

How schlieren affects beam transmissometers and LISST-Deep: an example from the stratified Danube River delta, NW Black Sea

A.P. KARAGEORGIS¹, D. GEORGOPOULOS¹, W.D. GARDNER², O.A. MIKKELSEN³ and D. VELAORAS¹

¹ Hellenic Centre for Marine Research, Institute of Oceanography, 46.7 km Athens-Sounio Avenue, 19013 Anavyssos, Greece

² Department of Oceanography, Texas A&M University, College Station, TX 77843, USA

³ MacArtney A/S, Gl. Guldagervej 48, 6710 Esbjerg V, Denmark

Corresponding author: ak@hcmr.gr

Handling Editor: Elina Tragou

Received: 29 October 2014; Accepted: 9 March 2015; Published on line: 5 May 2015.

Abstract

The term ‘schlieren’ describes angular deflection of a light ray when it passes through a fluid region characterized by refractive index inhomogeneities. These inhomogeneities in the marine environment are generally caused by density variations, i.e. salinity and temperature anomalies. The influence of schlieren on transmissometers and the *in situ* particle sizer LISST-Deep of Sequoia Scientific are examined in the Danube delta during October 2007. A seasonal pycnocline driven mainly by an intense temperature gradient was identified as a major hydrological feature. It was associated with high buoyancy frequency values and minor changes of the refractive index of seawater. Measurements of two 25-cm path-length transmissometers (660 nm and 470 nm) showed distinguishable peaks at the pycnocline. LISST also uses a 5-cm transmissometer (670 nm), which proved to be very sensitive in both cases. This is mainly due to its very small acceptance angle, which enables enhanced light scattering outside the lens, thus increasing beam attenuation. Subsequently, LISST falsely predicts abundance of large particles within the pycnoclines. A buoyancy frequency N of 0.01 s^{-1} is the new proposed threshold for schlieren occurrence.

Keywords: light scattering, pycnocline, buoyancy frequency, refractive index, acceptance angle.

Introduction

Gradient disturbances of inhomogeneous transparent media due to small refractive index differences are described by the term ‘schlieren’ (Töpler, 1867; Scharadin, 1942). These inhomogeneities are generally caused by density variations; in the marine environment primarily by salinity and temperature anomalies (Karpen *et al.*, 2004). In deltaic systems, density stratification due to surface heating (and fresh water/salt water mixing) is commonly observed. Styles (2006), in flume experiments, demonstrated that density gradients cause small angle forward scattering patterns that are indistinguishable from particle scattering. Beam attenuation is one of the inherent optical properties (IOP; properties that depend only on the water and other substances that are dissolved or suspended in it) measured routinely for many decades, and schlieren may affect the behavior of instruments measuring light scattering and transmission. Andrews *et al.* (2011) conducted similar experiments in oligotrophic waters and identified that schlieren results in dramatic underestimation of small ($<20 \mu\text{m}$) particles. In the oligotrophic Eastern Mediterranean schlieren artifacts have been recorded during autumn 2008 and were associated with pronounced pycnoclines (Karageorgis *et al.*, 2012). This contribution aims to study the schlieren

interferences in two commercial beam transmissometers and a third one, which is part of the particle sizer LISST-Deep (Laser In Situ Scattering and Transmissometry; Agrawal & Pottsmith, 2000). The data set assessed was obtained in the Danube River delta, during October 2007 in the framework of SESAME-IP Project; detailed information on particulate matter distribution, composition and sources are given in Karageorgis *et al.* (2014).

Regional setting

The Danube River’s delta is the largest in the Black Sea and covers 5800 km^2 (Panin, 1999). At the head of the delta, also referred to as ‘Mile 44’, the Danube River splits into Chilia (length 116 km) and Tulcea branches, and the latter bifurcates $\sim 17 \text{ km}$ downstream into the Sulina, and Sfântu Gheorghe branches, 63, and 109 km in length, respectively (Fig. 1). During the period of the cruise, in October 2007, water discharge was $4980 \text{ m}^3 \text{ s}^{-1}$ and suspended solids (SS) load 197 kg s^{-1} , the latter corresponding to SS concentration of 40 mg l^{-1} .

Materials and Methods

Measurements derive from a cruise of the R/V Aegaeo in the Danube delta (NW Black Sea), in the period 5-12 October 2007. In total, 17 stations were occupied at shallow depths (10-53 m) in the Danube delta area (Fig. 1).

