dT_i}{dt} = \sigma_{i+1}(T_{i+1} - T_i)\, w_* - \sigma_i(T_i - T_{i+1}) w_*, \qquad (A10)$$

$$V_i \frac{dS_i}{dt} = \sigma_{i+1}(S_{i+1} - S_i)\, w_* - \sigma_i(S_i - S_{i+1}) w_*, \qquad (A11)$$

with $i=2,\ldots,n$. The equation of evolution of the position of the internal layers is

$$\sigma_i \frac{dh_i}{dt} = Q_2^D(M). \qquad (A12)$$

The equations for the heat and salt in the bottom layer are

$$V_i \frac{dT_i}{dt} = \sigma_2(T_2 - T_1)\, w_* + (T_1 - T_2^D(M))Q_2^D(M). \qquad (A13)$$

$$V_i \frac{dS_i}{dt} = \sigma_2(S_2 - S_1)\, w_* + (S_1 - S_2^D(M))Q_2^D(M). \qquad (A14)$$

In the above, T_i, S_i, V_i, σ_i, h_i are the temperature, salinity, volume, area and thickness of the i-th layer, respectively (for the bottom layer $i=1$); n is number of internal layers plus bottom layer; $T_2^D(M)$, $S_2^D(M)$ are the temperature and salinity of the Aegean waters out-flowing from the lower layer of the Dardanelles Strait into the Marmara Sea, respectively; q_T is the temperature flux to the Marmara Sea, and w_* is the rate of internal mixing. The density is related to the salinity and temperature by the linear equation of state $\rho=\rho\,(T, S)$.

Two different regimes of the evolution of the Marmara Sea SML were modelled: the "entrainment regime", when the growth of thickness of the SML is caused by the turbulent entrainment of the lower layer as a result of wind and convective mixing, and the "detrainment regime", when the turbulence in the SML decays and a new surface mixed layer is formed with smaller thickness, while the previous SML joins the "staircase" of the internal layers (Turner & Kraus, 1967). For entrainment regime the model of Resnyansky (1976) is described in detail by Maderich & Konstantinov (2002). The temperature flux in the Marmara Sea is calculated using Haney (1971) formula $q_T = a(T_u - T_a)$, where T_a is the surface air temperature and a is an empirical parameter. The values of w_* and a are: $w_* = 3\cdot10^{-8}$ m s^{-1}, a$=6\cdot10^{-8}$ m s^{-1}. The parameters of the entrainment model are given by Maderich & Konstantinov (2002). The SML absorbs internal layers in entrainment regime. The bottom layer thickness is limited by the depth of penetration of winter convection in SML.

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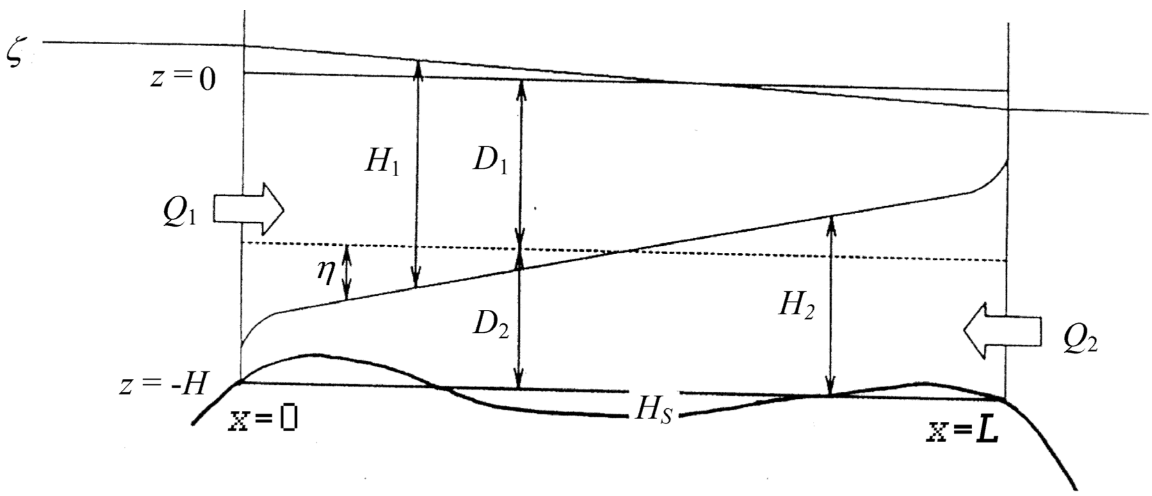

Fig. A1: Definition sketch showing along-strait section (Maderich & Konstantinov, 2002)