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HELLENIC MATHEMATICAL SOCIETY

### *International Journal for Mathematics in Education*

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VOLUME 4, 2012  
SPECIAL ISSUE

Commission Internationale pour l'Etude et l'Amélioration de  
l'Enseignement des Mathématiques

**Mathematics Education and Democracy:**  
*Learning and teaching Practices*  
**Education en Mathématiques et Démocratie:**  
*Les pratiques d'enseignement et d'apprentissage*



CIEAEM 64 Rhodes-Greece 2012  
Proceedings - Actes

Editors  
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## GREEK MATHEMATICAL SOCIETY

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The Hellenic Mathematical Society (HMS) decided to add this Journal, the seventh one, in the quite long list of its publications, covering all aspects of the mathematical experience. The primary mission of the HMS International Journal for Mathematics Education (*HMS i JME*) is to provide a forum for communicating novel ideas and research results in all areas of Mathematics Education with reference to all educational levels.

The proposals must be written almost exclusively in English but may be admitted, if necessary, also in French, in German or perhaps in Spanish.

The proposals could concern: research in didactics of mathematics, reports of new developments in mathematics curricula, integration of new technologies into mathematics education, network environments and focused learning populations, description of innovative experimental teaching approaches illustrating new ideas of general interest, trends in teachers' education, design of mathematical activities and educational materials, research results and new approaches for the learning of mathematics.

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**NOTES FOR AUTHORS**

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**Hellenic Mathematical Society**  
**International Journal for Mathematics in Education**

*Editors' Introduction*

More than a century after the list of problems to be solved by *David Hilbert* and almost half a century after the international reform attempt of the *modern mathematics*, the international landscape of the mathematics as of their teaching are different, both richer and more diversified: the unity of mathematics is no longer related to the study of the foundations, the power of mathematics is more indisputable than ever, the research for *links between the evolution of the mathematics, of the brain and of the human civilizations* leads to extraordinary educational results.

The study of the *phenomenology* of learning mathematics, the *interdisciplinary* research on the science of education as well as the *technological impact* on the representations of the reality have made the field of math education a new field of cognition, of research and of engineering, very important and particular in his complexity.

Greece, as the birthplace of the science of the mathematics, of the philosophy, of the theatre, of the democracy, but also of the bonds between them in the republic, has often been chosen to organise international meetings in math education. We note, indicatively, the historical international meeting (*OCDE, Athens*) for the teaching of modern mathematics in 1963 (1), more recently the Conference (*PME-33, Thessaloniki*) on the theoretical frame state of the research in math education in 2009 (2) and this year the international meeting (*CIEAEM-64, Rhodes, 2012*) on the links between teaching mathematics and democratic education (3).

The *Hellenic Mathematical Society* since its foundation in 1918 participates continuously in this international scientific, pedagogical and technological evolution of the math education. It systematically organizes conferences, training activities, competitions and Mathematics Olympiads. It publishes three pedagogical magazines for students, three scientific revues for its members, for the teachers' training and for all those who love mathematics,

it is the official editor of two international journals, the *Bulletin* and the *Journal for Mathematics in Education (JME)*.

The decision for the publication of the Proceedings of the International Conference CIEAEM-64 in this special issue of JME is a choice joining three dimensions.

The first is related to the recognition of the important role of this *International Commission for the Study and Improvement of Mathematics Education*. Since its foundation by eminent mathematicians, psychologists and pedagogues in 1950, date that a new age has started in Europe after the barbarism of the Nazis and the War, the meetings and the *cieaem's* documents have always had an important influence in the research, the innovation and the practice of teaching mathematics.

The second dimension is related to the importance that the *Hellenic Mathematic Society* brings in the *reinforcement of teaching mathematics in all academic degrees*, from kindergarten to high school included the cycle of higher education. In this same framework the *JME* makes a great effort to build consistent and creative links among international researchers and to encourage the comparative studies and the publication of joint reports.

The third dimension of this choice is related to *the particular theme of this cieaem-64 conference*. We live a financial crisis which is connecting structural and humanistic questions, related with the democratic local/global organisation of our world.

Just like in the modeling of the environmental crisis, there is no great difference if its description is oriented from the local to the global (Greece, Europe and planet) or in the opposite direction.

The important is to understand that there is a systemic crisis that puts in question the ability of our mind and of our societies to conceive the interactions, to formulate the unpredictable, to predict the uncertain, to think in the paradox, to include the dispersion and the infinite in the concepts of periodicity and recursion, to search for new types of order in the phenomenology of the disorder. All this conduce to mathematics concepts!

This crisis emerge the need to think with more complexity and to act with more responsibility. It is urgent to connect the math education with the democratic concern and the universal humanitarian values, to find meaningful tools and teaching practices that result to this connection in an explicit way, in order to construct an educational system which is innovative, flexible and adaptable to the variety of the concrete situations.

Mathematics has faced historical ruptures and crises however the mathematical community has proved the capability to surpass the antinomies, not by using interdictions but by creating new concepts and integrating them in richer new theories.

The democratic concepts of the city and the republic have traversed the history and the variety of human societies and they have shown resilience and an extraordinary capacity of adaptation, leading us to the axiom “democracy knows no dead ends”.

The conjecture of this volume is that it is possible to improve mathematics education by joining the teaching practices with the principles of the democratic organization, of the liberty, of the respect and the social integration.

The editors

- Sonia Kafoussi, Chair of the International Programme Committee for CIEAEM64
- Chrysanthi Skoumpourdi, President of the Local Organization Committee CIEAEM64
- François Kalavasis, Editor in Chief of the *International Journal for Mathematics in Education*



## ***Editors' Introduction***

Plus d'un siècle après la liste des problèmes à résoudre de *Hilbert* et près d'un demi siècle après la tentative internationale de la réforme des *mathématiques modernes*, le paysage international actuel des mathématiques et de leur enseignement est différent, plus riche et plus diversifié. L'unité des mathématiques ne se pose plus au niveau des fondements, la puissance des mathématiques est plus incontestable que jamais, la recherche des liaisons entre les mathématiques, le cerveau et les civilisations humaines conduit à des résultats pédagogiques extraordinaires.

L'étude de la phénoménologie de l'apprentissage mathématique, les recherches interdisciplinaires sur les sciences de l'éducation ainsi que l'impact technologique sur les représentations de la réalité ont fait du domaine de l'enseignement des mathématiques un nouveau domaine de recherche, d'ingénierie et de cognition, de plus en plus important.

Grèce, lieu natal de la science mathématique et de la philosophie, du théâtre, de la démocratie mais aussi des liens entre eux dans la république, elle a été souvent choisie pour l'organisation des rencontres internationales sur l'éducation mathématique. Nous pouvons indiquer la fameuse rencontre internationale de 1963 (1) sur l'enseignement des mathématiques modernes (*OCDE, Athènes*), plus récemment en 2009 (*PME-33, Salonique*) la conférence sur la recherche de théories cohérentes d'éducation mathématique (2) et cet été 2012 (*CIEAEM-64, Rhodes*) la rencontre internationale sur les liaisons entre l'enseignement mathématique et la démocratisation de l'éducation (3).

La *Société Mathématique Hellénique* depuis sa fondation en 1918 participe de façon continue à cette évolution scientifique, pédagogique et technologique de l'éducation mathématique. Elle organise systématiquement des conférences, des activités de formation, des concours et des Olympiades Mathématiques. Elle édite trois magazines pédagogiques pour les élèves, trois revues scientifiques pour ses membres, pour la formation des enseignants et pour toutes celles et tous ceux qui aiment les mathématiques, enfin elle est l'éditeur officiel de deux journaux internationaux de recherche, le *Bulletin* et le *Journal for Mathematics in Education (JME)*.

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La décision de la publication des Actes de la conférence internationale CIEAEM-64 dans ce numéro spécial de *JME* est un choix à trois dimensions.

La première est liée à la considération du rôle important de *la Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques*. Depuis la fondation de cette Commission en 1950 par des éminents mathématiciens, psychologues et pédagogues, dans une Europe qui reprenait sa voie démocratique après les barbaries nazies et la seconde guerre mondiale, les rencontres et les documents de la *cieaem* ont une influence importante à la recherche, aux innovations et dans les pratiques de l'enseignement mathématique.

La seconde dimension est liée à l'importance qu'apporte la *Société Mathématique Hellénique* au renforcement de l'enseignement des mathématiques dans tous les degrés scolaires, de la maternelle jusqu'au lycée et au cycle des études supérieures. Dans ce même cadre le *JME* fait un grand effort de construire des liens internationaux consistants et créatifs entre les chercheurs, d'encourager les études comparatives et la publication des rapports communs.

La troisième dimension de ce choix est liée *au thème particulier de cette conférence cieaem-64*. Nous vivons une crise financière joignant des questions structurales et humanitaires, liée aux problèmes de l'organisation locale/globale de la démocratie. Comme c'est le cas des crises environnementales, peu importe si la description du modèle est orientée du local vers le global (Grèce, Europe, planète) ou au sens inverse.

L'important est de se rendre compte qu'il s'agit d'une crise systémique qui met en question la capacité de notre esprit et des nos sociétés de concevoir les interactions, de formuler l'imprévisible, de prévoir l'incertain, de raisonner dans le paradoxe, d'inclure la dispersion et l'infini dans les concepts de périodicité et de récursivité, de chercher des nouveaux types d'ordre dans la phénoménologie du désordre. Tout cela conduit aux concepts mathématiques !

De cette crise émerge le besoin de réflexion plus complexe et d'action plus responsable. Il est urgent de faire connecter l'éducation mathématique avec le souci démocratique et les valeurs humanistes universelles, de trouver les moyens et les pratiques d'enseignement qui aboutissent à cette connexion de

façon explicite, pour construire un système éducatif innovant, flexible et adaptable à la variété des situations concrètes.


La communauté mathématique a connu des crises et des ruptures épistémologiques mais elle a fait preuve d'une capacité de surmonter les antinomies, non pas par l'interdiction mais par la construction de nouveaux concepts et par l'intégration des contradictions dans des nouvelles théories plus riches. Les concepts démocratiques de la cité et de la république ont parcouru l'histoire et la variété des sociétés humaines et ils ont montré une résilience et une extraordinaire capacité d'adaptation, nous conduisant à l'axiome que «la démocratie ne connaît pas d'impasses».

La conjecture de ce volume est qu'il est possible d'améliorer l'enseignement des mathématiques en le joignant aux principes de l'organisation démocratique, de la liberté, du respect et de l'intégration sociale.

The editors

- Sonia Kafoussi, Chair of the International Programme Committee for CIEAEM64
- Chrysanthi Skoumpourdi, President of the Local Organization Committee
- Francois Kalavasis, Editor in Chief of the International Journal for Mathematics in Education

- (1) Howard.F.Fehr, André Revuz, *Mathématiques modernes: Guide pour enseignants. Rapport d'une session d'étude internationale sur les nouvelles méthodes d'enseignement des mathématiques*. Athènes, 1963, Edition OCDE, Paris, 1964.
- (2) PME 33, *33rd Conference of the International Group for the Psychology of Mathematics Education, «In Search for Theories in Mathematics Education»*, Thessaloniki, 2009, Eds. Marianna Tzekaki, Maria Kaldrimidou, Haralambos Sakonidis
- (3) CIEAEM 64, «*Mathematics Education and Democracy: learning and teaching practices*», Rhodes, Greece 2012

 <p>COMMISSION INTERNATIONALE POUR L'ÉTUDE ET L'AMÉLIORATION DE L'ENSEIGNEMENT DES MATHÉMATIQUES</p> <p>INTERNATIONAL COMMISSION FOR STUDY AND IMPROVEMENT OF MATHEMATICS EDUCATION</p>	<p><b>CIEAEM 64</b></p> <p>University of the Aegean Faculty of Humanities Department of Sciences of Preschool Education and of Educational Design <b>Rhodes, Greece, 23-27 July 2012</b> <b>Rhodes, Grece, 23-27 Juillet 2012</b></p>
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***Proceedings of the CIEAEM64-Actes de la CIEAEM64***

***Mathematics Education and Democracy:  
learning and teaching practices***

***Education en Mathématiques et Démocratie:  
les pratiques d'enseignement et d'apprentissage***

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## ***DISCUSSION PAPER***

### **Mathematics Education and Democracy: learning and teaching practices**

Our theme for 2012 continues and extends recent CIEAEM conference topics (1979 “Mathematics for all and for everyone”, 1985 “Mathematics for all ... in the computer age”, 1999 “Cultural diversity and Mathematics Education”, 2005 “Changes in the society: a challenge for mathematics education”). The theme asks us, furthermore, to revisit the manifesto (2000) for 50 years of CIEAEM, in particular with respect to globalization and the interaction of mathematics education and democracy.

Democracy as a goal to strive for can be defined as an ideal form of social organization that establishes a free and equal practice of political self-determination including values, norms and behaviors that promise an optimal combination of economic, cultural and institutional structures for a whole population (Valero, 1999). An international goal of mathematics education still remains to overcome the history of limited opportunities for some populations (marginalized groups), and democracy’s promise of equal participation by all members of society suggests a variety of possibilities for mathematics education in supporting the goals of democracy itself. Moreover, the challenge about the role of mathematics education for a critical citizen today could be the focus of research. However, globalization determines in a new way the notion of democracy in society, raising questions about the complexity of national political systems and their relationship with global, economic forces, challenging local assumptions about culture, identity, and the importance of specific mathematical skills for a given curriculum, and even requiring any study of democracy to include a self-critique of the cultural assumptions and traditions that inform our working hypotheses about the potential and limitations of democracy as an ideal form of social organization.

#### *1. Democracy in mathematics curriculum: How does school mathematics contribute to critical thinking and decision-making in the society?*

In general, the mathematics curricula which exist in the countries of the world appear to be remarkably similar. Whether these similarities exist by reasoned choice or are a result of various waves of cultural imperialism is not clear, but they certainly do not appear to reflect any differences in socio-cultural context (Bishop, 2009). Within the democratic classroom students should see themselves in the curriculum and link mathematics to their everyday lives (Malloy, 2002).

Moreover, students are facing a world shaped by increasingly complex, dynamic and powerful systems of information and ideas (English, 2002). Mathematics curricula must broaden their goals to include concepts and processes that will maximize all students’

opportunities for success in society. Students should have the opportunity to think critically about world issues and their environment through mathematics.

- What are the values of a democratic mathematics curriculum?
- What are the goals and the contents of a democratic mathematics curriculum?
- What is the nature of the relationship between mathematical knowledge of the workplace settings and in life contexts in general and school mathematics?
- How can the new technologies help the design of a democratic mathematics curriculum?

2. *Democracy in mathematics classroom practices: What kind of mathematics classroom practices would enact a particular set of humanitarian values (e.g. social justice, respect and dignity)?*

Many researchers have developed the notion of “democratic access” to mathematical ideas by the learners. This notion is related to:

- the creation of the appropriate learning conditions where all the students develop the ability to solve and understand increasingly challenging problems,
- students’ mathematical knowledge, skills and understanding in order to cope with problems in our society and to shape their personal futures. Students have to become critical thinkers and decision makers in the classroom setting, but also in their life conditions.

Moreover, democracy in mathematics classroom practices designates the possibility of entering a kind of mathematics education that contributes to the consolidation of democratic social relations (Skovsmose & Valero, 2002). The mathematics classroom is a micro society in which democratic relationships among their members can be established during their communication. Research in contemporary multicultural classrooms has revealed many factors that influence learning and teaching of mathematics.

-What kinds of learning environments are needed to promote a democratic access to mathematical ideas for all students?

- What is the nature of the critical process into which learners must be initiated?
- What are the forms of a democratic discourse in the mathematics classroom?
- What values are students expected to learn?
- What are the criteria for assessment?

3. *Democracy in mathematics teacher education*

Mathematics teachers should provide students with a mathematics education that will serve them throughout their lifetimes. In doing so, they have to be acting subjects in identifying, planning and implementing the curriculum. Teachers have to be sensitive not only about the mathematical content, but also about the development of democratic social relationships among the members of the classroom.

Teachers, mathematics educators, researchers, and policy-makers are all participants in the systems of schooling. Interaction among them is necessary for the improvement of mathematics teaching and learning. Moreover, teachers’ development of mathematics

teaching is most effective when it takes place in a supportive community of inquiry through which knowledge can be developed and evaluated critically (Jaworski, 2003).

- What understandings, values and strategies do teachers need in order to promote democracy in mathematics classroom?
- What are the characteristics of a (pre-service or in-service) teacher education program for democracy in mathematics classroom?
- How does collaboration among policy-makers, mathematics educators, researchers and teachers support this goal?

#### *4. Democracy in research on mathematics education*

In mathematics education different actions and different concerns have produced different theories; over the past decades a number of research methodologies have been developed, and other, more innovative methods continue to be developed as we work. Consistent with a democratic orientation, this variety is inevitably enriched by the contributions and participation of differently-oriented research, realized in different societies where the influence of contrasting societal norms can be significant.

- How can mathematics educators more effectively act as “public Intellectuals” within a democracy, enabling a democratic public education about and with mathematics education in the public sphere?
- What are the criteria for a critical assessment of research theories and methodologies?
- How does a ‘local’ theory affect the researchers’ theory building?
- How can a democracy enact a democratic discussion about mathematics education issues that is informed by research and practice?

## **DISCUSSION DU THÈME**

### **Éducation en mathématiques et démocratie: les pratiques d’apprentissage et d’enseignement**

Notre thème pour 2012 développe les récents sujets abordés par les conférences CIEAEM (1979 “Mathématiques pour tous et chacun”, 1985 “Mathématiques pour tous ... dans l’ère de l’ordinateur”, 1999 “Diversité culturelle et éducation en mathématiques”, 2005 “Changements dans la société: un défi pour l’éducation en mathématiques”). Le thème nous demande de revisiter le manifeste (2000) qui a été publié pour les 50 ans de la CIEAEM, en particulier par rapport à la globalisation et l’interaction de l’éducation en mathématiques et de la démocratie.

On peut définir la démocratie, selon Valero (1999), comme un but à atteindre, une forme idéale d’organisation sociale qui établit une pratique libre et égalitaire de l’auto-détermination politique incluant les valeurs, les normes et les comportements qui promettent une combinaison optimale des structures économiques, culturelles et institutionnelles pour toute une population. L’éducation en mathématiques a pour but international de surmonter une série d’opportunités limitées pour certaines populations (groupes marginalisés) : or la promesse de la démocratie d’une participation égale par tous les membres de la société suggère diverses possibilités d’éducation en mathématiques. D’ailleurs, le défi du rôle de l’éducation en mathématiques pour former l’esprit critique du



citoyen d'aujourd'hui pourrait constituer un foyer de recherche. La globalisation détermine différemment la notion de démocratie en société, soulevant des questions au sujet de la complexité des systèmes politiques nationaux et leur relation avec les forces économiques globales, remettant en question les idées locales au sujet de la culture, de l'identité et de l'importance de certaines habiletés mathématiques pour un curriculum donné. La globalisation exige même que toute étude de la démocratie comporte une autocritique des notions et traditions qui informent nos hypothèses de travail sur le potentiel et les limites de la démocratie comme forme idéale d'organisation sociale.