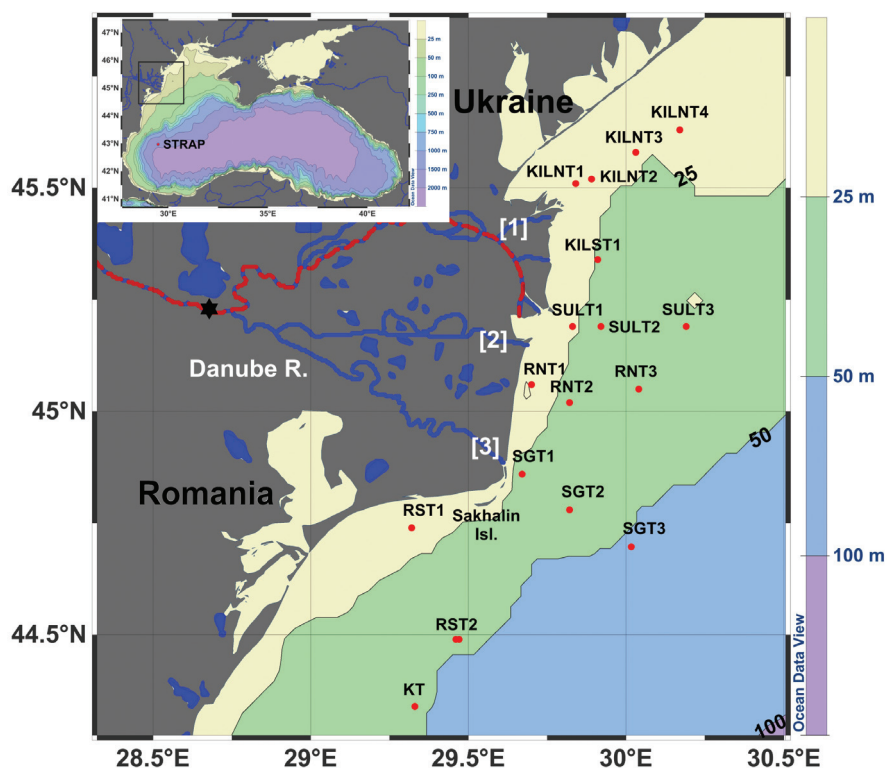


Fig. 1: Study area location map and sampling stations (red dots). Red dashed line indicates national borders. The Danube River main branches are [1] Chilia, [2] Sulina, and [3] Sfântu Gheorghe. Black star: gauging station 'Mile 44'.

Standard CTD measurements were obtained with a Sea-Bird Electronics 11*plus* CTD interfaced with a General Oceanics rosette with twelve 10-litre Niskin bottles. Light transmission was measured by two 0.25-m path-length transmissometers emitting at 470 nm-blue (Chelsea ALPHATRACKA MKII, acceptance angle 0.91°) and 660 nm-red (WET Labs C-Star, acceptance angle 1.2°).

Particle volume concentration, and particle-size distribution measured by 32 ring-detectors (the inner rings detecting the largest particles and the outer rings detecting the smallest), corresponding to 32 classes in the range 1.5-250 µm, were determined with an autonomous LISST-Deep, which also measured beam attenuation, c at 670 nm-red, and pressure (Agrawal & Pottsmith, 2000). The major difference of LISST-Deep and the more widely used LISST-100X is the special aluminium housing, which allows deployment to 3000 m depth, whereas the 100X is limited to sampling down to 300 m depth. The wavelengths of the WET Labs C-Star and LISST-Deep are nearly identical, but the acceptance angles are very different (1.2° and 0.0269°, respectively). The differences in the path-lengths of the two instruments (0.25 m and 0.05 m, respectively) is equalized in determining c from the raw output of the instrument. LISST-Deep was placed horizontally to the side of the CTD frame and deployed simultaneously with the other sensors with a profiling speed of 0.7 m s⁻¹.

Finally, a sonar altimeter was coupled with the CTD, providing accurate distance from the bottom, and thus enabling measurements to within 1–2 m of the seabed.

Sensors were factory calibrated prior to the cruise. Optical windows were rinsed with MilliQ water and wiped carefully prior to each cast. All data were binned into 0.1-dbar intervals after quality control of raw data.