*1. Curriculum de la démocratie en mathématiques: Comment les mathématiques à l'école contribuent-elles à la pensée critique et à la prise de décision en société?*

Les curricula en mathématiques sont remarquablement similaires à travers le monde. Que les ressemblances soient le fait de choix conscients ou résultent de vagues successives d'impérialisme culturel n'est pas clair mais elles ne semblent pas refléter les différences de contexte socioculturel (Bishop, 2009). Pourtant, les étudiants devraient se reconnaître dans les curricula suivis en classe démocratique et faire le lien entre les mathématiques et leur quotidien (Malloy, 2002).

De plus, les étudiants font face à un monde façonné par des systèmes d'information et d'idées de plus en plus complexes, dynamiques et puissants (English, 2002). Les curricula en mathématiques doivent élargir leurs buts en incluant des concepts et processus qui maximiseront, pour tous les étudiants, leurs opportunités de réussite sociale. Ils devraient avoir l'opportunité de développer leur esprit critique au sujet des questions mondiales et de leur environnement à travers les mathématiques.

- Quelles valeurs un curriculum démocratique en mathématiques doit-il proposer?
- Quels en sont les buts et les contenus?
- Quelle est la nature de la relation entre la connaissance mathématique des environnements de travail et de vie en général et les mathématiques en classe?
- Comment mettre les nouvelles technologies au service d'un curriculum démocratique en mathématiques?

*2. Démocratie dans les pratiques mathématiques en classes: quelles pratiques permettraient de vivre un ensemble particulier de valeurs humanitaires (ie justice sociale, respect et dignité)?*

Plusieurs chercheurs ont développé la notion "d'accessibilité démocratique" aux notions mathématiques par les apprenants. Cette notion est reliée à:

- La création de conditions d'apprentissage appropriées où tous les étudiants peuvent développer l'habileté à comprendre et résoudre des problèmes complexes.
- Aux connaissances mathématiques, habiletés et compréhensions en vue de faire face aux problèmes de nos sociétés et de préparer leur avenir personnel. Les étudiants doivent devenir des penseurs critiques et des décideurs en contexte de classe certes, mais aussi dans leurs contextes de vie.

La démocratie dans les pratiques mathématiques en classe ouvre la possibilité de commencer une éducation mathématique qui contribue à la consolidation de relations

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sociales démocratiques (Skovsmose & Valero, 2002). La classe constituera ainsi une microsociété où les communications feront naître des relations démocratiques. La recherche sur les classes multiculturelles contemporaines révèle plusieurs facteurs qui influencent l'apprentissage et l'enseignement des mathématiques.

- Quelles sortes d'environnements d'apprentissage pourraient promouvoir un accès démocratique aux idées mathématiques pour tous les étudiants?
- Quelle est la nature du processus de pensée critique auquel les apprenants doivent s'initier?
- Quelles sont les formes d'un discours démocratique en classe de mathématiques?
- Quelles sont les valeurs que les étudiants doivent apprendre?
- Quels sont les critères d'évaluation?

### *3. Démocratie en formation des enseignants en mathématiques*

Les enseignants en mathématiques devraient fournir une éducation en mathématiques qui servira les étudiants durant toute leur vie et, ce faisant, être actifs dans l'identification, la planification et la mise en œuvre du curriculum. Ils seront bien entendu sensibles aux contenus mathématiques mais aussi au développement de relations sociales démocratiques entre les étudiants en classe.

Les enseignants, les éducateurs en mathématiques, les chercheurs et les formulateurs de politiques sont tous des participants dans le système scolaire et l'interaction entre eux est nécessaire à l'amélioration de l'enseignement et de l'apprentissage des mathématiques. Le développement de l'enseignement mathématique est plus efficace lorsqu'il a lieu dans une communauté de support à la recherche qui développe et critique les connaissances (Jaworski, 2003).

- Quelles compréhensions, valeurs et stratégies requièrent les enseignants afin de promouvoir la démocratie dans la classe de mathématiques?
- Quelles sont les caractéristiques d'un programme d'éducation des enseignants (avant et pendant la classe) prônant la démocratie en classe de mathématiques?
- Comment la collaboration entre formulateurs de politiques, éducateurs en mathématiques, chercheurs et enseignants peut-elle supporter ce but?

### *4. Démocratie en recherche en éducation mathématique*

Différentes actions et préoccupations ont donné naissance à diverses théories en éducation mathématique; au cours des dernières décennies un nombre de méthodologies de recherche ont été développées et d'autres méthodes plus innovatrices sont en développement. En accord avec une orientation démocratique, cette variété s'enrichit de contributions de recherche aux orientations différentes, réalisées dans des sociétés où l'influence de normes sociétales contrastantes peut être significative.

- Comment les éducateurs en mathématiques peuvent-ils agir davantage comme "intellectuels publics" à l'intérieur d'une démocratie, favorisant une éducation publique en mathématiques démocratique et centrée dans la sphère publique?
- Quels sont les critères pour une évaluation critique des théories de recherche et des méthodologies?
- Comment une théorie "locale" affecte-t-elle l'élaboration de théories des chercheurs?

-Comment une démocratie peut-elle mettre en oeuvre une discussion démocratique au sujet des questions d'éducation en mathématiques qui soit informée par la recherche et la pratique?

**REFERENCES**

- Bishop, A. (2009). Researching family mathematical practices: background research and future challenges. *3<sup>rd</sup> Conference of CARME*, 43-53, Rhodes.
- English, L. (2002). Priority themes and issues in international research in Mathematics Education. In L. English (Ed.), *International Research in Mathematics Education* (pp. 3-15). LEA
- Jaworski, B. (1993). Research practice into influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics*, 54, 249-282.
- Malloy, C. (2002). Democratic access to mathematics through democratic education: an introduction. In L. English (Ed.), *International Research in Mathematics Education* (pp. 17-25). LEA
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (Ed.), *International Research in Mathematics Education* (pp. 383-408). LEA
- Valero, P. (1999). Deliberative Mathematics Education for Social Democratization in Latin America. *ZDM*, 1, 20-16.

## Pleenary Speakers

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### Mathematics Education and Democracy: An on-going challenge

Ole Skovsmose and Miriam Godoy Penteado

**Abstract:** Different groups of challenges can be associated to a mathematics education for democracy, and we address challenges related to: (1) *Mathematics*. What roles might mathematics be playing in society, technology, and daily-life practice? (2) *Mathematics education*. What kind of disciplining might be exercised through this education? What alternatives are possible? (3) *Foregrounds, meaning and possibilities*. How to characterize students' foregrounds? What could be considered meaningful mathematics education? What could a construction of possibilities mean?

What a mathematics education for democracy could mean is an open question without any specific answers in sight. Thus, we always have to keep in mind that “democracy” is an explosive concept.

*Key-Words:* mathematics education for democracy, school mathematics tradition, prescription readiness, student's foreground, meaning, possibility.

#### 1. Introduction

The discussion about mathematics education and democracy is old.<sup>1</sup> However, it is continuously transformed into new discussions, as interpretations of and conditions for democracy are ever changing.

One can characterise “democracy” as an explosive concept.<sup>2</sup> By this we understand a concept that can be defined only through concepts just as open

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<sup>1</sup> See, for instance, Skovsmose (1990).

<sup>2</sup> The notion of explosive concept has been described in Skovsmose (2005).

and vague as itself. Thus, democracy is related to equity, justice, and inclusion. We will not try to provide any defining clarification of democracy. But we will use the notion, recollecting Wittgenstein's observation in *Philosophical Investigations*, that the meaning of a notion is its use. So we hope to bring some meaning to "democracy" by using it.

Democratic ideals were expressed in Antiquity in Greece, and since then they have been ignored, forgotten, revived and transformed. They have been incorporated in parliamentary systems. They have been formulated as part of liberal political thinking. They have inspired the formulations of very different constitutions, the fundamental principles according to which a state is governed.

Accompanying the formulation of democratic ideals, one also finds expressed the counter idea that democracy is only for somebody. Thus, in 1910 in the English Parliament it was explicitly stated by Arthur James Balfour (Prime Minister 1902-1905 and Foreign Secretary 1916-1919) in a debate addressing Egypt that democracy was not for the "Orientals".<sup>3</sup> As a consequence, Balfour could present as an obligation for England to rule all these many people not apt for democratic systems. The democracy-is-not-for-them argument has been applied again and again. It has been basic to the apartheid regime in South Africa: Blacks had to be excluded. The same argument barred woman from participating in democracy for long periods, thus women got the right to vote in United Kingdom in 1928, in France 1944, in Switzerland in 1971, while they in Saudi Arabia are promised to be allowed to vote in 2015.

Democratic ideals also concerns institutions, organisations and work places. And also here one meets the democracy-is-not-for-them argument. Thus it is often claimed that institutions like the military, church, hospitals cannot be based on democratic principles. But what about the educational system? For a long period democracy was not considered relevant for this system. But it has been contested, for instance by the so-called 1968 students' movement. In fact the democracy-is-not-for-them argument is continuously being challenged, for instance through many non-governmental movements and, recently, through the Arabic Spring.

Democracy also functions as a useful label, and even the most brutal form of dictatorships has decorated themselves with the notion of democracy. Thus the official name of East Germany was *Deutsche Demokratische Republik* (*German Democratic Republic*). It should also be noted that even

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<sup>3</sup> See Said (1979, pp. 31 ff.)

if it is generally agreed that a political system, where women and blacks are not allowed to vote, can hardly be called a democracy one traditionally decorates our historical past and think of, say, England as being a democracy also before 1928.

Let us now concentrate on education. We will address different groups of challenges that can be associated to a mathematics education concerned about democratic ideals. We will address challenges related to:

1. *Mathematics*. What roles might mathematics be playing in society, technology, and daily-life practises?
2. *Mathematics education*. What kind of disciplining might be exercised through this education? What alternatives are possible?
3. *Foregrounds, meanings, and possibilities*. How to characterise students' foregrounds? What could be considered meaningful mathematics education? What could a construction of possibilities mean?

Through the discussion of foregrounds, meaning and possibilities we will get closer to characterise a mathematics education inspired by democratic ideals. However, let us take the issues in their proper order.

### **1. Mathematics**

We will characterise two different attitudes towards mathematics. The first we refer to as the *modern perspective on mathematics*.

It has been part of Modernity to celebrate mathematics, and this celebration includes different elements. First, it is emphasised that mathematics provides a unique insight in nature. This claim was crucial for the so-called scientific revolution, and it has accompanied the natural sciences ever since. In fact the mathematised natural sciences have been considered a scientific ideal that other sciences, as economy and medicine for instance, have to pursue. Thus mathematics has been nominated as the language of sciences, if not the rationality of science. Second, it has been emphasised that mathematics, through its broad range of applications represent the rationality of progress. Thirdly, mathematics has been celebrated as a pure science, which stands out due to its objectivity and neutrality.

The Modern celebration of mathematics is accompanied by the assumption that there exists an intrinsic harmony between mathematics and democratic ideals. In other words: by being an ambassador of mathematics one is also promoting the rationality of democracy. This assumption has been related to the observation that the axiomatic organisation of geometry

and the formulation of democratic ideals took place during the same period and in Antique Greece. The assumption claimed an intrinsic connection between the two ways of thinking.

One could contrast the modern perspective with a *critical perspective on mathematics*. According to this it is important not to address mathematics with any pre-set values, as the mathematical rationality can operate with all kinds of qualities. Mathematics represents a powerful rationality. It integrates knowledge and power. It constitutes a knowledge-power unity, which can be used for any purpose. As a consequence, mathematics needs to be critically examined, and not to be celebrated in general.

The critical outlook does not assume that there are any intrinsic harmony between democratic ideals and mathematical forms of thinking. Instead it is claimed that mathematics could come to serve many different political and socio-economic interests, also the most undemocratic types.

To explore further a critical perspective on mathematics it is important to consider the roles that mathematics does play in society. One finds mathematics brought in action in all different forms of technology. Here we interpret technology in the broadest possible way. Thus, we do not only refer to technological artefacts: washing machines, TV-sets, robots, electronic networks, etc. By technology we also refer to forms of automatisisation, procedures for financial transactions, forms of surveillance and control, etc. As technology, so all forms of daily practices become mathematised. Our life-worlds become continuously defined and re-defined through mathematics in action.<sup>4</sup>

Let us just refer to some of the domains where mathematics-based technological development has a tremendous impact on our life-worlds:

(1) One can consider a computer being a mathematical algorithm, materialised in an electronic format. In this sense a computer is a mathematical construct. We find such constructs in all forms of modern information and communication technology. All kinds of new equipment, like mobiles and tablets, are now flooding the market. Thus social practices become changed in huge scales due to implementations of mathematical constructs.

(2) Processes of production take new forms due to automatisisation. Any production process – of cars, airplanes, washing machines – is established

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<sup>4</sup> For a discussion of mathematics in action, see, for instance, Skovsmose (2009). For a discussion of processes of robotting and surveilling, see Skovsmose (2012a). See also Gellert and Jablonka (2009).

through a certain balance between automatic processes, technical management, and manual labour. However, this balance is always changing as new technology makes new forms of automatisisation possible. Such changes always reflect particular interests: The market conditions, the level of salaries, the possibility of exercising more control of the production process, etc.

(3) Processes of management and decision-making take new forms due to mathematics. As an example, one can think of the new conditions for analysing implications of not-yet implemented initiatives. Through more and more complex mathematical models, one tries to provide economic forecasting. This way new “necessities” become identified: “necessary economic actions”, “necessary reductions”, “necessary restrictions”, etc. Through mathematical models one identifies a range of necessities which frame our daily life and accompany us into the future.

(4) In medicine mathematics-based techniques play new crucial roles, both with respect to diagnostics and treatment. As one particular example, one can just think of the technologies used in cancer treatment. The machine through which radiotherapy is conducted is now a highly automatised instrument. All relevant parameters that are defining the treatment of a particular person become activated when she or he is treated. And more and more medical treatments are conducted through mathematical algorithms.

(5) All kind of advanced war technology is based on mathematics in action. As an example one can think of the “drones”, the unmanned aircrafts through which particular targets become attached. The possible targets become subjected to a taxonomy, including algorithmic decision-making of whether to fire or not to fire. This way the whole process of “targeting” becomes mathematised and automatised.

The hectic mathematics-based development of technology includes many tensions and conflicts. We do not have to do with any universal form of progress. This development brings about new forms of risks, exploitation, domination, control, surveillance, robotting. It brings about new forms of capitalising profits. There is no intrinsic connection between applications of mathematics and democratic forms of thinking.

There is no basis for a general celebration of mathematics. The modern perspective on mathematics has turned obsolete. Mathematical rationality has to be addressed critically in all its many instantiations. It is an important challenge for mathematics education to develop further a critical perspective on mathematics.



## 2. Mathematics education

Let us now consider what role mathematics education might be playing in society. One could assume that this education in general makes part of a democratic endeavour, and that there is an intrinsic connection between mathematics education and democratic thinking. One could claim as done by Hannaford (1998) that his “Mathematics Teaching *is* Democratic Education”.

Our point, however, is that one cannot make any such assumption. We find instead that mathematics education, as education in general, can be acted out with reference to any form of political interests. We do not find that mathematics education contain any democratic essence. Mathematics education operates as a socio-economic contingency: It could come to operate as a democratic force, but also as a disciplining device – to use a term explored by Foucault.<sup>5</sup>

Let us consider the school mathematics tradition. By this tradition we refer to some features of much mathematics education: (1) The activities in the classroom are first of all defined through the chosen textbook. The teacher makes exposition of a particular topic from the textbook, which defines the tasks for the students. This way, the “acted out” curriculum becomes defined through the textbook. (2) The mathematical exercises play a dominant role, as solving pre-formulated exercises is considered crucial for the learning of mathematics. These exercises demonstrate particular characteristics: All the information given is exact, and should not be questioned. It is necessary for solving the exercises, but no other information is relevant. The exercises have one and only one correct solution. (3) One important feature of the classroom practice is to eliminate errors. There are many possibilities of errors: An algorithm can be conducted wrongly; a wrong algorithm can be applied; an exercise can be copied wrongly from the textbook; the wrong exercise can be calculated, etc. All forms of errors have to be eliminated, as “doing things correctly” is considered equivalent to “learning mathematics”. (4) The students’ performances have to be evaluated. This can be done through the teacher’s questioning approach in the classroom; through the teacher’s control of the students’ solutions to exercises; and through different forms of tests. It can be completed through the exams by the end of the school year.

One feature of a disciplining through mathematics education might be captured by the notion of prescription readiness (see Skovsmose, 2008).

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<sup>5</sup> See, Foucault (1991).

Considering the *content* of the many exercises, defining for the school mathematics tradition, one can hardly claim that working with such exercises provide any deeper understanding of mathematics. However, one can pay attention, not to the content, but to the *form* of these exercises. They operate as a long sequence of commands: “Construct a triangle...!” “Solve the equation...!” “Calculate the distance between...!”, etc. Taking a command as given is a defining element of a logic of commands. Following a command accurately is another element. Furthermore, the implementation of a command has to be carefully controlled. In general, a logic of commands cultivates a prescription readiness.

However, a mathematics education concerned about democratic ideals needs to be associated to others approaches reaching beyond the school mathematics tradition. And there are in fact several approaches to be considered. One can think of responsive mathematics education which tries to make students able to respond to forms of domination as critical citizens (see for instance Greer, Mukhopadhyay, Powell, and Nelson-Barber (Eds.), 2009); one can think of mathematics education for social justice which put a particular attention to the notion of equity and inclusion (see, for instance, Sriraman, (Ed.), 2008); one can think of many ethnomathematical approaches as exemplifying an education concerned about democracy (see, for instance, D’Ambrosio, 2006); one can think of forms of inclusive education as we will return to later. More generally, one can consider all such approaches as representing the broad family of critical mathematics education (see, for instance, Alrø, Ravn and Valero (Eds.), 2010; and Skovsmose, 2011a).<sup>6</sup>

Furthermore we want to emphasise that the discussion of mathematics education and democracy cannot just concentrate on school education. Learning mathematics can take place in many different other contexts, and everywhere one can consider what it could mean to try to pursue democratic ideals.

### 3. Foregrounds, meaning and possibilities

Let us just summaries: There are no intrinsic connection between, one the one hand, mathematics and mathematics education, and, on the other hand, democratic aspirations. Possible relationships are contingent.

As a consequence, one cannot assume that mathematics education can acknowledge democratic ideals through well-defined methodologies.

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<sup>6</sup> See also Gutstein (2006); Appelbaum with Allan (2008); and Skovsmose (2011b, 2012b).

Relationships between mathematics education and democracy may be constructed in very different ways in different contexts.

In this section we will try to express aspirations of a mathematics education in terms of foregrounds, meanings and possibilities. In this way we will try to present the very open nature of the challenges.

### ***3.1 Foregrounds***

We see the notion of foreground as important for interpreting learning and for understanding what learning could mean when democracy is taken into consideration.<sup>7</sup>

The foreground of a person refers to opportunities which the social, political, economic and cultural situation provides for her or him. Naturally, the foreground is not composed of any well-defined space of possibilities. The statistics through which some opportunities in life become depicted only represents tendencies: some might appear almost deterministic, while others may be weaker likelihoods. Thus, we see a foreground as formed through propensities. However, a foreground is not only established by statistical parameters. It is also an experienced phenomenon. It represents the person's expectations, hopes, and frustrations. Thus, a foreground is formed through both social configured propensities and personal experiences and interpretations of possibilities and constrains. The foreground of a person is closely related to his or her background. However, there is no deterministic relationship between them. They are different entities.

One can think of foregrounds in individual terms, but it might be more appropriate to think of them in collective terms as the interpretation of possibilities is elaborated through interaction and communication. People from the same community can be submitted to the same overall propensities and somehow have similar foreground. Foregrounds are collective, but still including individual features.

When only desolate possibilities are experienced by a person or a group of persons, we talk about a ruined foreground. A foreground can be ruined through social, economic, political or cultural acts. During the apartheid regime in South Africa the destruction for black people's possibilities was an integral part of the system. Political suppression of different groups is a general destroyer of foregrounds. Poverty as well. However, also affluent environments could bring about ruined foregrounds: thus young people

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<sup>7</sup> For a presentation of the notion for foreground, see, for instance, Skovsmose (2011a, in print).

might experience a overwhelming indifference if they do not perceive opportunities that they could identify with. There are ranges of particular examples of foreground being configured through processes of social exclusion, thus Baber (2007) has explored foregrounds of immigrant students in Denmark.

Foregrounds form motives for learning. Thus students' aspirations and hopes provide energy to the learning process. Intentions for learning constitute important elements of the learning process; and intentions are directed towards the person's foreground. If, however, a foreground is ruined, there is not much there that intention could be directed towards. Thus a ruined foreground can turn out to become a most devastating learning obstacle. When considering achievements of different groups of students, one might observe differences; however we find it important not to personalise such possible differences, but instead to politicise them. It is crucial to consider how political and economic factors might ruin the foregrounds of different groups of students.

One important feature of an education concerned about democracy has to do with creating opportunities in life. It has to do with adding new elements to students' foregrounds. What this could mean depends on the context. Foregrounds are dynamic: elements can be added through education. Foregrounds include interpretations, and interpretations can change. A foreground is not just there spreading out in front of a person. It is an evolving phenomenon.

### **3.2 Meanings**

Let us try to relate the notion of foreground to the notion of meaning. The meaning of an action has to do with aiming at something, and meaningfulness in learning has to do with the directedness of the learning process. It has to do with the students' intentionalities pointing towards elements in their foregrounds.

Let us imagine that we have to do with activities in the mathematics classroom like: (1) solving quadratic equations; (2) addressing the mathematics of surfing; (3) considering what could be pilots' mathematics. What meaning could be associated to such activities?

If one considers meaning of a classroom activity as defined first of all through its relationships to the students' backgrounds and to what they already are familiar with, then such activities cannot be expected to make much sense to, say, students not interested in mathematics; students that have not ever seen the ocean; students that never did a trip by airplane; Our

point, however, is that experience of meaning also has to do with emerging intentionalities.

As part of a project taking place in a poorer neighbourhood in Rio Claro, we were teaching a group of students considered the most difficult students in the classroom. They were always causing problems by being noisy, by fighting, and by obstructing the teaching. They had not shown any interest whatsoever in mathematics. We meet with these 12 students in the computer room. It turned out that it was not that difficult to engage them in working with computers. For many, the school was the only place where they could touch a keyboard. And what kind of activities could make sense to these students? Many! For instance, working with dynamic geometry. They were fascinated by the movement on the screen, dragging a corner of a triangle here and there. Then experienced many things for the first time: like the magic that in any triangle the sum of internal angles result always in  $180^0$ . This kind of activities seemed to make sense for them. But why? Again, one need not think of meaning as having to do with familiarity. Meaning has to do also with engaging students' foregrounds. Working with computers might bring about new imaginations of possibilities: "This could also be for me."<sup>8</sup>

What to think about surfing, then? Let us quote from *An Invitation to Critical Mathematics Education*:

In a public school in Rio Claro, a city in the interior of the São Paulo State, the teacher wanted to introduce project work in mathematics. She asked what topics the students wanted to work with, and one suggestion was surfing and surf boards. The teacher did not find this to be a good possibility. The school was located in a poor neighbourhood. Most likely the students had never been at the beach and never seen the ocean. How could working with surfing make sense to them? If one relates meaning merely to already established experiences and to the background of the students, surfing does not appear meaningful to the students. However, one might miss some important aspects of what meaningfulness might include. It could very well be that surfing makes part of the students' foreground, and, as a consequence, elaborating a project about surfing could be extremely meaningful. (Skovsmose, 2011a, p. 29)

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<sup>8</sup> See also Penteadó and Skovsmose (2009) and Skovsmose and Penteadó (2011, in print).

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What can be experienced as meaningful also has to do with aspiration, hopes, and imagination.

Concerning the pilots mathematics let us also quote from *An Invitation to Critical Mathematics Education*:

For a long period of time, I [Ole] participated in a mathematics education project in South Africa.<sup>9</sup> We struggled with the challenge of what could make sense to children living in a village “beyond the mountains”? As mathematics educator one could try to investigate activities in the village through a mathematical archaeology and try to identify how mathematics might be part of the way the field work is conducted, how the crops are divided, how the cooking is done, etc. One could identify mathematical activities as related to these everyday activities, and on this basis try to organise a meaningful mathematics education for the children. But we need not be surprised in case meaningful mathematics education for these children could be developed around what we might call “pilots mathematics”. The children from the village beyond the mountains may only have experienced the airplane in the form of a thin, white, downy line high up there in the sky. But piloting might nonetheless be part of their foreground. (Skovsmose, 2011a, p. 29)

We find that meaningfulness is related to students’ foregrounds. It relates to their aspirations and hopes, and to their emerging intentionalities. Exploring the properties of a triangle, considering the mathematics of surfing, and working with the pilots’ mathematics may be meaningful to many children. It is an on-going challenge to a mathematics education to establish meaningfulness.

### **3.3 Possibilities**

We think of a mathematics education in terms of construction of possibilities. Such a construction is a complex political and socio-economic task. But it is also an educational task. The construction of possibilities concerns the students’ foregrounds. It concerns what could become their foregrounds. It has to do with possibilities that the students’ intentions could be directed towards.

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<sup>9</sup> See Vithal (2010).

When we did work with dynamic geometry and the students felt captivated, it cannot be due to their familiarity with the topic. Their engagement, however, demonstrated that they brought intentions into their activities, and intentions means directedness. Working with the software might have brought them to consider new possibilities; they might have experienced openings of their foregrounds.

Similar situation has also been investigated as part of the Epura project (Unesp, Rio Claro, Brazil) which aims at constructing possibilities for those who are excluded of the conventional learning environment. Some examples: Lessandra Marcelly S. Silva (2010) as part of her Master studies developed resources that can be used in the same class by blind and visual students. Renato Marcone de Souza (2010) investigated the case of a student that become blind during his undergraduation in Mathematics. It is discussed how teachers reacted to this and the strategies were used in order to organise the teaching. As part of his PhD, Denival Biotto Filho is working with teenagers in problematic family situation, attended by a local institution specialized in such cases. Based on the concept of foreground, Biotto Filho wants to study what possibilities mathematics education out of the school environment can open for the group. Furthermore, as part of Epura project, there are research addressing deaf students (Elielson Sales), students with intellectual disabilities (Carla Saullo), teacher education (Raquel Milani and Vanessa Cintra) and elderly people (Luciano Lima).

These references have to do with new possibilities emerging within the horizon. This is an important quality of a mathematics education concerned about democracy.

#### **4. Almost too difficult**

We started this paper by not defining democracy. And we continue not defining it. What we have done, however, is to relate mathematics education concerned about democracy to the notions of foreground, meaning, and possibility. This, naturally, does not bring any solidity to the discussion, as foreground, meaning, and possibility are just as open as the very notions of mathematics education and democracy.

Working for a mathematics education in such perspective means facing a range of challenges. All of them almost too difficult. Naturally, it need not be presuppose that democracy is well established in society, in the educational system in general, nor in the school environment in particular. In fact it is relevant to work for democratic ideals in contexts where there is no democracy in sight. Mathematics education concerned about democracy is a

challenge, and one can face this challenge in all situations and in all contexts. Certainly there is no guarantee for any success. But there are concrete things to work for in order that “this could also be for me” is experienced through mathematics education by more and more people.

A mathematics education for democracy is an on-going challenge, and also an on-going challenge for itself. What such an education could mean is an open question without any specific answers in sight. Thus, we always have to keep in mind that “democracy” is an explosive concept.

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### References

- Alrø, H., Ravn, O. and Valero, P. (Eds.) (2010). *Critical Mathematics Education: Past, Present, and Future*. Rotterdam: Sense Publishers.
- Appelbaum, P. with Allan, D. S. (2008). *Embracing Mathematics: On Becoming a Teacher and Changing with Mathematics*. New York: Routledge.
- Baber, S. A. (2007). *Interplay of Citizenship, Education and Mathematics: Formation of Foregrounds of Pakistani Immigrants in Denmark*. PhD thesis. Aalborg: Aalborg University
- D’Ambrosio, U. (2006). *Ethnomathematics: Link between transitions and modernity*. Rotterdam: Sense Publishers.
- Gellert, U. Jablonka, E. (2009). The demathematising effect of technology: Calling for critical competence. In P. Ernest, B. Greer, and B. Sriraman (Ed.s), *Critical issues in mathematics education (19-24)*. Charlotte, NC: Information Age Publishing.
- Greer, B. Mukhopadhyay, S., Powell, A. B. and Nelson-Barber, S. (Eds.) (2009). *Culturally responsive mathematics education*. New York: Routledge.
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York and London: Routledge.
- Foucault, M. (1991). *Discipline and punish: The birth of the prison*. London: Penguin Books. (First French edition 1975).



- Hannaford, C. (1998). Mathematics Teaching *is* Democratic Education. *Zentralblatt für Didaktik der Mathematik*, 98(6), 181-187.
- Penteadó, M. G. and Skovsmose, O. (2009). How to draw with a worn-out mouse? Searching for social justice through collaboration. *Journal for Mathematics Teacher Education*, 12 (3), 217-230.
- Said, E. W. (1979). *Orientalism*. New York: Vintage Books.
- Skovsmose, O. (1990): Mathematical education and democracy. *Educational Studies in Mathematics*, 21, 109-128.
- Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.
- Skovsmose, O. (2008). Mathematics education in a knowledge market In: E. de Freitas and K. Nolan (Eds.), *Opening the research text: Critical insights and in(ter)ventions into mathematics education* (159-174). New York: Springer.
- Skovsmose, O. (2009). *In doubt: About language, mathematics, knowledge and life-Worlds*. Rotterdam: Sense Publishers.
- Skovsmose, O. (2011a). *An invitation to critical mathematics education*. Rotterdam: Sense Publishers.
- Skovsmose, O. (2011b). Critique, generativity, and imagination. *For the Learning of Mathematics*, 31(3), 19-23.
- Skovsmose, O. (2012a). Symbolic power, robotting, and surveilling. *Educational Studies in Mathematics*. Published online: 28 February, 2012.
- Skovsmose, O. (2012b): Critical mathematics education: A dialogical Journey. In A. A. Wager and D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with mathematics educators*. NCTM, National Council of Mathematics Teachers, USA.
- Skovsmose, O. (in print). Justice, foregrounds and possibilities. In Cotton, T. (Ed), *Towards an Education for Social Justice: Ethics applied to Education*.
- Skovsmose, O. and Penteadó, M. G. (2011). Ghettoes in the classroom and the construction of possibilities. In B. Atweh, M. Graven, W. Secada and P.Valero. (eds.), *Mapping equity and quality in mathematics education* (77-90). New York: Springer.
- Skovsmose, O. and Penteadó, M. G. (in print). Issues on teacher education: From the perspective of critical mathematics education. In L. J. Spielman and J. Mistele (eds.), *Mathematics teacher education in the public interest*.
- Silva, Lessandra M.S. (2010). *As história em quadrinhos adaptadas como recuso para ensinar matemática para alunos cegos e videntes*. Master

- 
- Dissertation. Graduate Program in Mathematics Education. Universidade Estadual Paulista (Unesp) at Rio Claro, São Paulo.
- Souza, Renato M. (2010). *Aprendendo a partilhar experiências*. Master Dissertation. Graduate Program in Mathematics Education. Universidade Estadual Paulista (Unesp) at Rio Claro, São Paulo.
- Sriraman, B. (Ed.) (2008). *International Perspectives on Social Justice in Mathematics Education*. The Montana Mathematics Enthusiast, Monograph 1. Charlotte, NC: Information Age Publishing, Inc.
- Vithal R. (2010). Democratising mathematics education doctoral research teaching and learning: Undoing the North-South divide. In H. Alrø, O. Ravn and P. Valero, P. (eds.), *Critical mathematics education: Past, present and future* (197-207). Rotterdam: Sense Publisher.
- Wittgenstein, L. (1958). *Philosophical investigations*. Translated by G. E. M. Anscombe, Second edition. Oxford: Basil Blackwell. (First published 1953.)

## **Aiming for 21<sup>st</sup>. Century Skills**

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### **Introduction**

There is a saying, “In education everything happens 50 years later”. If true, this may have been not too problematic in older times, when changes were slow, but in times like ours where changes are extremely fast, it will be disastrous. Instead of being behind, education should be ahead. Primary-school, for instance, will have to prepare today’s students for their entrance in society, which will be about twenty years from now. However, this is not the case. Instead, Tony Wagner (2008) speaks of an “achievement gap” between what schools (in the USA) are teaching and what students will need to succeed in today’s global knowledge economy. He argues that, “students are simply not learning the skills that matter most for the twenty-first century” (ibid, 8-9). And he goes on to say that, “Our system of public education—our curricula, teaching methods, and the tests we require students to take—were created in a different century for the needs of another era. They are hopelessly outdated.” (ibid, 8-9). If we focus on employability, we may observe, that future employees in countries such as the USA will have to compete with colleagues in other countries with similar skills who work for lower wages. But, more importantly, the skills that the current and future jobs require anywhere, differ significantly from what current education offers. Wagner interviewed numerous CEO’s of large companies and he found a strong communality in what they look for in new employees; as one of them phrases it: “First and foremost, I look for someone who asks good questions.” (ibid, 2). The underlying rationale is that these employees will have to function in dynamic organizations. Employees today continuously have to learn new things. From this perspective, asking the right questions, is an important skill. In this respect, there is, however, a huge gap between what society demands and what schools offer. Prevailing language educa-

tion, for instance, aims at technical reading skills, but does not ask students to think about what they read. Mathematics instruction also often focuses on skills, instead of understanding. Schools, Wagner (ibid) goes on to say, are not designed to support students in learning to think. The reason for that, he argues, is that we as a society never asked schools to teach students to think. This is reflected in national tests that are not designed to assess the ability to reason and analyze. The point is, the world has changed, schools have not.

Next to the dynamics of the modern workplace, there is also a shift in the type of work people do. Empirical research by the economists Levy and Murnane (2006) shows that employment involving cognitive and manual routine tasks in the USA dropped between 1960 and 2000, while employment involving analytical and inter-active non-routine tasks has grown in the same period. This change especially concerned industries that rapidly automatized their production. Parallel to the development in industry, similar changes occurred in other areas where a strong computerization took place. This change happened on all levels of education. Jobs with a high routine character are disappearing. Jobs that will be offering good prospects for the future, are jobs which concern non-routine tasks, which are tasks that require flexibility, creativity, problem solving skills, and complex communication skills. Autor, Levy, and Murnane (2003) refer to examples such as reacting to irregularities, improving a production process, or managing people. The jobs of the future are the ones that ask for flexibility, creativity, lifelong learning, and social skills. The latter are jobs that require communication skills, or face-to-face interaction—such as selling cars or managing people. These changes do not only affect the decline or rise specific jobs; existing jobs are changing as well. Secretaries, and bank employees for instance have got more complex tasks since word processors and ATM's have taken over the more simple tasks.

Earlier, Friedman (2005) pointed to the fact that the effects of computerization and globalization overlap and reinforce each other. Routine tasks can easily be outsourced, since information technology enables a quick and easy worldwide exchange of information. The latter also makes it possible to outsource business services, such as call-centers, or the work of accountants and computer programmers. Another effect of globalization is that it forces companies to work as efficient as possible. This requires companies to immediately implement computerization and outsourcing when it is economically profitable and strengthens the market position of the company. It also demands of the company to be on the lookout for opportunities to im-

prove efficiency. As a consequence, working processes will have to be adapted continuously. This in turn, puts high demands on the workers, who have to have a certain level of general and mathematical literacy to be able to keep up.

In summary we may conclude that schools will have to change to comply with the requirements imposed by our rapidly changing society. In this paper we will especially look into the consequences for mathematics education. We will start by taking a closer look at the 21<sup>st</sup> century skills. This will reveal the need for changes in the nature of the instructional process, and we will discuss what this means for mathematics education by reviewing the main characteristics of, annex requirements for, problem-centered, interactive, mathematics education. Next we will turn to the content of mathematics education, which also has to change. Will we start with the 21<sup>st</sup> century skills.

### ***21<sup>st</sup> century skills.***

Voogt & Pareja Roblin (2010), who reviewed the literature around five theoretical frameworks on 21<sup>st</sup> century skills, observe that the need for 21<sup>st</sup> century skills is mostly addressed by private or business initiatives, while educational leaders, practitioners and the educational community do not actively participate in the debate. The frameworks they reviewed concern: the 'Partnership for 21st century skills' (Partnership for 21<sup>st</sup> century skills, 2008), 'EnGauge' (North Central Regional Educational Laboratory and the Metiri Group, 2003), 'Assessment and Teaching of 21st Century Skills' (ATCS) (Binkley, Erstad, Herman, Raizen, Ripley & Rumble (2010), 'National Educational Technology Standards' (NETS) (Roblyer, 2000), and 'Technological Literacy for the 2012 National Assessment of Educational Progress' (NAEP). Dede (2009) compares similar frameworks in his review of 21<sup>st</sup> century skills. Both review studies show that the frameworks appear to strongly agree on the need for skills in the areas of communication, collaboration, ICT literacy, and social/cultural awareness. We may take Wagner's (2008) list of what he calls, 'the new survival skills', as exemplary:

1. critical thinking and problem solving
2. collaborating and leading by influence
3. agility and adaptability
4. initiative and entrepreneurship
5. effective oral and written communication
6. accessing and analyzing information
7. curiosity and imagination.

We may also follow Wagner (ibid) in arguing that the 21<sup>st</sup> century skills are not just about employability<sup>1</sup>. They will also have to include broader goals, such as becoming a responsible, active and well-informed citizen. Wagner (ibid) typifies this broader goal by asking if the students would eventually be well-equipped for acting as a member of a jury within the US legal system: “*Would they know how to distinguish fact from opinion, weigh evidence, listen with both head and heart, wrestle with the sometimes conflicting principles of justice and mercy, and work to seek the truth with their fellow jurors?*” (Wagner, 2008, xvi-xvii). This characterization resonates with the theme of this conference: “*Democracy in mathematics curriculum*”, and the corresponding issue of contributing to critical thinking and decision-making in the society. This begs the question of how to translate these lofty goals into instructional practice.