Results and discussion

Hydrology

The surface waters (3-dbar) near the Danube delta area are characterized by temperatures ~18 °C and salinities between 8.5 and 14, and form a narrow belt near the coast. At Chilia (northern branch) relatively colder waters extend towards the north, probably influenced by freshwater of other rivers discharging in the area. Off-shore, both temperature and salinity increase gradually, with the highest values recorded SW of the delta (19.7 °C and 17.5, respectively). According to Karageorgis *et al.* (2009), the preferred water and particle transport pathway when northerlies prevail, is along a narrow strip near the coast, with a south-southwest direction. As northerlies prevailed prior and during the cruise in October 2007, this pattern appears to be dominant.

The water column near the coast exhibits two pycnoclines; the upper pycnocline at 5-7 dbar and the deeper, seasonal pycnocline between approximately 19 and 24 dbar (Fig. 2a). The upper pycnocline is observed at stations near the coast (e.g. Stn. SGT2; Fig. 2a) and is associated with the low salinity freshwater inputs from the Danube River. The deeper, seasonal pycnocline is attributed to the over-

all warmer surface Black Sea layer (19.2-20.4 °C), which overlies cooler, more saline waters characteristic of the cold intermediate waters of the Black Sea (Figs. 2a, 3a).

Optical measurements

Profiles of the three transmissometers used in the survey for Stn. SGT2, which represents the coastal area near the mouth of the Danube (Fig. 2b), reveal that c_{660} and c_{470} : (a) decrease rapidly with depth but do not exhibit any pronounced peak at the upper pycnocline; (b) show a small increase between 11 and 19 dbar; and (c) gradually decrease in the deeper pycnocline (20-23 dbar), and then slightly increase in the uniform bottom 20 m. In contrast, the 5-cm path-length transmissometer of LISST (c_{670}) shows a peak in the upper pycnocline (9.0 m⁻¹), a prominent peak at 20.1 dbar (21.9 m⁻¹), and a distinct secondary peak at 22 dbar (10.2 m⁻¹); below the pycnocline, c_{670} measurements reach background values (Fig. 2b).

Similar observations at Stn. KT, which characterizes the offshore area of the Danube delta, show almost constant temperature (19.74-19.78 °C) and salinity values (17.49-17.55) from surface down to 27 dbar. Between 27 and 35 dbar a pronounced pycnocline is developed, due to a severe temperature decrease (from 19.74 °C to 9.38 °C), despite a slight increase in salinity. The optical instruments in this case are all showing peaks between 30.4 and 33.8 dbar ($c_{660} = 0.59$ m⁻¹; $c_{470} = 0.39$ m⁻¹; $c_{670} = 6.4$ m⁻¹). Overall, at all stations occupied where a deep pycnocline was developed, both transmissometers and LISST were influenced by multiple light scattering - schlieren.

The main cause of schlieren is density gradient(s) (a function of temperature, salinity and pressure), which

have varying refractive indices, thereby causing light scattering (Styles, 2006). However, the different behavior of transmissometers in inhomogeneous transparent media is also dependent on the acceptance angle of the instrument (Boss *et al.*, 2009).

In order to assess the effect of stratification or vertical density gradient, the buoyancy frequency or Brunt-Väisälä frequency, N , was computed using Sea-Bird's data processing software according to the equation:

$$N = \sqrt{-\left(\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}\right)}$$

where g is the gravitational acceleration, ρ_0 is the average density at the time the station was occupied, and $\partial \rho / \partial z$ is the vertical density gradient. A buoyancy window of 1 dbar was used to calculate the frequency from the 0.1-dbar bin-averaged data.

In the case of the coastal station SGT2, maximum N was estimated to be 0.105 s⁻¹ at 20 dbar, whereas for the offshore station KT, maximum buoyancy frequency was 0.073 s⁻¹ at 30.5 dbar (Figs. 4a and 5a, respectively). The highest value recorded in the Danube delta area was 0.265 s⁻¹, at Stn. SGT3 at 2 dbar, followed by 0.257 s⁻¹ at Stn. RNT1 at 8.9 dbar. It is noteworthy that at Stn. SGT2, where N in the deeper pycnocline is amongst the highest of the data set, c_{660} and c_{470} do not exhibit pronounced peaks, compared to Stn. KT, which exhibits lower N , but visible peaks for c_{660} and c_{470} (Figs. 2b, 3b).