### **Mathematics education for 21<sup>st</sup>. century skills**

Most publications on 21<sup>st</sup> century skills do not discuss how these goals might be achieved in education. It seems fair to say, however, that adopting 21<sup>st</sup>. century skills as educational goals will primarily affect the way instructional practice is shaped. Mathematics education seems to be a good place to incorporate these goals. Dam, & Volman (2004), for instance, conclude on basis of a review study that critical thinking is best developed in conjunction with science and mathematics education. But also in a more general sense, it may be argued that interactive, problem-centered, mathematics education will contribute to most 21<sup>st</sup>. century skills. However, if we want to integrate the objective of fostering 21<sup>st</sup>. century skills with attaining the regular goals of mathematics education, we will have to find a way by which problem solving, collaborating, communicating etc. serves the mathematics goals. In theory this is not too problematic, since the literature on reform mathematics points to this very type of activities—problem solving, collaborating, communicating etc.—as necessary conditions to enable students to construct meaningful mathematics. “In theory”, because we know that creating mathematics classrooms which can be characterized as interactive and problem-centered is not easy. We will discuss this issue by reviewing the main characteristics of, annex requirements for, problem-centered, interactive, mathematics education.

### ***Local instruction theories***

By reform mathematics we mean an approach to mathematics education that assumes that students have to construct or reconstruct mathematics by

themselves with help of the teacher and the textbook. Instead of reconstructing, one may also speak of reinventing—following the tradition of realistic mathematics education (RME) (Gravemeijer, 2008). One of the prerequisites for fostering reinvention—which has been a focus of RME research—is the availability of local instruction theories that serve the need for guidelines about the routes along which students might reinvent pieces of mathematics. A local instruction theory consists of a theory about a possible learning process for a given topic, and the means of supporting that process. Mark that a local instruction theory should not be confused with a scripted textbook. The local instruction theory is meant to function as a framework of reference for teachers.

### *Hypothetical learning trajectories*

On the basis of such a framework of reference a teacher may design so-called ‘hypothetical learning trajectories’ (Simon, 1995) for the actual instructional activities in his or her classroom at a given moment in time. With the term hypothetical learning trajectory, Simon (1995) refers to the notion that a teacher has to deliberate on what mental activities the students might engage in as they participate in the instructional activities, he or she is considering. A decisive criterion of choice then will be how those mental activities relate to the chosen learning goals. He emphasizes the hypothetical character of those learning trajectories; the teachers are to analyze the reactions of the students in light of the stipulated learning trajectory to find out in how far the actual learning trajectory corresponds with what was envisioned. Based on this information the teacher has to construe new or adapted instructional activities in connection with a revised learning trajectory.

### *Classroom social norms*

In addition to a sound planning of the instructional activities, another prerequisite for inquiry mathematics concerns the participation of the students. For the point is that students do not easily engage in problem solving and reasoning in regular classrooms (Desforges & Cockburn, 1987). Cobb & Yackel (1996) argue that this is not surprising, because students are often familiar with a classroom culture in which the classroom social norms (Cobb & Yackel, 1996) are that the teacher has the right answers, that the students are expected to follow given procedures, and that correct answers are more important than one’s own reasoning. In this type of classrooms teachers usually ask questions of which they already know the answer. Apart from being used to this situation, students have learned what to expect

and what is expected from them. In relation to this, Brousseau (1988) speaks of an implicit 'didactical contract'. Significant, however, is that the students have learned this by experience, not because the teacher told them so.

In contrast to what is common in school mathematics, the classroom has to work as a research, annex learning, community. To make this happen, the students have to adopt classroom social norms that fit an inquiry-oriented classroom culture. These encompass, the obligation to explain and justify one's solutions, to try and understand other students' reasoning, and to ask questions if one does not understand, and challenge arguments one does not agree with. In addition to those social norms, Cobb & Yackel (1996) observe that the teacher also has to establish socio-mathematical norms, which relate to what mathematics is. This is expressed in norms about issues such as, what counts as a mathematical problem, what counts as a mathematical solution, and what counts as a more sophisticated solution. In regard to the latter, we may argue that socio-mathematical norms lay the basis for the intellectual autonomy of the students, as it enables them to decide for themselves on mathematical progress.

### ***Task orientation***

In addition to appropriating inquiry-based norms, students also have to be inclined to invest effort in solving mathematical problems, discussing solutions, and discussing the underlying ideas. Students may engage in learning activities for different reasons. The attitude of students in a mathematics classroom can be broadly divided in two categories, *ego orientation* and *task orientation* (Jagacinski & Nicholls, 1984). Ego orientation implies that the student is very conscious of the way he or she might be perceived by others. Ego-oriented students are afraid to fail, or to look stupid in the eyes of their fellow students, or the teacher. As a consequence, they may choose not to even try to solve a given problem, in order to avoid embarrassment. Task orientation on the other hand implies that the student's concern is with the task itself, and on finding ways of solving that task. Research shows that task orientation and ego orientation can be influenced by teachers.

Cobb, Yackel & Wood (1989) report on a study on a socio-constructivist classroom, where task orientation was fostered. Part of their approach was to change the classroom culture from one of competition, where students compare themselves with each other, and with the criteria set by the teacher, into a classroom culture, where students would measure success by comparing their results with their own results earlier. We may think of the latter perspective as one similar to that of an amateur painter or amateur musician.



An amateur musician would not think of comparing him- or herself with others; there would always be many people performing better. Instead an amateur musician would be pleased if he or she would master a piece, which he/she could not play some time ago. A similar situation is possible in a mathematics classroom, where the goal for the students would be personal growth. Here, experiencing an ‘Aha-Erlebnis’, for instance, may function as an incentive. In such a classroom, students might even protest to be given ‘the solution’, for that would deprive them from the satisfaction of figuring out things for themselves. The aforementioned research of Cobb et al. (1989) shows that a classroom culture that emphasizes the exchange of ideas, and the development of mathematical understanding as a collaborative endeavor, may foster the task orientation of the students.

### ***Mathematical interest***

In connection with student motivation, we may add that next to the obvious “pragmatic interest”, which realistic problems appeal to, students will also have to develop “mathematical interest” (Gravemeijer & Van Eerde, 2009). This concerns the preparedness of the students to the investigate solution procedures, concepts and so on, from a pure mathematical perspective. This is a necessary condition for construing more sophisticated mathematics. Mathematical interest will rarely come naturally, but has to be cultivated by the teacher by asking questions such as: What is the general principle here? Why does this work? Does it always work? Can we describe it in a more precise manner? We may assume that the teacher can foster the students’ mathematical interest by making mathematical questions a topic of conversation, and by showing a genuine interest in the students’ mathematical reasoning.

### ***Framing topics for discussion***

Another essential role of the teacher concerns the orchestration of productive whole-class discussions. For this the teacher has to identify the differences in mathematical understanding which underlie the variation in student responses (Cobb, 1997). Next, he or she has to frame these underlying mathematical issues as topics for whole-class discussions. And finally, he or she has to orchestrate a productive whole-class discussion on those topic in order to foster higher levels of understanding.

In a similar feign, Stein, Engle, Smith, & Hughes (2008), espouse five practices for promoting productive disciplinary engagement: anticipating students’ mathematical responses, monitoring student responses, purpose-

fully selecting student responses for public display, purposefully sequencing student responses, and connecting students responses. The first two overlap to some extent with Simon's hypothetical learning trajectory, whereas the last three are related to identifying, framing, and discussing mathematical issues. Although a difference may be that Stein et al. (2008) seem to try to address both mathematical ideas and the solution strategies of the students, while Cobb (1997) emphasizes the mathematical issues that underlie student solutions, and tries to steer away from a focus on solution strategies as such.

### **21<sup>st</sup> Century mathematical skills**

The publications on 21<sup>st</sup> century skills mostly list general skills. Elaborations of what changes are needed in the content of the various topics that are taught, are very scarce. With some exceptions, there is also little attention is given to the content of the mathematics curriculum. One of the reasons may be that one of the effects of computerization is that mathematics becomes invisible. Mathematics is hidden in integrated systems, such as spreadsheets, automatic cashiers, and automated production lines. However, a consequence of this is that many people become "mathematics consumers", as Levy and Murnane (2006, 19) put it. They argue that people who use computerized systems are expected to make decisions on the basis of the output of hidden mathematical calculations. The computerized equipment does the actual calculation. However, they go on to say, if the decision maker does not understand the underlying mathematics, he or she is very vulnerable to serious errors of judgment. If we follow this line of reasoning, we may discern two seemingly conflicting tendencies. On the one hand, we appear to need less and less mathematics, since various apparatus take over a growing number of mathematical tasks. On the other hand, we will need more mathematics as we develop into 'mathematics consumers', who become increasingly dependent of the quantitative information and mathematical models—which we ought to understand.

The aforementioned hidden character of mathematics in the information society makes it difficult to establish, what mathematics students will have to learn to become sensible mathematics consumers. To answer this question, we will follow two tracks. The first concerns research of mathematics at the workplace, the second concerns an analysis of the role of computers as interface between the physical world and the computer user.

### ***Techno-mathematical literacy's***

When trying to relate the goals of mathematics education to the requirements of the workplace, a complication is that the mathematics that is used at the workplace is strikingly different from conventional mathematics (Hoyles & Noss, 2003; Roth, 2005). To describe this specific kind of mathematics, Hoyles and Noss (2003) coined the term “techno-mathematical literacy's”—or TmL's for short. TmL's are defined as idiosyncratic forms of mathematics that are shaped by work-place practices, tasks, and tools. Acting successfully at the workplace is dependent on a combination of mathematical knowledge and contextual knowledge.

In a recapitulatory report, Hoyles, Noss, Kent & Bakker (2010) mention as one of their key findings, that artefacts comprising symbolic information in the form of numbers, tables and graphs are often understood as “pseudo mathematics”. This means that these representations are conceived as labels or pictures, without an understanding of the underlying mathematics. They argue that, “Symbolic information thus failed to fulfill its intended role in facilitating communication across ‘boundaries’ between communities within and beyond the workplace” (ibid, 196). Important mathematical skills that come to the fore in their research are: reading tables and graphs, identifying and measuring key variables, reasoning about models in terms of the key relationships between variables, and representing and interpreting data.

We may further note that the role of TmL's does not simply boil down to applying mathematical knowledge. Students will have to be able to flexibly adapt their existing mathematical knowledge or adopt new knowledge. This asks for experience with a variety of non-canonical forms of mathematics.

### ***Quantitative models of reality***

Apart from studying mathematics at the workplace, we may trace mathematical skills that are needed in a computerized environment, by analyzing the role computers and computerized appliances play as interfaces between the users and the concrete reality. In relation to this, we may discern the following processes:

- reality is quantified to make it accessible for computers—as computers only work with numbers;
- these numbers are processed by the computer on basis of models that describe interdependencies between variables;
- the output, which often has some mathematical form, is interpreted.

Thus in order to understand how a computer deals with reality, one must,

- have some idea of what quantifying (or measuring) entails;

- understand at some level, what a variable is, how we can reason about interdependencies between variables, and how computer models represent reality;
- has to be able to interpret the output of computers.

An important aspect of a measuring is that there will always be some inaccuracy and uncertainty involved. In general students do not realize that there will always be some measurement error, or that a repeated measurement may result in a different outcome. Nor do they realize that variance is part of industrial production, even though they will be aware of the phenomenon of natural variance in nature. We would argue that it is important for students to develop this kind of understanding, for many of the numbers they will have to work with will be the result of sampling. We may further follow Jones (1971) in his observation that measuring an object comprises assigning a value to a variable; what is measured is not the object but a property of that object. The significance of this observation comes to the fore when we look at co-variation. In such cases we do not compare the individual lengths and weights of certain persons, but we study how “length” varies with “weight”.

Fortunately information technology offers eminent possibilities for creating educational tools for helping students to come to grips with these ideas. Hoyles et al. (2010), for instance, report on the use of interactive computer tools that present symbolic information such as graphs, models expressed in algebraic symbols, or numerical measures. Experiments with such computer tools (which they call, technology-enhanced boundary objects) show that they can be employed successfully in enhancing the meanings of the symbolic information by making them more visible and manipulable, and therefore more accessible. We may note that this use of computer tools is in line with Kaput’s, faith in the educational potential of information technology that allows for the use of *dynamic representations* (Kaput & Schorr, 2007). Computers can show the numerical results or graphical representations of measuring activities real time on a screen, but also offer representations of simulations that can be manipulated at will.

We may mention two examples of the educational use of dynamic representations in service of the mathematical objectives mentioned above. One concerns a simulation of a train on a rail track, which is accompanied by a graphical representation of the speed of the train, which can be used to foster an understanding of co-variation and informal calculus (Galen & Gravemeijer, 2010). A second example concerns an introduction to explora-

tory data analyses with the help of so-called computer minitools (Gravemeijer & Cobb, 2006), that allow for a variety of representations and ways of structuring of data sets. While analyzing data with the help of these minitools, students may develop a qualitative notion of a distribution of data as an object with certain characteristics, such as shape and spread.

### **Conclusion**

We started by observing that there is a significant gap between what students learn at school and what modern society demands. Computerization and globalization result in a change in job requirements that involves a shift from routine tasks towards non-routine tasks. The jobs of the future require flexibility, creativity, problem solving, life-long learning and complex communication skills. Our fast changing information society not only put new demands on workers, it also affects our roles as learners and citizens. In relation to this, we referred to Wagner's (2008) benchmark that the students eventually would have to become well-equipped for acting as a member of a jury in the US legal system.

Similar to Wagner, many publications in this area use the term, "21<sup>st</sup> century skills"; skills that may be summarized as flexibility, critical thinking, problem solving, collaborating, and communicating. We argued that these skills fit very well with problem-centered, interactive, mathematics education, and we reviewed the most important characteristics of this type of mathematics education.

We further discussed what changes in the content of mathematics education would be needed. Both research on mathematics in the workplace, and an analysis of the role of computers as interface between the physical reality and the user, point to topics such as, measuring, tables, graphs, variables, models of relationships between variables, and elementary statistics.

Finally we indicated how computer tools may help students in coming to grips with the aforementioned mathematics, by offering dynamic representations. In conclusion, we want to stress the urgency of a reconsideration of the goals of mathematics education in light of the demands of the 21<sup>st</sup> century, and the need to start experimenting with educational practices that may foster these goals.

### **Notes**

1. Mark that most publications on 21<sup>st</sup> century skills stem from Western countries, which implies a certain bias. We may argue, however, that the 21<sup>st</sup> century skills that are identified will be or become significant for other countries as well.

### References

- Autor, D., F. Levy, & R. Murnane (2003). The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics*, 118 (4): 1279-1333.
- Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., & Rumble, M. (2010). *Defining 21st century skills*. Retrieved from <http://atc21s.org/wp-content/uploads/2011/11/1-Defining-21st-Century-Skills.pdf>.
- Brousseau, G. (1988). Le contrat didactique: le milieu. *Recherche en didactique des mathématiques* 9(3), 33 – 115.
- Cobb, P., 1997. Instructional design and reform: Locating developmental research in context. In M. Beishuizen, K.P.E., Gravemeijer, & E.C.D.M van Lieshout, (Eds.) (1997). *The Role of Contexts and Models in the Development of Mathematical Strategies and Procedures*. Utrecht: CdB Press.
- Cobb, P. & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31, 175-190.
- Cobb, P., Yackel, E., and Wood T. (1989). Young children's emotional acts while doing mathematical problem solving. In D. McLeod and V. M. Adams (eds), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 117–48). New York: Springer.
- Dede, C. (2010). Comparing Frameworks for 21st Century Skills. In J. Belanca & R. Brandt, Eds, *21st Century Skills* (pp. 51-76). Bloomington, IN: Solution Tree Press.
- Dam, G. ten, & Volman, M. (2004). Critical thinking as citizenship competence: teaching strategies. *Learning and Instruction* 14, 359-379.
- Desforges, Ch. and A. Cockburn (1987), *Understanding the Mathematics Teacher, A Study of Practice in First School*, London: The Falmer Press.
- Friedman, Th. L. (2005). *The World Is Flat*. London, England: Penguin Books (pp. 5-12).
- Galen, F. van & Gravemeijer, K. (2010). *Dynamische grafieken op de basisschool*. Utrecht: Ververs Foundation & Freudenthal Instituut.

- Gravemeijer, K.P.E. & Cobb, P.(2006). Design research from a learning design perspective. In: Akker, J., Gravemeijer, K., McKenney, S., Nieveen, N.(Eds). *Educational Design Research*. London: Routledge, Taylor Francis Group, pp. 45-85.
- Gravemeijer, K. (2008). RME Theory and Mathematics Teacher Education. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education: Vol.1. Knowledge and beliefs in mathematics teaching and teaching development* (pp. x-xx). Rotterdam, the Netherlands: Sense Publishers, 283-302.
- Gravemeijer, K. & Van Eerde, D. (2009) Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *Elementary School Journal, Volume 109* (5), p.p. 510-524.
- Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F. Leung (Eds), *Second International Handbook of Mathematics Education (Vol. 1)*, Dordrecht, the Netherlands: Kluwer Academic, pp. 323–349.
- Hoyles, C., Noss, R., Kent, Ph. & Bakker, A. (2010). *Improving Mathematics at Work, The Need for Techno-Mathematical Literacies*. TLRP's Improving Learning Series. New York: Routledge.
- Jones, L. V. (1971), The nature of measurement. In R.L. Thorndike (Ed.) *Educational Measurement (2<sup>nd</sup> Edition)*. Washington: American Council on Education.
- Kaput, J. & Schorr, R. (2007). Changing representational infrastructures changes most everything: The case of SimCalc, Algebra, and Calculus. In G. Blume & K. Heid (Eds.) *Research on technology in the learning and teaching of mathematics: Syntheses and perspectives* (pp. 211-253). Mahwah, NJ: Erlbaum.
- Levy, F. & R.J. Murnane (2006). *How computerized work and globalization shape human skill demands*. Downloaded November, 2, 2006, from <http://web.mit.edu/flevy/www/>.
- Jagacinski, C. M., Nicholls, J. G. (1984). Conceptions of ability and related affects in task involvement and ego involvement. *Journal of Educational Psychology, Vol 76*(5), 909-919.
- North Central Regional Educational Laboratory and the Metiri Group. (2003). *enGauge, 21st century skills: Literacy in the digital age*. Naperville, IL: NCREL.
- Partnership for 21st Century Skills (2008). *21st Century Skills Education & Competitiveness*. A resource and policy guide,

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[http://www.p21.org/documents/21st\\_century\\_skills\\_education\\_and\\_competitiveness\\_guide.pdf](http://www.p21.org/documents/21st_century_skills_education_and_competitiveness_guide.pdf) Downloaded at 19-10/2010

- Roblyer, M. D. (2000). The National Educational Technology Standards (NETS): A Review of Definitions, Implications, and Strategies for Integrating NETS into K-12 Curriculum. *International Journal of Instructional Media*, v27 n2 p133-46
- Roth, W-M. (2005). Mathematical Incriptions and the Reflexive Elaboration of Understanding: An Ethnography of Graphing and Numeracy in a Fish Hatchery. *Mathematical thinking and Learning*, 7(2), 75–110.
- Simon, M.A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26, 114-145
- Steen, L.A. (2001). Data, Shapes, Symbols: Achieving Balance in School Mathematics. In: Steen, L.A. (ed.), *Mathematics and Democracy: The Case for Quantitative Literacy* (pp. 53-74). Princeton, NJ: National Council on Education and the Disciplines.
- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008). Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell. *Mathematical Thinking and learning*, 10: 313-340.
- Voogt, J. & Pareja Roblin, N. (2010). *21<sup>st</sup> Century Skills – Discussion Paper*. Enschede (The Netherlands): University of Twente. Retrieved 27 May 2010 from <http://onderzoek.kennisnet.nl/onderzoeken-totaal/21stecentury>
- Wagner, T. (2008). *The Global Achievement Gap, Why even our best schools don't teach the survival skills our children need – and what we can do about it*. New York: Basic Books.