To calculate the refraction index of seawater, n , at 670 nm the empirical equation of Quan & Fry (1995) was used:

$$n(S, T, \lambda) = n_0 + (n_1 T + n_2 T^2) S + n_3 T^2 + \frac{n_4 + n_5 S + n_6 T}{\lambda} + \frac{n_7}{\lambda^2} + \frac{n_8}{\lambda^3}$$

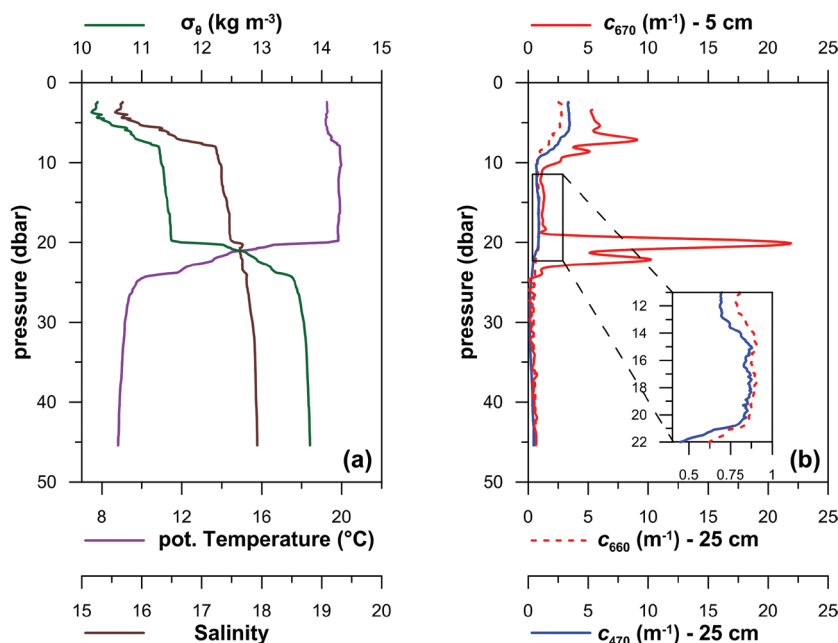


Fig. 2: (a) Water-column profile (Stn. SGT2) of salinity, pot. temperature and density (sigma-theta), showing the two pycnoclines characteristic of the coastal area surrounding the Danube delta; (b) beam transmission profiles of c_{660} (WET Labs C-Star, path-length 25 cm), c_{470} (Chelsea ALPHatracka MKII, path-length 25 cm), and c_{670} (LISST-Deep, path-length 5 cm).

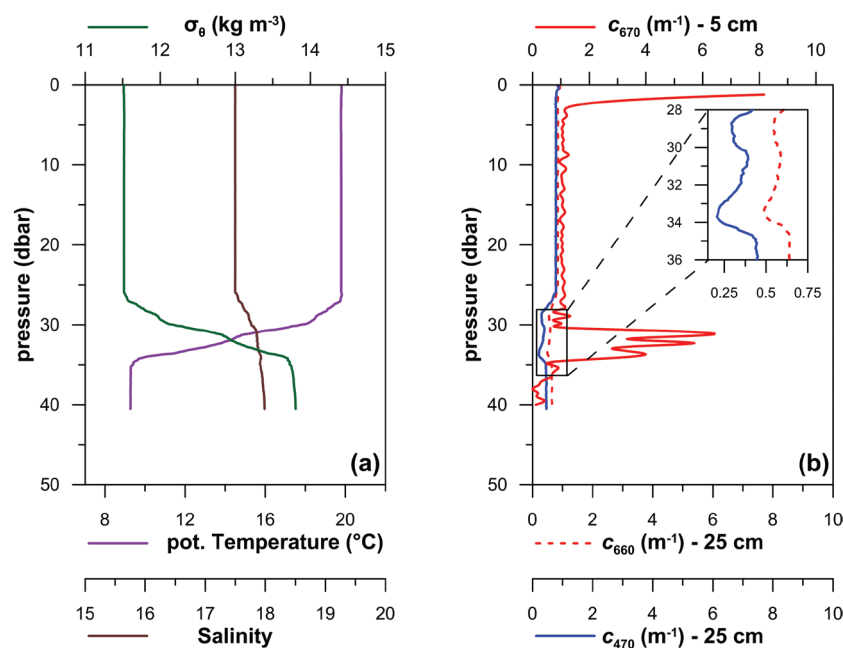


Fig. 3: (a) Water-column profile (Stn. KT) of salinity, pot. temperature and density (sigma-theta) showing the deep pycnocline, characteristic of the offshore area surrounding the Danube delta; (b) beam transmission profiles of c_{660} (WET Labs C-Star, path-length 25 cm), c_{470} (Chelsea ALPHATRACK MKII, path-length 25 cm), and c_{670} (LISST-Deep, path-length 5 cm).

where S is the salinity in ‰, T is the temperature in degrees Celsius and λ is the wavelength in nanometers.

The coefficients have the following values:

$$n_0 = 1.31405, n_1 = 1.779 \times 10^{-4}, n_2 = -1.05 \times 10^{-6}, n_3 = 1.6 \times 10^{-8}, n_4 = -2.02 \times 10^{-6}, n_5 = 15.868, n_6 = 0.01155, n_7 = -0.00423, n_8 = -4382, n_9 = 1.1455 \times 10^6.$$

At both stations SGT2 and KT, n exhibits small changes within the deeper pycnocline, from 1.334 to 1.335, but with notable vertical variability (Figs. 4a and 5a, respectively). It is possible that fluctuations of the refractive index of seawater are much higher (e.g., to mm-scale), but cannot be captured by typical CTD measurements. Small changes of the refractive index n should exist in parallel with turbulence at that depth to develop buoyancy frequency variations and consequently to trigger schlieren. According to Mikkelsen *et al.* (2008), the schlieren effect is a combination of the density gradient, the shear in the pycnocline, and the profiling speed of the instruments. Station SGT2 shows high c_{670} (21.9 m⁻¹) when compared to Stn. KT (6.4 m⁻¹), although N and index of refraction are virtually equal. This pattern can be explained by the presence of smaller particles, which are more affected by schlieren (Andrews *et al.*, 2011).

Schlieren side effects

Beam attenuation coefficient is not the only IOP affected by schlieren. LISST-Deep volume scattering function (V_{SF} ; a fundamental IOP that characterizes the intensity of scattering as a function of angle) measurements appear artificially increased in pycnoclines. Likewise, total volume concentration (VC) and median particle size (D_{50}) measurements obtained from the LISST are biased. At Stn. SGT2

below the upper pycnocline, VC varies between 2 and 3 $\mu\text{l l}^{-1}$, and rapidly increases to 221 $\mu\text{l l}^{-1}$ within the deeper pycnocline (Fig. 4b). Similarly, D_{50} varies between 20 and 30 μm , and then in the deeper pycnocline, and deeper it shows great variability (120–190 μm ; Fig. 4b). At Stn. KT, in the homogeneous upper part of the water column VC varies between 3 and 5 $\mu\text{l l}^{-1}$, D_{50} varies from 35 to 80 μm , and within the pycnocline fluctuates abruptly between 40 and 185 μm (Fig. 5b). Similar patterns, i.e. very high particle volume concentrations and large particle diameters, were observed at all stations where a pycnocline was developed, attributed to artificial particles due to schlieren.

Suggested N threshold for schlieren occurrence

Mikkelsen *et al.* (2008) found maximum values of N in the range 0.22–0.24 s⁻¹ in the strong pycnocline of the Hudson River that coincided with large variability of both temperature and salinity. The same authors, found N to be an order of magnitude lower (~ 0.03 s⁻¹) in Cardigan Bay in the Irish Sea, UK, and concluded that buoyancy frequencies exceeding 0.025 s⁻¹ could be considered as the threshold for schlieren influence. Apparently, N maxima recorded in the Black Sea are an order of magnitude higher than the suggested threshold, thus schlieren artifacts would certainly affect the measurements in pycnoclines. However, the Black Sea threshold values, where buoyancy frequency increases can be lower than 0.025 s⁻¹, (e.g. at Stns. SGT2 and RNT2), suggest that schlieren may be triggered at lower N than the values proposed in the literature. To further investigate this notion we calculated the first derivative of the index of refraction, n' , which expresses the rate of change of n over depth. A plot of