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## **LA DÉMOCRATIE EN ÉDUCATION**

### ***Les mathématiques : terreau propice pour la démocratie***

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*La démocratie est ce lieu où tout le monde se  
croit compétent pour parler de tout. La vérité  
ne se décide pas au suffrage universel.*

Platon

#### **INTRODUCTION**

J'aimerais avant toute chose remercier les organisateurs de ce colloque pour avoir pensé à moi pour une plénière. Je suis parmi les plus anciens de la CIEAEM ici dans cet amphi aujourd'hui, j'aimerais prendre un instant pour rendre hommage aux personnes que j'ai côtoyées depuis que je suis entré à la CIEAEM, des personnes qui ont donné, qui ont tout fait pour que la CIEAEM soit ce qu'elle est aujourd'hui. Malheureusement ils ont quitté cette terre. Je pense à Lucienne FÉLIX, à Zofia KRIGOWSKA, à Hans FREUDENTAHL, à Paulo ABRANTES, à René MÉTRÉGISTE et à Filippo SPAGNOLO.

Je suis entré à la CIEAEM à Budapest en 1986, à une commission restreinte et depuis, je suis resté membre actif mais en changeant de nationalité tantôt j'étais français, tantôt canadien, tantôt québécois et même tchèque.

Je suis très heureux de venir parler de la démocratie en Grèce, le berceau même de la démocratie moi qui suis originaire d'un pays qui s'appelle la République Démocratique du Congo, là où il n'y a vraiment pas de démocratie comme dans d'autres pays qui s'affublent le titre de République Démocratique de... quelque chose et où ce parfum de démocratie est plutôt

l'illusion de démocratie, un faux-semblant qui est pire qu'une absence de démocratie.

Le thème que je prends aujourd'hui est très large, complexe et difficile. Comment lier ou concilier les mathématiques et la démocratie ? Telle est l'une des questions que je me suis posé avant de préparer cette adresse. Sous quel angle traiter la démocratie et les mathématiques, deux concepts qui peuvent converger selon certains et complètement diverger selon d'autres.

Avant de commencer, je voulais partager avec vous cette réflexion de Tozzi (2005) que je fais mienne. «Le miracle grec, c'est la connaissance (l'émergence simultanée), au niveau politique de la démocratie, au niveau juridique du procès moderne, au niveau épistémologique de la rationalité (notamment sous ses deux formes majeures, la science et la philosophie).

La démocratie, c'est le droit égal pour tout citoyen de prendre la parole sur l'agora pour débattre et décider des affaires de la cité et même d'être tiré au sort pour exercer les fonctions clefs. Avec les limites que l'on sait pour l'époque : un citoyen doit être Grec, homme, libre et de plus de trente ans. Mais pour la première fois dans l'histoire des hommes, la légitimité d'une parole n'est pas fondée sur la transcendance d'un dieu ou la puissance d'un chef, mais sur la seule capacité à convaincre son auditoire par cette parole. Le débat argumentatif entre hommes libres et égaux est fondateur de l'espace public démocratique.

Du même mouvement se développe *le procès*, c'est-à-dire le débat contradictoire entre deux parties, dont chacune, présumée victime ou/et coupable doit être entendue, et peut faire entendre sa voix en vue d'une décision judiciaire. C'est la parole exprimée, argumentée, partagée, antagonique et arbitrée.

C'est aussi le passage du *mutos* (le mythe imaginaire et symbolique) à *l'épistémè* (un exigeant rapport au savoir et à la vérité), la *raison* comme moyen et arbitre pour établir des connaissances et valider des thèses, sous la forme par exemple de la démonstration mathématique ou de la réflexion philosophique.

C'est cet étroit nouage entre le débat et ses formes d'expansion, l'espace citoyen public, l'espace juridique du procès et l'espace de confrontation des deux formes occidentales de la raison (science et philosophie), que consacre le miracle grec, matrice de l'histoire institutionnelle et culturelle que tout le monde veut adopter ».

Dans le sous-thème 2 : *Démocratie dans les pratiques mathématiques en classe : quelles pratiques permettraient de vivre un ensemble particulier de valeurs humanitaires ?* Une question de fond est posée : Quelles sont les

formes d'un discours démocratique en classe de mathématiques ? Les pays où il n'y a pas de démocratie, comment voulez-vous qu'on pratique la démocratie en éducation et en mathématiques ? Souvenez-vous en Argentine au temps de la dictature des généraux de 1976 à 1983, les mathématiques modernes étaient interdites car elles proposaient des concepts comme ensemble qui donnait l'idée de regroupement, réunion et pouvaient donner l'idée de réunion des gens pouvant renverser le régime, de vecteur, car il donnait l'idée de maladie vénérienne.

Par ailleurs, dans une perspective historique des relations entre démocratie et éducation, il semble capital de réfléchir aux enjeux d'éducation pour favoriser l'exercice démocratique. Je vais centrer mon propos sur l'enjeu d'éducation dans la société pour favoriser l'exercice démocratique à l'heure où les citoyens se détournent de plus en plus de la participation politique, où on assiste à une montée des extrémismes, à l'intimidation dans les écoles, à la violence, à l'entrée des intervenants à l'école (des policiers)

À chaque pays, notent Panassier et Pugin (2011), correspond un système éducatif qui reprend les grands axes du projet politique de société et de ce fait d'éducation. Cela dépend des pays, en France, la référence est encore largement l'École de la République, l'École de l'unité de la nation, laïque, gratuite et obligatoire. Elle est définie et mise en œuvre pour ancrer les principes démocratiques contre la monarchie et promouvoir la raison et les sciences contre la religion. Il faut noter qu'en France aujourd'hui au 21<sup>e</sup> siècle, la démocratie et la laïcité sont des acquis qui ne sont plus remis en cause. Panassier et Pugin soulignent également que progressivement et avec la constitution « *d'une société des individus* », la mission principale de l'École de former des citoyens éclairés s'est déplacée vers une réponse aux attentes individuelles de formation d'autant plus exigeantes dans un contexte d'insertion sociale et professionnelle difficiles. Aujourd'hui, toute question collective est d'abord visitée sous le prisme de l'individu ou des intérêts particulier au détriment de l'intérêt général et de la cohésion du vivre ensemble. Dans un tel contexte notent Panassier et Pugin (2011), l'enjeu d'éducation pour réinvestir une dimension collective apparaît essentiel.

Au Québec, souligne le Programme de Formation de l'École Québécoise (PFÉQ, 2001), l'école compte parmi les lieux importants de transmission entre les générations des acquis de la société. Par le biais de ses activités de formation, elle crée un environnement dans lequel l'élève s'approprie la culture de son milieu, poursuit sa quête de compréhension du monde et du sens de la vie et élargit l'éventail de ses moyens d'adaptation à la société. L'école québécoise a le mandat de préparer l'élève à contribuer à l'essor

d'une société démocratique et équitable.

D'une façon générale, au Canada, la vaste majorité des programmes scolaires (de la maternelle à la 12<sup>e</sup> année) qui enseignent la citoyenneté mettent l'accent soit sur la qualité du «*bon citoyen*», soit sur «*le fonctionnement du gouvernement*».

Pourtant, selon Westheimer (2011), la réforme actuelle de l'éducation limite les façons dont les enseignants peuvent développer le genre d'attitudes, de compétences, de connaissances et d'habitudes nécessaires à l'épanouissement d'une société démocratique. Ses constats font craindre que les pratiques actuelles incitent davantage à plaire à l'autorité et à réussir aux examens qu'à développer et à défendre des convictions. Ceci a pour conséquence d'avoir une population qui maîtrise mal les questions d'intérêt public et qui peine à analyser des décisions politiques complexe d'un point de vue critique.

Westheimer (2011) souligne également que la plupart d'entre nous tiennent pour acquis que les valeurs démocratiques s'enseignent à l'école mais celui-ci montre que ce n'est toujours pas le cas. La plupart des programmes qui veulent encourager la citoyenneté mettent l'accent sur le bénévolat, la charité et l'obéissance plutôt que sur la démocratie. La citoyenneté dans une collectivité démocratique va au-delà de la bonté, note-t-il.

La Belgique, qui est ma puissance coloniale, aussi loin que peuvent remonter mes souvenirs, il n'y a eu aucun cours au Congo sur la démocratie ou sur la citoyenneté. On avait même pas de citoyenneté, on était presque apatrides. Jamais je n'ai entendu prononcer le mot «*démocratie*» à l'intérieur du cadre scolaire.

Finalement, dans tous les pays du monde même dans ceux qui ne sont pas démocratiques, théoriquement l'école a pour mission d'éduquer les enfants dans un esprit démocratique et de le préparer à leur rôle de citoyen actif. De ce fait, apprend-on les rudiments de la démocratie, la liberté et l'autonomie à nos élèves ? Mais plutôt la facilité, le laisser-aller et le laxisme qu'on confond complaisamment souligne Aktouf (1999) avec la responsabilisation et l'émancipation.

En suivant l'histoire des systèmes éducatifs à travers le monde, on peut constater que la compétition reste l'élément constant chez les acteurs de l'éducation (élèves, enseignants, parents, etc.). Peut-on imaginer un système éducatif sans compétition ? La question est posée par Jacquart (1999) qui pense non sans raison que «*Être dans une société démocratique signifie que tout individu, membre du peuple, a le droit de s'exprimer et que sa voix*

compte. Pour cela, bien sûr, il faut qu'il soit capable de s'exprimer. C'est ce qu'il faut chercher à l'école.

Or justement, souligne encore Jacquart (1999), une société basée sur la compétition ne peut pas être démocratique, puisqu'elle est basée sur la lutte, et qu'alors il y a tous ceux qu'on n'écoute pas et que, progressivement, on met sur le bord de la route, ce sont les exclus, tous les en-trop. Une société qui laisse quelques élèves sur le bord doit se réformer. Jacquart propose de remplacer « décrocheur » par « décroché », il y a une différence entre un voleur et un volé : le décrocheur a été décroché, il a peut-être mis du sien, mais c'était un enfant qu'on devait accrocher.

### **VERS UNE DÉFINITION FONCTIONNELLE DE LA DÉMOCRATIE**

Les deux concepts en jeu ici sont *les mathématiques* et *la démocratie*. Nous sommes tous à la CIEAEM, il va sans dire que je ne vais pas perdre votre temps à définir les mathématiques, leur enseignement et leur apprentissage mais je vais plutôt m'attarder à définir la démocratie pour qu'on soit au moins au même diapason.

### **LA DÉFINITION DE LA DÉMOCRATIE SELON QUELQUES RECHERCHES ET AUTEURS**

Selon Valero (1999) cité dans la discussion du thème de la seconde annonce de la CIEAEM-64, la démocratie est un but à atteindre, une forme idéale d'organisation sociale qui établit une pratique libre et égalitaire de l'auto-détermination politique incluant les valeurs, les normes et les comportements qui permettent une combinaison optimale des structures économiques, culturelles et institutionnelles pour toute une population.

Popper cité par Corvi (1997) ne distingue que deux types de régimes politiques : la démocratie et la tyrannie. Il va définir la démocratie comme le gouvernement du peuple. La question classique depuis Platon « Qui doit gouverner ? » est rejetée par Popper comme étant essentialiste. À ce problème, il propose d'en substituer un plus réaliste : « Existe-t-il des formes de gouvernement qu'il nous faille rejeter pour des raisons morales ? Et inversement : existe-t-il des formes de gouvernement qui nous permettent de nous débarrasser d'un gouvernement sans violence ? Popper qualifie de démocratique, un régime dans lequel les dirigeants peuvent être destitués par les dirigés sans effusion de sang. Tout autre gouvernement dans lequel la destitution des dirigeants ne peut passer que par la violence pourra être qualifié de tyrannique.

Le problème majeur de Popper sera de penser à l'organisation de la démocratie de telle sorte que celle-ci permette au mieux la destitution des dirigeants. Il rejette dès lors la démocratie directe et le scrutin proportionnel. En effet se justifie-t-il, avec la démocratie directe, le peuple est responsable devant lui-même, ce qui est une contradiction : le peuple ne peut se destituer lui-même. Avec le scrutin proportionnel, la plupart des partis sont nécessairement représentés dans les assemblées dans une ou moins grande proportion, quoi qu'il arrive lors des élections, et les partis majoritaires sont souvent forcés de devoir gouverner avec eux en créant les coalitions, ce qui signifie en clair que certains partis pourraient toujours participer au pouvoir et ne jamais être destitués.

Ainsi Popper préfère plutôt la démocratie représentative avec scrutin majoritaire, et ce en raison de ce qu'il pense être les faiblesses de la démocratie directe et du scrutin proportionnel. Popper marque une nette préférence pour le bipartisme, où le parti opposant a la charge de critiquer les hypothèses formulées par le parti majoritaire, et inversement. Le système des primaires internes aux partis permet de rajouter une autocritique des hypothèses à l'intérieur même des partis.

### DÉMOCRATIE ET ÉDUCATION

Meirieu a énoncé 32 principes possibles pour une éducation démocratique dont le principe 13 : « l'éducation à la citoyenneté par la découverte de l'histoire de l'émergence de la démocratie et l'expérimentation de ses principes et de ses modes de fonctionnement dans les cadres et sur les objets adaptés au niveau de développement de l'enfant ». Le 19 : « L'école doit lutter à la marchandisation des savoirs scolaires, en particulier à travers un combat contre l'hégémonie des notes. La prise en compte des progrès de chacun et du développement de sa personnalité doit entrer systématiquement en ligne de compte ». Le principe 20 stipule : « Pour lutter contre la marchandisation des savoirs scolaires, l'École doit promouvoir « la pédagogie du chef d'œuvre » : les activités scolaires doivent être finalisées par des travaux personnels ou collectifs qui, poussés au plus degré d'exigence, permettent, à travers des tâches dans lesquelles les élèves s'investissent pleinement, de rencontrer des obstacles et d'élaborer des savoirs. Les enseignants accompagnent cet investissement en étant attentifs aux progrès réalisés par chacun. Ils peuvent utiliser des échelles de progression afin de permettre à l'élève de se situer au regard des exigences qui lui sont manifestées. En aucun cas, un travail ne doit « être payé d'une mauvaise note » et abandonné. Tout travail imparfait doit être repris et poussé à son terme ».

Il faut souligner que les règles de fonctionnement de l'École sont semblables aux règles de fonctionnement de la société. Si la société n'est pas démocratique, il serait étonnant que l'école soit démocratique. Les règles de fonctionnement de l'École comportent nécessairement une part non négociable (les missions de l'institution, les programmes, l'interdit de la violence, le respect des biens collectifs), mais elles comportent également une part négociable avec les élèves dans le cadre de dispositifs pédagogiques structurés et organisés par le maître (« Conseil d'élèves »). Les adultes ont ici pour mission d'aider les élèves à construire « le bien commun » et à identifier les moyens de le faire respecter.

Reboul (1997) souligne que « les problèmes pédagogiques ont une dimension politique. On peut penser d'ailleurs qu'il existe une relation entre le régime politique d'une société et la pédagogie qu'elle utilise en enseignant. Mais la relation n'est pas à sens unique, car si l'enseignement est déterminé par la société globale, il la détermine à son tour, il la fixe ou il la change. À partir de là, il se pose la question de savoir : que doit être un enseignement dans une société démocratique ? Si la réponse est difficile, cela veut dire que le mot « *démocratie* » est loin d'être univoque, je dirai même polysémique. On a qu'à remarquer dans la formule « *démocratiser l'enseignement* ». Certains la traduisent par : donner plus de liberté, plus de responsabilité aux élèves eux-mêmes, dans ce sens le produit de la démocratisation est alors pédagogique avant tout. D'autres traduisent la formule tout autrement : rendre tous les enfants égaux devant l'enseignement, soit en donnant à tous les mêmes chances, soit ce qui est logique, en donnant plus de chances à ceux qui sont moins favorisés au départ, par des classes moins nombreuses, des cours de soutien, etc.

On peut remarquer que pour les premiers, l'enseignement peut rester tout à fait inégalitaire, puisque rien n'empêche la pédagogie démocratique de confirmer les différences, donc les clivages entre plus et moins doués, entre futurs chefs et futurs subordonnés dans les termes de Meirieu (1987).

Pour les seconds, l'enseignement peut rester autoritaire, l'essentiel étant que personne ne soit défavorisé. Vous comprenez qu'une démocratie ne peut sacrifier ni la liberté ni l'égalité.

Pour démocratiser l'enseignement, Reboul (1997) pose deux principes majeurs:

1. **Principe 1** : Comme l'avait bien montré John Dewey : « une société n'est réellement démocratique que si l'école forme réellement des démocrates. Une pédagogie autoritaire risque de faire des êtres sou-

mis ou des révoltés, une pédagogie laxiste des irresponsables. C'est pourquoi l'enseignement doit se fonder autant que possible sur l'autorité du contrat, par exemple la gestion commune de projets adoptés en commun. En tout cas la démocratie exige que les élèves acquièrent dès que possible l'autodiscipline, le sens de la coopération, le respect de l'autre qui sont au principe même de son fonctionnement.

2. **Principe 2** : est que l'enseignement fondamental dure le plus possible. Une société, en effet, n'est pas démocratique si elle contraint la plupart des jeunes à entrer très tôt dans le monde du travail ou de la formation professionnelle et laisse à une minorité le loisir de se cultiver, car cette culture là ne sera plus alors que celle d'une élite dirigeante. Bref, que tous reçoivent une culture de base complète que possible, donc aussi longue qu'il le faut.

Pour Audigier (2005), « la démocratie est à la fois une forme d'organisation du politique, c'est-à-dire des pouvoirs, et une conception de la société, c'est-à-dire des droits et libertés de chacun. Une de ses originalités tient au fait qu'elle prend en compte la diversité qui caractérise nos sociétés modernes, diversité des opinions, des croyances, des intérêts, des savoirs, des attentes, et plus encore qu'elle fait de cette diversité un atout pour son propre développement. ». En fait, il fait une différence majeure entre la démocratie sociétale et la démocratie politique qu'il faut travailler à retrouver les chemins d'une formation qui intègre ces deux dimensions et marque clairement les différences et les complémentarités ».

Enfin, Breton (2005) souligne que : « Avant d'être un système d'organisation politique, la démocratie est une pratique de la parole organisée socialement dans le but de prendre des décisions ».

## DÉMOCRATIE ET MATHÉMATIQUES

Un esprit critique et indépendant est une condition nécessaire pour prendre sa place dans la société et participer aux processus démocratiques et aux institutions d'enseignement,

(Karlheinz, 2003).