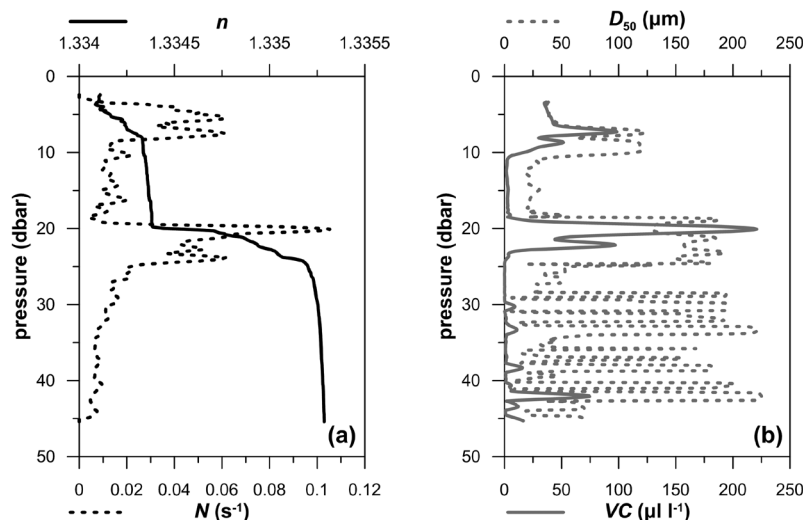


Fig. 4: (a) Water-column profile (Stn. SGT2) of the index of refraction of seawater (n) and Brunt-Väisälä frequency (N); (b) LISST-Deep profiles of particle median diameter D_{50} and particle volume concentration VC .

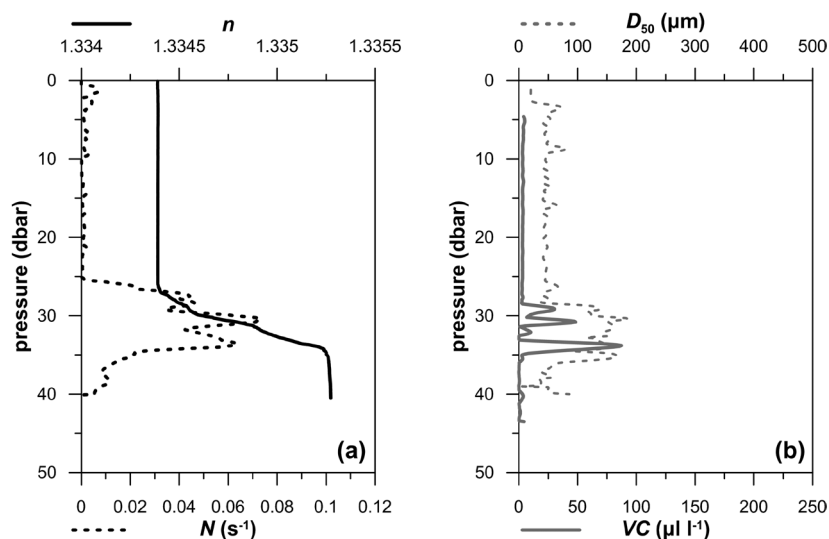


Fig. 5: (a) Water-column profile (Stn. KT) of the index of refraction of seawater (n) and Brunt-Väisälä frequency (N); (b) LISST-Deep profiles of particle median diameter D_{50} and particle volume concentration VC .

N against n' shows that variables are positively and significantly correlated ($r = 0.739$, no samples = 4550), suggesting that changes of n increase with stratification, thus enhancing schlieren occurrence (Fig. 6). Superimposing D_{50} values on the plot, we observe that values of $N > 0.025 s^{-1}$ (Mikkelsen *et al.*, 2008) correspond to large particle median diameters in the range 100-250 μm , which we attribute to schlieren. Those high values correspond to almost 20% of the particle-size data collected during the cruise. Another 7% of the data with D_{50} values greater than 100 μm fall within the range $0.025 < N < 0.01 s^{-1}$ and are also suspect of schlieren artifacts. These findings support that a new threshold value of buoyancy frequency N of $0.01 s^{-1}$ should be considered when interpreting LISST measurements obtained in high density gradients, such as river deltas, open-sea frontal boundaries or lakes. In such cases, the LISST artifacts cannot be avoided, but they can be assessed and removed manually after the estimation of N , and comparison with threshold values.

The discrepancy observed here between our results and those of Mikkelsen *et al.* (2008) could be related to the profiling package itself; in a stratified environment there are three plausible factors that will influence the degree to which schlieren will affect the measurements: (i) Richardson gradient (i.e., velocity differences across the pycnocline, which by itself would cause turbulence); (ii) profiling speed - fast profiling would cause more turbulence than slow profiling; and (iii) profiling package design - a bulky package would cause more turbulence than a sleek and slim package. It could be that the single-most important factor in creating schlieren in stratified environments is the profiling speed and/or the design of the profiling package, i.e. how much it disturbs the water (thus causing turbulence and schlieren) as it is descending. It is evident that Mikkelsen *et al.* (2008) did not observe schlieren at buoyancy frequencies below $0.025 s^{-1}$, whereas here the effects were observed from around $0.01 s^{-1}$. Conceivably, one could minimize the effect of schlie-