## **LA DÉMOCRATIE À TRAVERS LE PROGRAMME DE FORMATION DE L'ÉCOLE QUÉBÉCOISE AU PRIMAIRE ET AU SECONDAIRE**

À travers *Vivre ensemble et citoyenneté*, communauté d'apprentissage et microcosme de la société, l'école québécoise accueille des individus de provenance sociales et culturelles diverses. Elle constitue, à ce titre, un lieu privilégié pour apprendre à respecter l'autre dans sa différence, à accueillir la pluralité, à maintenir des rapports égaux et à rejeter toute forme d'exclusion. Elle place les élèves dans des situations qui les amènent à relever quotidiennement les défis de la coopération dans un esprit d'entraide, de solidarité, d'ouverture à l'autre et le respect de soi. Elle leur permet ainsi de faire l'expérience des principes et des valeurs démocratiques sur lesquels se fonde l'égalité des droits dans la société québécoise. Cette préparation à l'exercice de la citoyenneté s'appuie également sur des apprentissages cognitifs, notamment ceux qui sont réalisés dans le cadre des disciplines appartenant au domaine de l'univers social.

Le PFÉQ (2001, 2003) permet aux élèves et aux étudiants de participer à la vie démocratique de l'école ou de la classe et de développer des attitudes d'ouverture sur le monde et de respect de la diversité. Pour appuyer cette intention éducative, il a fixé trois axes de développement :

### *1. Valorisation des règles de vie en société et des institutions démocratiques*

Processus démocratique d'élaboration des règles dans la vie scolaire, municipale et nationale; acteurs de la vie démocratiques (individus, représentants, groupes d'appartenance) et respect des personnes dans leur rôle; droits et responsabilités liés aux institutions démocratiques. À cet effet, les élèves et les étudiants assistent souvent aux débats et aux périodes des questions à l'Assemblée Nationale.

### *2. Engagement dans l'action dans un esprit de coopération et de solidarité*

Principe, règles et stratégies du travail d'équipe; processus de prise de décision (consensus, compromis. Etc.); établissement de rapports égaux; recours au débat et à l'argumentation; leadership; dynamique d'entraide avec les pairs; projets d'action liés au vivre-ensemble.

### *3. Culture de la paix*

Interdépendance des personnes, des peuples et de leurs réalisations; égalité des droits et droit à la différence des individus et des groupes; conséquences négatives des stéréotypes et autres formes de discrimination et

d'exclusion; lutte à la pauvreté et à l'analphabétisme; sensibilisation aux situations de coopération et d'agression; résolution pacifique des conflits; modalité d'entente ou de contrat.

Le PFÉQ, dans sa partie de l'univers social (histoire et éducation à la citoyenneté) visent beaucoup plus à inculquer le respect de la loi qu'à préparer les élèves à contribuer démocratiquement à la société.

### **LA DÉMOCRATIE À TRAVERS LES PROGRAMMES DES MATHÉMATIQUES DU PRIMAIRE ET DU SECONDAIRE**

Le PFÉQ en mathématiques ne fait aucun lien avec celui de l'univers social en ce qui concerne la démocratie. Et pourtant, c'est un programme qui se dit d'amener l'élève à établir des liens entre ses apprentissages scolaires et sa vie quotidienne et de lui offrir l'occasion de comprendre différents contextes de vie, de se construire une perception nuancée de ces contextes et d'envisager d'actions dans des situations données.

### **L'APPORT DES MATHÉMATIENS À LA DÉMOCRATIE**

Plusieurs mathématiciens ou apparentés mathématiciens ont contribué à l'évolution de la démocratie. L'apport conceptuel de certains d'entre eux mérite que je m'y attarde. Le choix est orienté par mes lectures. Pour Aristote, le père de la théorie du syllogisme, cité par Cauquelin (1994), la démocratie évite deux écueils qui font sombrer les autres régimes : la sédition et la corruption. Cette absence de faiblesse est conditionnelle : le peuple n'est pas incorruptible mais seulement moins accessible à la corruption. Une sédition des riches n'est pas non plus impossible. Aristote souligne que la menace par excellence est la sédition. L'insurrection est le danger qu'il faut éviter à tout prix. Or ce risque est moindre en démocratie que dans les autres régimes parce qu'il y a plus de pauvres que de riches et donc le pouvoir des indigents fait plus de satisfaits que d'insatisfaits. Pour Aristote, le pire des régimes est la tyrannie et le meilleur des régimes est la monarchie. Mais tout régime risque de dégénérer et ses défauts seront alors proportion de ses qualités. Ainsi, si la monarchie dégénère, comme ses qualités sont maximales, les défauts de la forme dégénérée (tyrannie) seront les pires. La politeia parce qu'elle est le moins bon des bons régimes dégénère en démocratie moins mauvais des mauvais régimes. Et donc, il faut opter pour la démocratie.

Pour différencier la monarchie de la démocratie, Aristote utilise cette métaphore médicale : « La monarchie, c'est la meilleure santé possible au risque du choléra. La démocratie est un simple rhume et peut-être vaut-il mieux garder son rhume que prendre le risque du choléra. La démocratie est

donc le régime de la prudence.»

Il faudrait souligner que la justification formelle du *suffrage universel* a été produite pour la première par le marquis Nicolas de Condorcet en 1795. À ce niveau, Condorcet reprenait l'argument de Jean-Jacques Rousseau dans le *Contrat social*, selon lequel l'opinion de la majorité est légitime car elle exprime la volonté publique. Condorcet a démontré la pertinence de cette vision à partir d'un argument probabiliste : en effet, dans la mesure où le citoyen moyen a moins d'une chance sur deux de se tromper, la somme de tous les votes des citoyens a très peu de chance d'être erronée. Cette démonstration est connue sous le nom du *Théorème du jury de Condorcet*.

Descartes souligne que « Il y a un mal fou à comprendre la démocratie... à la comprendre de l'intérieur, du dedans de notre être. Pourtant on y est réellement depuis suffisamment longtemps. Du mal avec la démocratie, parce que chacun veut avoir raison mais que la démocratie suppose que personne n'a raison en soi, et que la vérité n'existe que du mélange et naît des conflits, des heurts et des compromis »

### ***LA DÉFINITION NAÏVE DE LA DÉMOCRATIE SELON LES ÉLÈVES ET LES ÉTUDIANTS***

Au Québec, le PFÉQ a inscrit dans son curriculum le domaine d'univers social comprenant la géographie, l'histoire et l'éducation à la citoyenneté. Nous faisons l'hypothèse qu'en principe, le concept de démocratie devrait être familier à nos élèves et étudiants. Afin de connaître les conceptions de ces apprenants, nous avons soumis un questionnaire anonyme à 50 sujets (10 élèves du primaire, 10 du secondaire, 15 étudiants du collégial et 15 étudiants de l'université en didactique des mathématiques). Le questionnaire comportait trois questions dont :

1. D'après vous qu'est ce que la démocratie ?
2. Pouvez-vous donner deux exemples de la présence ou de l'absence d'une démocratie ?
3. Pensez-vous qu'on puisse apprendre la démocratie dans un cours de mathématiques ? Justifiez votre réponse.

L'analyse des réponses de ces sujets à la question 1 montre pour 40 d'entre eux que la démocratie est réductible au vote, aux partis politiques et au pouvoir décisionnel de la majorité des citoyens.

La définition qui revient le plus souvent est « *La démocratie est une forme de parti politique servant à voter des lois, à représenter les citoyens. Chacun a le droit à son opinion. Le dirigeant du parti n'a pas droit de veto* ». Pourtant souligne Robert (1999) : « La démocratie, dans un sens plus

large, ce sont aussi des juges indépendants, des forces de police subordonnées, l'égalité de tous devant la justice civile, la liberté des associations, des partis et de la presse, des règles équitables pour la fiscalité, l'accès aux soins et à l'éducation, des disponibilités qui empêchent la corruption, le respect des individus et des minorités, de la vieillesse, de la jeunesse, de la féminité, des différences culturelles, l'existence de règles protectrice en faveur des salariés, etc.. ».

Quant aux exemples de la présence de la démocratie, la majorité des sujets, cite souvent les pays occidentaux comme lieux de démocratie (Canada, USA, France, etc.) même s'ils y perçoivent des poches de dictature. En ce qui concerne l'absence de démocratie, l'exemple qui revient le plus souvent c'est le manque de droit de parole en Irak, les pouvoirs tyranniques en Syrie, à Cuba, en RDC, etc.

À la troisième question sur l'apprentissage de la démocratie dans un cours de mathématiques, les statistiques sont citées à l'occasion des élections par le calcul des pourcentages. Le droit de donner son opinion dans un cours de mathématiques ou de donner le droit de vote pour choisir une activité mathématique. Certains sujets soulignent qu'on ne peut pas apprendre la démocratie en mathématiques, car celles-ci concernent le raisonnement et la logique, or la démocratie est sujette aux émotions et aux opinions sur lesquelles sont fondées les interactions sociales et politiques.

### **L'APPORT DES MATHÉMATIQUES À LA DÉMOCRATIE**

Les résultats de ce questionnaire nous conduisent à poser la question suivante : En quoi la société a-t-elle intérêt à former mathématiquement et démocratiquement tous les individus qui la composent ? Cette perpétuelle question a été posé moult fois dans les milieux de l'éducation mathématique. Duperret (2008) a tenté d'y répondre.

Une première réponse qu'il avance est que l'apprentissage des mathématiques est une forme d'apprentissage de la démocratie, en mettant les élèves en « activité mathématique ». Celle-ci commence en général par une recherche personnelle, défi entre le problème et l'élève, démarche intellectuelle intime qui développe et construit la pensée. Celle-ci continue dans une communauté scientifique, la classe dans l'enseignement, communauté qui permet successivement le débat en soumettant aux preuves et réfutations les diverses possibilités de solutions, puis l'assurance de la certitude partagée. On retrouve dans cette démarche la longue histoire qui lie les mathématiques et la démocratie depuis les grecs.

Une seconde réponse qu'il propose est liée à nature même de l'activité

mathématique : résoudre des problèmes, c'est-à-dire se mettre dans une constante confrontation au non savoir. Et là se développe des comportements « experts », avec la recherche de la meilleure stratégie, du modèle le plus pertinent, comportements tout à fait transférables à d'autres champs d'action que les mathématiques.

Une troisième réponse, souvent cataloguée « mathématiques du citoyen », est la formation à l'analyse et au traitement de l'information. Les mathématiques vont développer des aptitudes à trier, ranger, transformer des informations en s'appuyant sur de fréquents changements de registre : textes, tableaux, graphiques, résultats numérique, etc. On trouve ici le rôle social des mathématiques : la lecture, l'interprétation, l'utilisation de diagrammes, tableaux, graphiques, leur analyse critique aident l'élève à mieux comprendre les informations qu'il reçoit, et en cela contribuent à son éducation civique et démocratique.

Orrill (2001) souligne que la méconnaissance complète des mathématiques, et en particulier du traitement de l'information, que les Étatsuniens appellent « innumeracy » rend les citoyens infirmes à même titre que l'alphabétisme, l'illiteracy. Inversement, un apprentissage du calcul, de la statistique et des probabilités constitue un bon atout pour se situer dans le présent et saisir les enjeux de l'avenir.

Il faut souligner une mauvaise note dans la démocratisation de l'enseignement, dans le sens qu'elle a remis en cause bon nombre de certitudes, entre autres sur le statut des mathématiques, de leur enseignement et du rôle pervers de sélection qu'elles ont joué avec le latin et le grec.

### **DÉBAT MATHÉMATIQUE ET DÉBAT DÉMOCRATIQUE**

Un enseignant des mathématiques qui n'est pas ouvert à la démocratie pratique généralement le culte de la pensée unique. Il n'est pas ouvert à la discussion donc n'est pas prêt à ouvrir un débat dans sa classe.

Le débat bien mené dans une classe de mathématiques, à n'importe quel niveau peut conduire les élèves ou les étudiants à un raisonnement à l'aide de concepts et des processus mathématiques (MELS, 2003) et travailler la notion d'implication qui est le pivot du raisonnement déductif et de la démonstration (Scotti, 2001).

En partant d'une étude menée par Mounier (2005), je peux ressortir les différences et les ressemblances entre les deux types de débat : le débat mathématique en classe et le débat démocratique.

*Les différences :*

En mathématiques, il y a une seule réponse (une vérité). Pas en démo-

cratie.

En mathématiques, ce n'est pas nécessairement la majorité qui a raison, contrairement à ce qui se passe en démocratie : il existe en mathématiques une solution validée par les experts qui reste vraie quelles que soient les représentations de la majorité sur le sujet.

En démocratie, il s'agit de convaincre, en mathématiques, il s'agit de prouver. En démocratie, on peut toujours discuter une décision, en mathématiques, cela n'a plus de sens lorsque cela a été démontré.

En démocratie le débat porte sur des valeurs, par exemple, le débat sur le droit ou non à l'avortement, euthanasie, pas en mathématiques.

*Les ressemblances :*

Chacun peut défendre son point de vue, chaque point de vue est pris en compte. Les règles d'écoute et de prise de parole sont les mêmes. Le débat permet à se confronter à l'altérité et permet d'avancer vers une solution. Enfin, il est dans les deux cas nécessaire d'argumenter.

Il faut souligner quelques règles particulières du débat en classe :

Celui qui parle le plus fort n'a pas nécessairement raison, celui qui a raison n'est pas nécessairement le meilleur de la classe.

On a le droit de changer d'avis, écouter les autres peut aider à trouver la solution. Se moquer de celui qui se trompe ne fait pas progresser le débat. En mathématiques, il faut formuler les propositions de façon précises, si on veut pouvoir se comprendre, sinon on ne sait pas de quoi on parle. En mathématiques (et ailleurs) une proposition et son contraire ne peuvent être toute les deux vraies.

Il existe plusieurs types de débat de la vie de classe dont le *débat réglé*. Vincent (2005) souligne que « Contrairement aux temps d'échange rencontrés au cours des activités scolaires (en mathématiques, en français, en sciences, etc.) qui s'appuient sur la confrontation des représentations, des démarches et une justification cognitive (liée à une connaissance), les temps de débat réglé se caractérisent par le fait qu'ils portent sur des questions relationnelles, sociales, morales, organisationnelles concernant l'environnement proche et impliquant la plupart du temps, une prise de décision et une action collectives.

Il existe un certain nombre de conditions caractéristiques pour que le débat réglé soit un débat démocratique :

- *L'institutionnalisation et la ritualisation*

Pour fonctionner démocratiquement les débats doivent être institutionnalisés, ritualisés, c'est-à-dire respecter un certain nombre de procédures et de règles claires concernant :

L'horaire de débat : il est fixé dans l'emploi de temps et est respecté; le lieu et la disposition spatiale : on ne débat pas assis les uns derrière les autres; les modalités explicites de fonctionnement : choix des thèmes, animation, prise de parole, suivi des décisions, etc.

- *Des objectifs pour une parole « vraie »*

1) Un objectif d'apprentissage des conditions et des enjeux du débat démocratique.

C'est un temps de proposition et d'écoute, de négociation et de prises de décisions concernant la vie de la classe ou de l'établissement scolaire. C'est un temps d'apprentissage des modalités de fonctionnement d'un débat démocratique.

2) *Un objectif de formation du jugement*

C'est un temps d'analyse et de confrontation de points de vue concernant l'ensemble des sujets qui concernent directement les élèves (vie de la classe) ou qui concernent les problématiques plus générales qu'elles soient politiques, économiques ou sociales.

## CONCLUSION

En guise de conclusion, je vais simplement souligner que l'école doit aujourd'hui instaurer, pour tous les élèves, un rapport plus démocratique aux savoirs et au rapport plus coopératif à la loi. Dans une perspective d'instruction républicaine, comme en France ou de socialisation et de formation du citoyen comme au Québec, l'école devrait mettre l'emphase sur les pratiques de la démocratie et faire en sorte que toutes les disciplines enseignées à l'école contribuent à la socialisation démocratique dans une approche interdisciplinaire. Elle devrait favoriser les débats scientifiques, la libre confrontation des opinions, le dialogue et les échanges.

De ce fait, les questions qui suivent semblent essentielles :

- Comment former les citoyens de demain pour favoriser l'exercice démocratique contre des formes de repli ou l'apparition de régimes politiques plus totalitaires?
- Quels savoirs et compétences sont en jeu?
- Comment faciliter la compréhension du monde dans lequel on vit, l'apprentissage à la prise de parole et l'expérience du vivre ensemble ?
- Comment faire des liens entre les compétences transversales et les compétences disciplinaires en mathématiques autour de l'apprentissage de la démocratie.

**BIBLIOGRAPHIE**

- Aktouf, O. (1999). L'alphabétisme, paradoxe de nos sociétés néolibérales, in Merhran Ebrahimi (dir.), *Éducation et démocratie, entre individu et société*, Québec, Isabelle Quentin éditeur.
- Audigier, F. (1999). *L'éducation à la citoyenneté*, Paris, INRP.
- Breton, Ph. (2005). D'abord une pratique..., *Cahiers pédagogiques*, la démocratie dans l'école, 433, p. 16-17.
- Cauquelin, A. (1994). *Aristote*, Paris, Éditions du Seuil.
- Corvi, R. (1997). *An introduction to the thought of Karl Popper*, London, New-York, Rodopi.
- Duperret, J.-C. (2008). *Mathématiques et valeurs*, IUFM-Reims.
- Jacquart, A. (1999). De l'individu à la personne ou à la métamorphose par l'éducation, in Merhran Ebrahimi (dir.), *Éducation et démocratie, entre individu et société*, Québec, Isabelle Quentin éditeur.
- Meirieu, Ph. (1987). ([www.Img.Ulo](http://www.Img.Ulo), Principes possibles pour une éducation démocratique)  
[www.meirieu.com/MANIFESTESETPROPOSITION/PRINCIPES.pdf](http://www.meirieu.com/MANIFESTESETPROPOSITION/PRINCIPES.pdf) (consulté le 12 janvier 2012).
- Orrill, O. (2001). Mathematics, Numeracy and Democracy, in L. A. Steen, *Mathematics and Democracy, The Case Quantitative Literacy*, NCED.
- Panassier, C. et Pugin, V. (2011). *Éducation et Démocratie, Millénaire*, Le Centre Ressources Prospective du Grand Lyon.
- Programme de formation de l'école québécoise (2001), *Éducation préscolaire, Enseignement primaire*, Québec, Ministère de l'éducation.
- Programme de formation de l'école québécoise (2003), *Enseignement secondaire*, Québec, Ministère de l'éducation du loisir et du sport.
- Reboul, O. (1997). Éducation et démocratie in *Philosophie de l'Éducation*, Paris, Presses Universitaires de France, Que sais-je ? pp. 77-80.
- Robert, F. (1999). *Le droit à l'école*, Issy-les-Moulineaux, ESF.
- Scotti, C. (2001). *Les règles du débat mathématique*, Mémoire professionnel inédit présenté à l'IUFM de Montpellier. p. 18-19.
- Tozzi, M. (2005). L'autorité démocratique : une provocation conceptuelle, *Cahiers Pédagogiques*, numéro 433, p. 45-67.
- Valero, P. (1999). Deliberative Mathematics Education for Social Democratization in Latin America, *ZDM*, 1, 20-16.
- Westheimer, J (2011). *Restoring the Public*, Ottawa, Public Education.