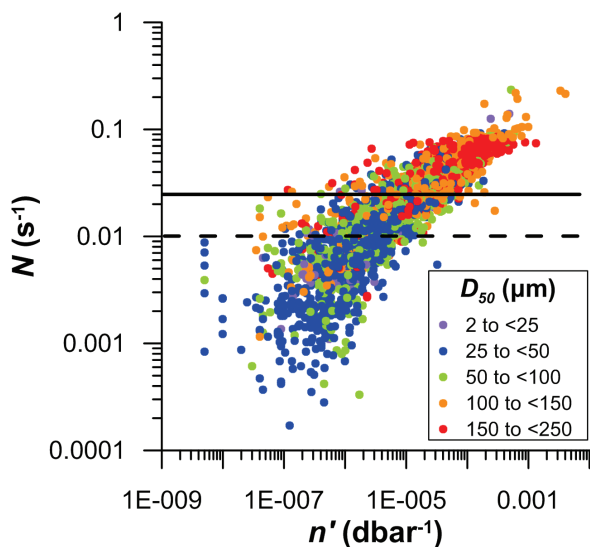


Fig. 6: Log-log plot of buoyancy frequency (N) vs. the first derivative (n') of the index of refraction (n). Colored dots represent particle median diameter (D_{50}) classified in five bins. Solid black line: N threshold (0.025 s^{-1}) for schlieren occurrence after Mikkelsen *et al.* (2008) and dashed black line: suggested new N threshold (0.01 s^{-1}).

ren in stratified environments by lowering the profiling speed through the pycnocline, in particular when using bulky instrument packages like CTD's.

Transmissometer acceptance angles

Comparing the three transmissometers used in the Danube delta survey, it may be deduced that acceptance angles play a primary role in the distortion of light scattering in pycnoclines, in agreement with the findings of Mikkelsen *et al.* (2008) and Boss *et al.* (2009). The acceptance angles of WET Labs C-Star, Chelsea ALPHATRACKA MKII, and LISST-Deep transmissometers are 1.2° , 0.91° , and 0.0269° , respectively. In practice, if light is scattered between 0.0269° and $\sim 1^\circ$, it will not be detected by LISST-Deep, thus increasing c_{670} , but it will fall inside the acceptance angle of the other transmissometers, leaving c unaffected. Our measurements show that LISST's c_{670} is much more sensitive in stratified waters, with maximum values $>21 \text{ m}^{-1}$. The other two transmissometers were also influenced, but to a lesser extent. This is in agreement with Boss *et al.* (2009) who compared the performance of eight different commercial transmissometers; their mean attenuation values differed markedly and in a consistent way with instrument acceptance angle: smaller acceptance angles provided higher beam attenuation values.

Summary and Conclusions

Beam attenuation (c_{660} and c_{470}) measurements of two commercial transmissometers and the particle sizer LISST-Deep were evaluated in the light of 'schlieren' disturbances, caused by density discontinuities in waters off the Danube delta. Shallow coastal stations' hydrology appeared to be af-

fected mainly by the freshwater (low salinity creates the upper pycnocline) inputs from the Danube River, and offshore deeper stations were influenced by the seasonal, temperature-driven deeper pycnocline. The optical measurements of the two 25-cm path-length transmissometers were largely unaffected within the salinity-controlled pycnocline, but conversely they exhibited peaks within the deeper temperature-controlled pycnocline. LISST-Deep uses the principle of laser diffraction to provide measurements of particle size distribution using a 5-cm (670 nm) transmissometer. The optical signal from the latter instrument was substantially affected by schlieren within both types of pycnoclines: (a) c_{670} was an order of magnitude higher than c_{660} and c_{470} ; (b) particle volume concentration increased abruptly; and (c) an artificial abundance of large particles with $D_{50} > 100 \mu\text{m}$ was recorded. The transmissometer of LISST-Deep, which has a very small acceptance angle, recorded high beam attenuation values, whereas the other transmissometers with higher acceptance angles showed small, but identifiable deviations only in the deeper, seasonal pycnocline. The maximum buoyancy frequency N within the seasonal pycnocline was 0.105 s^{-1} , higher than the threshold for schlieren appearance according to the literature. The proposed new threshold of N is 0.01 s^{-1} , more than 2-fold lower than values suggested in the literature. In such stratified environments, schlieren artifacts can be reduced if the optical package has a slim, hydrodynamic design and the profiling speed is low. Care must be taken when interpreting data from transmissometers or LISST instruments in high density gradients where schlieren effects should be considered. Ideally, an underwater video profiler (Picheral *et al.*, 2010) or other floc cameras (e.g. Mikkelsen *et al.*, 2006) could provide useful information in order to resolve the issue of the presence of large particles in strong pycnoclines.