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**WOMEN, MATHEMATICS, TECHNOLOGY AND  
OTHER DANGEROUS THINGS: A GENDERED READING OF  
DEVELOPMENT AS THE QUALITY/EQUITY DISCOURSE**

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**Abstract**

*Despite current political endeavours, technology and mathematics are still peripheral options for most females when they consider further studying or career up growth in the fields related to mathematical science. Specifically, in mathematics education, technology mediated teaching is assumed a means for development towards progressive policies for inclusion in both metropolitan (e.g. NCTM in US, 2000), and peripheral localities (e.g. DEPPS in Greece, 2003). Such curricula reforms have been largely rooted in pedagogical agendas for quality/equity (mathematics) education that aim to foster simultaneously self and society development in the realm of a 'new' information age. The present paper discusses the potential for an alternative theorising of female relation to school technoscience such as technology and mathematics related literacies. Based on a preliminary analysis of interview data the constructs of 'cyborg' and 'subaltern' are introduced as ways of disrupting stereotypic readings of a partial relation to technoscience as negative, passive or, even, dangerous<sup>1</sup>.*

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<sup>1</sup> The present paper is based on a chapter entitled 'Disrupting Development as the Quality/Equity Discourse: Cyborgs and Subalterns in School Technoscience' published in Atweh, B., Graven, M., Secada, W., and Valero, P. (eds) Mapping Equity and Quality in Mathematics Education. Dordrecht. Springer. (pp. 3-21).

## INTRODUCTION

Lakoff (1987) used the catch phrase ‘*women, fire, and dangerous things*’ as a title of a book concerned with how human thinking is totally immersed in metaphors and depended on their role to produce meaning in everyday talk. His choice to place the word ‘women’ next to ‘fire’ and next to ‘dangerous things’ intended to show the power of metaphor-use in language-use. It, also, served to produce a certain ‘image’ of the possible meanings concerning the category ‘woman’. First, a woman is a *thing* -not really a person. In addition, a woman, like fire, is a *dangerous thing*. The semantic categories of ‘mathematics’ and ‘technology’ along with those of ‘women’ and ‘fire’ in Lakoff’s choice of words seem to exemplify, when placed together, a similar ‘dangerous’ liaison, for good reasons, as will be shown in the sections below.

‘[M]athematics and technology are unfamiliar fields to women’ says Anita, a primary school teacher in her late 30s, whilst Giorgos, a young engineering student, argues that although some female students can cope well with what is required to do with technology during coursework, they lack a passion for it. Coping well with school subjects, including mathematics and technology, creates emotional conflicts for Afrodite, an adolescent Greek Gypsy girl, who senses that she will soon need to abandon school for an early marriage –repeating her parents’ story. Education, and specifically mathematics education, provides her with a promise of joining the desired ‘modern’ ways of imagining, organising and controlling her life. Simultaneously, this very desire soon becomes an unfulfilled promise, creating frustration, pain and feelings of failure. Schooling turns out to be an (almost) impossible path for Afrodite, who, despite being a successful learner, wonders what might be the real value of school for her. Schooling demands a cultural border crossing, and a constant compromise amongst conflicting ‘values’ related either to community or school formalities. Afrodite becomes ‘voiceless’, ‘hopeless’ or a ‘subaltern’ in Spivak’s (1992b) words as her struggle for recognition proves futile or un-ending.

Anita remembers being good at mathematics (geometry), when she says: ‘*I used to like mathematics and I was good at geometry. Yet, there is this oxymoron [...]. And, indeed, everybody believed –towards the end of secondary school- that I will become a scientist*’. Contrary to her family’s and companions’ belief in her capacities, chooses not to study mathematics since she feels that ‘science’ is not really suitable for her as a woman. Despite her choice not to engage in what was perceived as natural for her, she

recognises the fact that the ‘new’ generation has the potential to reverse such stereotypes if, as she argues, access to both resources and expertise is safeguarded. However, Giorgos, a young male who belongs to this ‘new’ generation, seems to espouse that women’s pursuit of science is not out of pure interest or passion but of mere necessity to acquire the skills required in modern society. He mentions: ‘[...] men are more into technology. They like it. Whilst women –those who get involved- because not all of them are involved, they do so, I believe, out of necessity. In other words, men have a passion (for technology), they buy magazines about technology [...] whilst women do not care much.’. Lack of passion and ‘pure interest’ show that women’s relation to technology is weak, subordinated and marginal. As such their pursuit of technology is taken as ‘different’ and becomes ‘other’.

Giorgos, like Anita, invests on hegemonic discourses which naturalise young women as non-passionate, non-dedicated participants in technoscientific practices arguing that they ‘get involved [...] out of necessity’. Taking into account the fact that the discourse of an intrinsic ‘*passion for science*’ is predominant when scientific creativity and innovations are taken into consideration (Turkle 2008), one easily concludes, as Anita does, that ‘*women are not really made for the worlds of mathematics and technology*’. In contrast, the case of Afrodite shows that passionate desire alone does not seem to safeguard a continuous participation to education (including mathematics education). Afrodite lives at the borders of two competing discourses; the one depicting school as ‘*the beginning of a new life*’ and the other emphasizing that ‘*it is a shame for a girl to attend school*’. Schooling represents the risky path towards a ‘new’, yet ‘uncertain’, life. In a similar vein, Anita rejects ‘uncertainty’ and chooses a safer area for study and work.

The narratives offered by Anita, Giorgos and Afrodite are inscribed within discourses that carry a ‘negative’ sense of female experience with technology, mathematics and education. Not only Giorgos, but also Anita and Afrodite seem to be captured within gendered discourses espousing a fixed view of women’s relation to technoscience. Their stories are not interpreted in a positive way, but instead perpetuate the projection of stereotypic images. According to Michel Foucault (1972, p.49) discourses function constitutively towards producing ‘truths’ which ‘*systematically form the object about which they speak*’. This approach explains how hegemonic discourses serve to reproduce women as having distinct ways of knowing (Belenky et

al 1986) or that technoscience<sup>2</sup> is mainly a masculine route towards realising the rational, modern 'self' and developing systemic societal change (Ellul 1964). Within this realm, school mathematics and technology are not seen a 'female' choice since women deal with techno-mathematics in 'different' ways –ways that potentially can work towards 'disrupting' commonly held assumptions and expectations. Women's relation to technoscience can be seen 'disruptive' as they embrace technology and mathematics without revealing a devoted passion. Instead, their engagement seems to be a continuous struggle towards fitting technoscientific materialities in the multiplicities of their everyday working, studying and living. Yet, women's struggling to appropriate technoscientific knowledge is often read as problematic. The issue of 'woman as a problem' has been discussed extensively in relation to technology (Wacjman 2007), but also in relation to mathematics (Fenemma and Leder, 1990). And, its assumed 'normality' can be oppressive as it does not allow 'other' subjectivities to emerge and does not voice alternative positioning(s).

Following Michel Foucault and Judith Butler, here, I attempt to re-read hegemonic discourses of female relation to techno-mathematics. Foucault (1972, p.151) observes insightfully how discourse '*obeys that which it hides*' and becomes '*the path from one contradiction to another*'. He argues that '*to analyse discourse is to hide and reveal contradictions; it is to show the play that they set up within it; it is to manifest how it can express them, embody them, or give them a temporary appearance*'. Discourse as contradiction comes close to notions of 'disruption' and 'trouble' as promoted by Butler (1990, 1997) arguing for the need to deconstruct what are often seen as 'normal' or 'natural' assumptions on agency, subjectivity and identification. Along these lines, the present paper aims to move beyond a negative interpretation of women's relation to technology and mathematics as passive, indifferent or marginal. It argues that female partial and at times marginal positionings could problematise technological determinism (Ellul 1964) and bring forward an alternative reading concerning our understanding of technoscientific practices where the complex incompatibility of using

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<sup>2</sup> According to wikipedia '*Technoscience is a concept widely used in the interdisciplinary community of science and technology studies to designate the technological and social context of science. The notion indicates a common recognition that scientific knowledge is not only socially coded and historically situated but sustained and made durable by material (non-human) networks*' (<http://en.wikipedia.org/wiki/Technoscience>).

technology and mathematics is not concealed but spoken out and negotiated. It is suggested, here, that an alternative reading might be closely related to disrupting assumed normalities of human-technoscience relation(s) by means of disrupting ‘development’ as the quality/equity discourse in technology mediated mathematics education.

Such an alternative approach to technoscience then becomes a dangerous gesture to development discourse(s). Danger is encountered at several levels. *First*, opting out technology and science is ‘dangerous’ for the ‘modern’ individual as it blocks the development of the rational subject and perpetuates the ‘savage’ and emotional self. *Second*, this very fact becomes dangerous as it holds up the development of modern systemic changes where ‘self’ and ‘society’ is interchangeably linked. Development here entails both a quality direction (i.e. towards becoming the rational subject) and an equity dimension (i.e. all subjects need to become rational or else technomathematically literate). However, narratives concerning women’s relation to school technoscience do not profess this very notion of development. Instead, they exemplify a ‘dangerous liaison’ as far as women instrumentally make use of their right to opt out, to resist, or to become marginal actors. The vision of an equitable future within mathematics education is, also, critically dependent on the potential of reworking what it means to assume a sense of ‘I’ – an agency that is subjectively negotiated. Specifically, women, instead of committing themselves to the ‘risky’ path of a ‘passionate’ experience with formal education and ‘new’ technologies they turn towards a ‘modest’ relationship carefully negotiating boundaries. Donna Haraway (1997, p.130), considers technoscience as the story of globalisation and argues for the significance of a ‘modest witness’ position as a space for feminist work – a space where technoscientific knowledge is regarded as situated “*deep and wide throughout the tissues of the planet, including the flesh of our personal bodies*”.

In this paper, I attempt to provide a type of bird-eye view over a very complex area that, at present, pressurises teachers and students towards adopting ‘new’ media, ‘new’ roles and ‘new’ identities. This intensity for change is being discussed under the caveat of development by providing *access for all* (i.e. equity) via technology-mediated mathematics education curricula. However, what do we really mean by development? And how do these relative links among school technoscience appropriation, development, quality and equity affect the daily lives of women and men and especially of women and men who belong to marginalised, oppressed and voice-

less groups? Taking into account the above, the following sections attempt to re-read hegemonic discourses on ‘development’ as ‘quality’ and ‘equity’ in mathematics education with an eye to disrupt assumed positioning(s) –or, in other words, to analyse how discourses can ‘hide and reveal contradiction’ (Foucault 1972, p. 151).

### **DEVELOPMENT AS QUALITY: INTENSITY FOR CHANGE**

Whilst equity, as Secada (1992, 1995) claims has been marginally explored in the research field of mathematics education, quality has been well emphasized. From the 1980s onwards, mathematics education research has greatly invested on promoting innovative curricula design in order to promote *quality* teaching and learning. Main sources for theorizing *quality* have been certain psychological perspectives based primarily on either constructivist or socio-cultural approaches to learning. Curricula innovations included the cognitively guided instructions for mathematics, contexts for authentic learning, realistic mathematics education etc (Shoenfeld 1994; Greeno 1998; Treffers 1987).

Issues of quality were mainly discussed in relation to the *micro-context* of mathematics classrooms taking into account primarily didactic and pedagogical aspects. Emphasis on how children develop mathematical skills and competences has led towards focusing on cognitive and meta-cognitive strategies as they relate to social interactions (teacher intervention, group work, classroom activity). Despite commonalities, constructivist and socio-cultural perspectives could hardly agree on fundamental principles concerning learner agency and knowledge status. On the one hand, a constructivist perspective<sup>3</sup> directs attention to the learner as an active autonomous subject who potentially reflects on and negotiates ideas by means of experimenting with suitable materials. Learning and knowledge development, thus, depend mainly on explorative activity, reflection, and active engagement with task parameters. A mainstream socio-cultural approach<sup>4</sup>, on the other hand, em-

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<sup>3</sup> The notion of ‘a constructivist perspective’ is used, here, in an excessive way, but one needs to keep in mind that more than one constructivist perspectives have been formed within the field of (mathematics) education, such as interactive, dialectic, radical, social etc (Chronaki 1992; Chronaki 1997).

<sup>4</sup> In a similar vein, one needs to mention diversification across a variety of socio-cultural perspectives ranging the emphasis from psychological to cultural, anthropological and critical approaches to learning and communicating (Kontopodis et al., under publication).

phasizes semiotic mediation, tool-use, and collective engagement with purposefully organized activity. The learner is conceived as a motivated subject who needs to actively interact with more knowledgeable others and to purposefully use tools that bridge the gap among past, present and future historical practices, forming zones of proximal development (Wertsch 1991).

Stressing the urgency for quality at the micro-level is not isolated from the macro-level reform agendas in mathematics education at national and international levels (TIMMS 2007). Certain curricula politics (i.e. prescriptions for content, skills and competences, assessment methods) act as 'ideological state apparatus' (Althusser 1971) that regulate behaviour at the micro-level of human interactions. In that way, reform implementation mediates the macro-level societal structuring and creates micro-spaces (e.g. didactic innovations) where self and society develop together. As such, the stress for quality in mathematics education curricula cannot be considered neutral. Mainstream constructivist and socio-cultural perspectives work synergistically towards this end and provide a language for re-producing and legitimizing discourses of 'quality'. Such discourses materialize by means of curriculum reforms and innovations in schools and classrooms (i.e. mathematical content and competences such as active learning, collaborative work, technology-use etc) *thus* producing fixed identities of the 'good' learner and teacher. Walshaw (2001, p.96) highlights that the learner is seen as constructing '*...viable theories of the ways in which the world works*', the teacher as facilitating and empowering learners to '*...give voice to their subjugated knowledge*' and that learner's personalized and localized knowledge '*...generate not only visibility but also are said to offer agency in terms of identity and position from which they might act for change*'. However, she critiques the view that subject agency can be easily fixed through suitable didactic interventions.

Within this realm, technology mediated mathematics learning enjoys a prominent position within recent curricula reforms in mathematics education. For example, there is evidence that certain digital tools suitable for dynamic geometry, computer algebra, data handling, statistics, programming and modelling can be instrumentally utilised towards encouraging the development of specific mathematical skills and competences such as visualising, representing and manipulating symbolic entities such as mathematical ideas. At the same time, they foster certain ways of working such as collaboration, reflection, active experimentation etc (NCTM 2000; Hershkowitz et al 2002; Ruthven et al. 2004). Technology-based mathematics edu-

cation becomes a political arena for teachers, learners and curriculum designers towards producing a particular collective identity change in the name of the 'new' math teacher who safeguards 'quality' learning. Specifically, the 'new' maths teacher is required to be a flexible facilitator of knowledge construction, as opposed to knowledge transmitter in the traditional paradigm (Chronaki 2000). Stressing the transformative role of 'new' technology is an old issue that reflects broader socio-economic politics in the so-called 'new' information age (Castells 1996/2000). The sense of 'new' becomes a reference to the most glamorous recent past and implies that 'new' equals 'better' and thus 'new' is associated with quality. The 'new' signifies 'the cutting edge', the avant-garde, the place for forward-thinking people to perform (and become) designers, producers and practitioners. Thus, discourses of 'change' tend to become avenues to 'new' and relate to long-lasting modernist views of social progress and development as smoothly delivered by technology (Lister et al. 2003).

Investment on such discourses emphasizes the revolutionary impact of technology towards producing profound transformations of maths teachers' everyday life in terms of evolving techniques, skills, relations, feelings, communicative practices, and organisational structures. The transformative impact of 'new' technologies has been mainly discussed as far as it concerns epistemological, pedagogic and didactic potential for change through the analysis of focused teaching experiments (Marriotti 2002). Despite the benefits outlined in such exemplary cases, and the high investment on time and economic resources, widespread technology integration in mathematics classrooms remains a challenge (Ruthven et al. 2004). In addition, a number of studies indicate how female teachers and students do not choose related fields to study and work and rarely report long-lasting transformative experiences (Wacjman 2007). In a similar vein, Anita, Afrodite and the female engineers position themselves in discourses that inscribe them as 'different' when compared to men on the basis of lacking not only passion, but also the flair for active engagement and the competence for deliberative decision making. However, the discourse on 'difference' can easily slip into discourses of 'gender gap' and 'female danger'. Although, Anita, Giorgos and Afrodite are different cases in terms of age, gender, race, and school role (teacher, undergraduate student, and secondary school student), they all seem to support the view that women's liaison with mathematics and technology is not only uneasy, but it can be a marginalised or a 'dangerous' one. How else, could one explain Anita's choice to withdraw from a successful



future in mathematics since she senses that aspects of her everyday life might be in danger, and Afrodite's conflicting experiences that lead her to consider quitting school? But also, how could one predict where young female engineering students might end up in their careers since they, according to Giorgos, lack a passion for technology and science?

In this realm, female teachers and students easily fall into the stereotypical image of resisting technology. An alternative reading is that some teachers and students do not resist technology itself but the stressful requirements for immediate 'change' towards a predefined quality agenda. They realise technology as a risky terrain and they set boundaries on technology use. Illich (1972) has argued that good and evil are not attributes of technology per se, but of technologies-in-use. For example, Anita realises how incompatible is for her to invest on mathematics or technology as a career pursuit. Similarly, female students reject a passionate relation to technology and concentrate, instead, on more pragmatic approach in specific localities. Whilst, males are believed to be passionately attracted by 'new' technology, as Giorgos, the young engineer states, their female counterparts, although competent, do not perform a passionate desire.

Instead of pursuing uncritically a path towards identity 'change' our data of women narratives urge us to consider the human-machine relation as situated in everyday practices. This view agrees with feminist perspectives on technoscience that alert for the importance to move away from a view of 'change' as development towards a full masculine self-realisation. Specifically, Haraway critiques a number of Marxist and psychoanalytic epistemological positions on feminism and turns to explore the complex production of woman/difference/other in relation to technoscience. Striving for a move away from dichotomies, dualisms or binaries situated in discourses that indicate a 'lack' (e.g. lack of passion, interest, competence) and reproduce gendered technological essentialism or technophobia, Donna Haraway introduces the notion of 'cyborg'<sup>5</sup> as a metaphor for a hybrid entity that blurs

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<sup>5</sup> Cyborg, short for cybernetic organism, is a term coined by the research scientists Manfred Clynes and Nathan Kline in the '60s as they tried to imagine the kind of augmented man that would be necessary for extra-terrestrial exploration or space flight. It refers most particularly to an imagined and actual mix of machine and organism so as to constitute an integrated information circuit. [...] The first cyborg, from Clynes and Kline's lab was a white lab-rat with an osmotic pump implanted to allow the researchers to inject chemicals to control and observe aspects of the rat's physiology. [...]. Donna Haraway has taken cyborg as a metaphor to draw together an array of critical questions about human-machine relations

the boundaries between organic and mechanic. The cyborg refers to the ontology of an enhanced command-control-communication-intelligence system (c3i) where human-machine organisms are integrated into a symbiosis that transforms both (Haraway 2004, 2009).

The cyborg, short for *cybernetic organism*, is taken to be the image of an 'augmented human' suitable for extra-terrestrial explorations, scientific experiments and science fiction narratives. But, Donna Haraway uses the term as a prime resource to imagine an alternative kind of material-semiotic world, an alternative perspective of identity politics, and in consequence an alternative optic of feminist technoscience. The notion of cyborg denotes that dichotomies and dualisms such as nature/culture, woman/man, body/mind can no longer be used to figure or create the other. She claims that; '[...] *Instead, the cyborg is resolutely committed to partiality, irony, intimacy, and perversity. It is oppositional, utopian, and completely without innocence. Cyborgs are not reverent; they do not re-member the cosmos. They are wary of holism, but needy of connection*' (Haraway 2000; Schneider 2005, p.64).