Acknowledgements

The study was funded by the European Commission, DG Research Sixth Framework Programme, Priority 6.3: Global Change and Ecosystems. Contract EU-036949, SESAME Integrated Project (Southern European Seas: Assessing and Modelling Ecosystem changes). Constructive comments of two anonymous reviewers are gratefully acknowledged.

References

- Andrews, S.W., Nover, D.M., Reuter, J.E., Schladow, S.G., 2011. Limitations of laser diffraction for measuring fine particles in oligotrophic systems: Pitfalls and potential solutions. *Water Resources Research*, 47, W05523, doi:10.1029/2010WR009837.
- Agrawal, Y.C., Pottsmith, H.C., 2000. Instruments for particle size and settling velocity observations in sediment transport. *Marine Geology*, 168, 89-114.
- Boss, E., Slade, W.H., Behrenfeld, M., Dall'Olmo, G., 2009. Acceptance angle effects on the beam attenuation in the ocean. *Optics Express*, 17, 3, 1535-1550.

- Karageorgis, A.P., Kourafalou, V.H., Anagnostou, C., Tsiaras, K.P., Raitos, D.E. *et al.*, 2009. River-induced particle distribution in the northwestern Black Sea (September 2002 and 2004). *Journal of Geophysical Research*, 114, C12003, doi:10.1029/2009JC005460.
- Karageorgis, A.P., Georgopoulos, D., Kanellopoulos, T.D., Mikkelsen, O.A., Pagou, K., *et al.*, 2012. Spatial and seasonal variability of particulate matter optical and size properties in the Eastern Mediterranean Sea. *Journal of Marine Systems*, 105–108, 123–134. doi:10.1016/j.jmarsys.2012.07.003.
- Karageorgis, A.P., Gardner, W.D., Mikkelsen, O.A., Georgopoulos, D., Ogston, A.S. *et al.*, 2014. Particle sources over the Danube River delta, Black Sea based on distribution, composition and size using optics, imaging and bulk analyses. *Journal of Marine Systems*, 131, 74–90. doi:10.1016/j.jmarsys.2013.11.013
- Karpen, V., Thomsen, L., Suess, E., 2004. A new ‘schlieren’ technique application for fluid flow visualization at cold seep sites. *Marine Geology*, 204, 145–159.
- Mikkelsen, O.A., Hill, P.S., Milligan, T.G., 2006. Single-grain, microfloc and macrofloc volume variations observed with a LISST-100 and a digital floc camera. *Journal of Sea Research*, 55, 87–102.
- Mikkelsen, O.E., Milligan, T.G., Hill, P.S., Chant, R.J., Jago, C.F. *et al.*, 2008. The influence of schlieren on in situ optical measurements used for particle characterization. *Limnology and Oceanography: Methods*, 6, 133–143.
- Panin, N., 1999. Danube Delta: geology, sedimentology, evolution. *Publications Association des Sédimentologues Français*, 29, 1–63., Paris.
- Picheral, M., Guidi, L., Stemmann, L., Karl, D.M., Iddaoud, G. *et al.*, 2010. The Underwater Vision Profiler 5: An advanced instrument for high spatial resolution studies of particle size spectra and zooplankton. *Limnology and Oceanography: Methods*, 8, 462–473.
- Quan, X., Fry, E.S., 1995. Empirical equation for the index of refraction of seawater. *Applied Optics*, 34, 18, 3477–3480.
- Schardin, H., 1942. Die ‘schlieren’ verfahren und ihre Anwendungen. *Ergebnisse Der Exakten Naturwissenschaften*, 20, 304–405.
- Styles, R. 2006. Laboratory evaluation of the LISST in a stratified fluid. *Marine Geology*, 225, 151–162.
- Töpler, A., 1867. Über die Methode der ‘schlieren’ beobachtung als mikroskopisches Hilfsmittel, nebst Bemerkungen zur Theorie der schiefen Beleuchtung. Poggendorf’s *Annalen der Physik und Chemie*, 127, 556–580.