This particular 'cyborg' point of view allows us to re-consider female relation with technoscience by appreciating its intense partiality. In this sense, Anita's and female engineers' experience as non-passionate, partial, disloyal could be considered as a 'cyborg' position. They can be 'augmented' human creatures as they appropriate, utilize and negotiate varied uses and productions of technology in everyday lives. Through cyborg a more nuanced and complex angle of vision is offered that sees the technoscientific as a field for the contestation of meaning and the possibility of remoulding and redirecting what looks repressive into something more subversive and even democratic. While fully aware of the fact that the image of the cyborg could be as much about global control and domination, or about pre-emptive strikes and imperialism masked as deterrence or defence, Haraway (1985) offers an alternative possibility; '*[A] cyborg world might be about lived social and bodily realities in which people are not afraid of their joint kinship with animals and machines, not afraid of permanently partial identities and contradictory standpoints. The political struggle is to see from both perspectives at once because each reveals both dominations*

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and varied embodied forms of technoscience as part of socialist feminism. Recently, the cyborg had emerged as a figure in popular culture and especially in science fiction (Clynes and Kline 1960; Haraway 1991).

*and possibilities unimaginable from the other vantage point*'. (Schneider 2005, p.72). As such, the cyborg is not only an image or figure, an entity in reality or imagination, but it is also a standpoint, a way of thinking and seeing.

Calling the late twentieth-century understanding of the relationship between organism and machine a 'border war', Donna Haraway (1997, 2006/2009) recommends instead a pleasure to be found in bringing about the destabilization of these boundaries and an accompanying heightened '*responsibility in their construction*'. This means that intensity for 'change' via discourses of 'quality' cannot be taken for granted as if it constitutes a 'normal', 'neutral' or 'static' path for development. Discourses of quality as identity change towards developing a 'fixed' list of goals promotes 'fixed' and 'static' identities and denies a 'cyborg' view on women's experience with technology. Specifically, it conceals the fact that 'development' happens in multiple, complex and hybrid ways where boundaries between humans and machines are disintegrated and destabilised. In the following section, development as equity will be discussed as the urgency *for all* to change with/in school maths.

### **DEVELOPMENT AS EQUITY: THE URGENCY FOR ALL TO CHANGE**

In the field of mathematics education, gender inequity has been, mainly, explored in two interrelated dimensions using, at large, comparative analysis; first in relation to boys' and girls' achievements in specific mathematical curriculum content areas, and second, in relation to male *vs* female participation in areas of study and work that require mathematical knowledge and competences. As far as achievement in particular curricular areas of mathematics (geometry, algebra, problem solving) is concerned, the quantitative data gathered during the last decade informs us that male-female differences have started not only to disappear but even to reverse, since in some countries (e.g. Iceland and Cuba) we, recently, witness some female advantage (Xin Ma 2008 based on a meta-analysis of regional and international studies on student assessment). A number of meta-analytic review studies concerning the relative interdependence of variables such as gender, class, achievement, attitudes, cognitive and meta-cognitive strategies seem to agree that the gender gap has gradually been eliminated (Hanna 2003; Xin Ma 2008). When the dimension of women's career paths is considered, recent research outcomes points out that although there is some considerable increase in the presence of females in areas of study, research and work,

their participation in scientific fields still remains unsatisfactory (Jutting et al. 2006). For example, the American Mathematical Society (AMS) reports a slight increase in the representation of women in academic editorials boards. Specifically, they explain that between 1994 and 2003 women representation rate has increased from 9% in 1994 to 16% in 2003.

The situation is far more devastating in countries of the so-called developing world, where women still have limited access to work and education. Dunne and Sayed (2002), for example, explain that, in southern African countries only 5% of all female students enrol in mathematics, computing and engineering. Frantzi (2008) investigating women who enrol in mathematics related higher degrees in Greece observes that whilst before the Second World War women mathematicians were a rare phenomenon and mainly came from the middle or upper classes, during the period 1940–1964, more women enrolled to study mathematics and they came mostly from a lower middle class background. Gender inequity, thus, is not an isolated phenomenon but rather greatly related to class, colonial, racial, and cultural constraints experienced by the individual as s/he struggles for access and participation in related practices.

One might observe that although there is noticeable increase in female achievement and participation, the gap between males and females continues to create social inequity. Even though female students are as competent as male and enjoy practicing technoscience, they continue not to choose the subject as a main field for study or work. The narrative of Afrodite, a young Gypsy Greek girl, as seen in the introductory vignette, indicates how both gender and race discourses prevent not only her continuous participation in schooling practices thus making her a case at risk, but also constitute her ‘voiceless’. Spivak uses the term ‘subaltern’<sup>6</sup> to talk about how certain colonial and postcolonial discourses constitute not really the ‘voiceless subject’, but the subject who realises the impossibility of ‘voice’. In exemplary cases of female struggles in imperial India she problematizes how the colonial world has always been defined by the West. According to Spivak

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<sup>6</sup>Gramsci has originally coined the term ‘subaltern’ in order to address the economically dispossessed, and today Ranajit Guha reappropriates Gramsci’s term in an effort to locate and re-establish a voice or collective locus of agency in postcolonial India. In her essay “Can the Subaltern Speak?”, Spivak acknowledges the importance of understanding the ‘subaltern’ standpoint but also criticizes the efforts of certain subaltern studies emphasis towards creating a ‘collective voice’ through westernised mediating practices.

(1999, 1992) civilization, progress, and even self-identity itself always eludes the subaltern. In other words, the West is defined by its differentiation between the 'present', 'past' and 'future' as well as a sense of the other. The colonial world has no such self-identity, at least as the Western viewer perceives it. The cry in Afrodite's diary-writing, perhaps, denotes exactly this awareness of the impossibility to speak and become heard about non-easily fixed, almost un-resolvable, issues.

Based on Spivak we realize how Afrodite becomes doubly the 'other' as a woman and as a gypsy woman and how she realizes her fragile and fractured self as she attempts to cross cultural borders amongst home and school. She struggles to live in-between two worlds that require her to continuously cope with conflicting choices and feels emotionally devastated as her diary-writing reveals (Dafermos 2005, p. 257-259). Anzaldúa (1987) argues that crossing 'borders' is not a simple but instead a process of learning to accept transformations and learning to tolerate contradictions and ambiguities – a 'mestiza' rhetoric in her words. Mendick (2005) refers to Anelia, a young Turkish Cypriot girl from a UK based immigrant family, who also experiences the home/school divide. Anelia, as Afrodite, has a passion for (mathematics) education, but she also perceives that making the choice to study could be incompatible for her life because she holds that 'mathematics' is not suitable for a woman. Participation in formal educational practices means for Anelia, as well as for Afrodite, engaging in identity-work that creates multiple contradictions in her life and leads her towards limiting the study of science or considering quitting school (see the case of Afrodite). While both women cope well with formal educational activities - including mathematics and technology- they risk being characterised as 'savage', primitive or other. Although this view is highly criticized by contemporary anthropological thinking for being an imposed 'gaze' at non-Western cultures (Appadurai 1996; Harding 1998, 2008), such differentiation serves to reinforce the epistemic chasm between savage and rational mind by perpetuating knowledge hierarchies. In addition, this chasm shows, as Spivak (1992b) argues, a concern for the processes whereby postcolonial studies rehearse neo-colonial imperatives of political domination, economic exploitation and cultural erasure –an issue referred to by Spivak as 'epistemic violence'. It can be claimed that this is due to the fact that 'development' for Afrodite and Anelia is counted on an imperialist conception of the world and of technoscience. Spivak's post-colonial critique addresses the western, male, privileged, academic, institutionalized discourses which clas-

sify the 'other' in the same measures used by colonial dominance that, ironically, seek to dismantle.

Most Gypsy girls do not perform as individual 'entrepreneurs of self', using Paul DuGay's words, as their decisions in life are depended on extended family and community values, needs or habits (DuGay 1996). Living between two cultures, Afrodite has to confront conflicting discourses about either 'attachement to community life' or 'pressure to lead a modern life' (Chronaki 2009). For her, it is not an either/or situation but instead a desire to be both and this very fact places her in a painful situation. Afrodite's dilemma whether to quit school or not is connected with pathologising her as incapable of making a sensible choice and as destined to remain subaltern. How could we, then, reconsider 'equity' in view of Afrodite? This means that gender equity cannot be simply viewed in terms of comparative studies of male's and females' skills and attitudes rooted in quantitative analysis or positivistic interpretations. Afrodite's urgency to develop is also linked to the urgency to move towards a certain quality of modern 'life' inscribed through masculine and imperialist agendas of development. Her case, in particular, urges us to consider this 'move' as an unfulfilled promise or as potentially unending.

A major consequence of hegemonic discourses of equity is the constitution of subjects as marginalised, oppressed, or voiceless. In an almost pessimistic tone Spivak concludes that the 'subaltern' cannot speak in the context of cultural imperialisms and moreover the 'subaltern' cannot be given a voice via a mediator. She specifically suggests that any attempt from the outside to grant subalterns a 'collective voice' is problematic as first, it assumes cultural solidarity among members of a heterogeneous group of people, and secondly it depends upon western intellectuals to 'speak for' their condition. Spivak argues that through such a process the subalterns, in fact, re-inscribe their marginalised and subordinate positions. Afrodite seems to fall into this category. As a gypsy woman she is required to perform a 'normal' gendered positioning as constructed by her community. Taking into account that 'normal' is a fictional category one can claim that there is no normal way for any gender to act. Gayatri Chakravorty Spivak optimistically argues that although we cannot 'give' a voice, we can clear the space for the subaltern to speak. She suggests that instead of urging for a 'collective voice' by means of the Western logos, it is preferable to focus on clearing the subalterns' path so that their voice can be heard. The subaltern, be it a Gypsy adolescent girl or a Western woman who, though competent with

computers and maths, chooses not to make them a priority in life, seems to live at the margins of hegemonic discourses of ‘development’. Clearing the path for them to be heard, in the context of this study, is closely related to troubling and disrupting ‘development’ within hegemonic discourses by revealing contradictions and taking seriously the contextual processes that constitute marginalised and voiceless positioning(s). Growth, progress, development all seem to safeguard quality. And access to *quality for all* is assumed to be the measure for equity. Quality and equity, thus, become the two opposite sides of the same coin called ‘development’.

### CONCLUSIONARY REMARKS

Technoscientific practices, and school technoscience in particular, are central to both self and society ‘development’. A recent anthropological study shows how ‘namba tok’ (number talk or the use of statistics by colonial officials) is coupled with ‘kaantri’ (country) creation in the consciousness of the Nimakot people of central New Guinea who see their lives changing from nomadic to settled inhabitancy (Wesch 2007). What we have come to call ‘modern’ society has emerged through production and appropriation of a variety of ‘technologies’ including arithmetic, archiving and spacing structural systems utilised to organise and control daily mind-body practices. Dunne (2008), based on Foucault (1977/1980, 1991) discusses the political role of mathematics and technology as core values of modernity and explains how both are utilized to define and fulfil goals of ‘development’. This happens simultaneously at two levels; *first*, they are used to measure the achievement of certain predicted economic, social, and educational outcomes, through a broad application of statistics, and *secondly*, by applying pressure, via local and national educational policies, mathematical and technological literacy is promoted. An imperialist (and sometimes a post-colonial) agenda of governmentality means that women’s access to and participation in these subject areas are measured and evaluated against that of men and western culture. In other words, the dominant discourse of development serves to legitimise ‘women’ as ‘others’ (i.e. women, as primitives, need to develop and progress).

From this point of view, technology-mediated mathematics education is not merely a tool for better understanding mathematical concepts, but can be seen as a tool for introducing learners to certain standards of ‘modern’ life – and for some (including women) this can be a risky, unsafe and uncertain terrain. Hegemonic discourses, based mainly on constructivist and sociocultural agendas, tend to overemphasize the ‘active’, ‘rational’, ‘autonomous’

learner who is able to instrumentally utilize any accessible technology and make timely choices and decisions. However such a view eschews the ideological underpinnings of an oversimplified adherence to modernist and neo-liberal ideologies. Valerie Walkerdine (1993) and Nikolas Rose (1999), amongst others, explain that discourses related to an impetus to govern modern life are based on the virtue of self-reliance (autonomy, self-regulation, self-efficacy etc) and reflect mainstream and conservative psychology or sociology. Rose (1999), in particular, supports that the burden of 'choice' conceals the broader social context in which jobs for life have disappeared leaving the fiction of life-long learning instead. Simultaneously, inability to choose, to act or to make appropriate decisions signifies inability to perform as an 'autonomous subject' which then results into lack of development and leads to marginalisation. As explained above, self /society development requires both a quality and equity dimension. Within the confines of imperialist, colonial and patriarchal discourses, development is taken to be equivalent to the construction of a fixed 'rationality' as the ultimate goal for quality. Rational development is also taken to be at the heart of technoscientific practices including mathematics and technology related literacies. Therefore, quality in mathematics education curricula and practices is taken to be a cornerstone for safeguarding quality/equity and minimising exclusion and marginalisation.

However, women often seem to either resist or embrace partially and without passion certain technoscientific practices affecting their daily life or work. Such a standpoint can be stereotypically interpreted as the 'woman as a problem' situation –an interpretation rooted in hegemonic discourses of quality/equity. As previously seen, on the one hand, some mainstream constructivist and mainstream socio-cultural perspectives strive to prescribe quality in mathematics education, and on the other hand, certain mainstream feminist perspectives focus on investigating gender inequity not only at the level of achievement, competences and attitudes but also at the level of access to and participation in mathematics and technology related fields. While constructivist and socio-cultural theorists emphasize quality curricula, feminists identify gender gaps. In simplified terms, it may seem that one's work serves (to sustain) the work of the other. In other words, when a gender gap becomes identified, a quality curriculum will be there to fill the gap. But life is not that easy.

In the realm of the present chapter, it has been argued that hegemonic discourses of quality/equity as means for self/society development need to



be approached through alternative perspectives that enable subjects to move beyond a pressurising emphasis to a singular ‘perfectionist’ relation to technoscience. Hegemonic discourses tend to read women as ‘others’ by their being considered, perhaps unintentionally, as the passionless and subordinate users of technoscience. By re-reading these stories we come to realise, that involvement in mathematics and technology in school practices is *neither* simply a matter of access to equitable sharing of resources, knowledge and support *nor* an issue of a particularly passionate interest and positive attitude towards the subject. Women and men seem to live in complex localities that require them to simultaneously appropriate not one but a number of discourses that often become competing forces in both personal and school lives. The notion of ‘cyborg’ induces a renewed vision of quality, as far as the subject’s involvement in technoscience is concerned, emphasizing partiality and hybridity. Women as ‘cyborgs’ can be fragile and fractured amalgams of a human-machine organism and can claim for themselves the right to ‘error’, to express ‘failure’, to demand ‘connectivity’ and to feel confident with ‘partiality’. According to Donna Haraway (2006/2009), it is not the ‘machine’ that women reject but the insecurity that comes as a result of communication breakdown. In other words, it is the fact that they do not seem to have control over the fluid relation which develops between humans and machines that requires a ‘holistic’ instead of a ‘connectivist’ relation to technology. The cyborg, thus, has the potential to become a way of thinking and re-working subjectivity as situated, hybrid and partial.

In addition, the notion of ‘subaltern’, as argued by Gayatri Chakravorty Spivak, offers an alternative optic on issues of difference or otherness as they affect marginalised, oppressed and voiceless subjectivities. She claims that by having limited access to cultural imperialism and by being constructed as ‘different’ or ‘other’, the subaltern can signify the ‘proletarian’ whose voice can not be heard as it is structurally deleted from the capitalist bourgeois narrative. Furthermore, she objects to the view that since the subaltern cannot speak, an advocate is required to speak for her, arguing: ‘*Who the hell wants to protect subalternity? Only extremely reactionary, dubious anthropologic museumizers. No activist wants to keep the subaltern in the space of difference [...] You don't give the subaltern voice. You work for the bloody subaltern, you work against subalternity*’ (Spivak 1992, p.46). The burden created by the organisation of ‘collective’ or ‘mediated’ voices for subalterns constitutes, according to Spivak, a rehearsal of a political domination of ‘voice’ via neo-colonial exploitation that ultimately exacerbates

‘epistemic violence’. Instead, she voices the need to seriously consider clearing the way for the subaltern to speak.

As it has already been shown, clearing the way is a process of disrupting the hegemonic discourses of development that either implicitly or explicitly nurture subalternity. While Haraway promotes a notion of the cyborg that opens up subjectivity to embrace situatedness, hybridity and partiality, Spivak enters the complexities of marginalised and voiceless subjectivity by encountering the subaltern’s voice. The impossible task of being heard signifies the impossibility of realising self as part of society or else the impossibility of belonging. Spivak doesn’t hesitate to criticise the post-colonial practices that assume a ‘voice’ can be given via the mediation of an advocate and passionately argues that the subaltern do not need to be given a ‘voice’ but instead we need to clear the way for them to walk and be heard. This very simple, but important gesture means that the responsibility of their having a ‘voice’ is simultaneously our responsibility of listening to their voices – a deeply dialogical gesture. As a final word, I would like to argue for the need to consider involvement in school technoscience as a risky gendered situation where subjects negotiate their positions by taking the boundaries and affordances of their localities into account. A situated notion of agency with/in technoscientific practices rejects the utopian and imperialist politics of ‘holism’, ‘advocacy’, ‘perfectionism’ and, instead, pursues ‘connectivism’ and ‘partiality’. For this reason, a turn towards post-structural and postcolonial theorising of female experiences with school technoscience may prove most valuable.

## REFERENCES

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- Althusser, L. (1971). Ideology and ideological state apparatuses. In L. Althusser (ed.), *Lenin and Philosophy and other essays* (translated by Ben Brewster) (121-176). London: New Left Books
- Anzaldúa, G. E. (1987). *Borderlands/La Frontera: The New Mestiza*. Aunt Lute Books
- Appadurai, A. (1996). *Modernity at Large: Cultural Dimensions of Globalization*. Minneapolis: University of Minnesota Press.
- Belenky, M., Clinchy, B., Goldberger, N. & Tarule, J. (1986). *Women’s Ways of Knowing: the development of self, voice, and mind*. New York: Basic Books

- 
- Butler, J.P. (1997). *Excitable Speech: A Politics of the Performative*. New York: Routledge.
- Butler, J.P. (1990). *Gender Trouble: Feminism and the Subversion of Identity*. New York: Routledge
- Castells, M. (1996/2000). *The Rise of the Network Society*. Malden, MA.: Blackwell
- Chronaki, A. (2009). An entry to dialogicality in the maths classroom: encouraging hybrid learning identities. In M. Cesar & K. Kumpulainen (Eds.), *Social Interactions in Multicultural Settings* (117-144). Rotterdam: Sense Publishers
- Chronaki, A. (Ed.). (2008). *Gender and Technology in the Body of Education*. Volos: University of Thessaly Press.
- Chronaki, A. (2000). Computers in Classrooms: learners and teachers in new roles. In B. Moon, S. Brown, & M. Ben-Peretz (Eds.), *International Companion in Education*. London: Routledge.
- Chronaki, A. (1997). Constructivism as an energiser for thinking. *Chreods: Journal on Philosophical Issues of Mathematics Education, Special Issue on Mathematics Education and Society: Radical Visions and Socialist Perspectives* (guest editor Peter Gates), 11, 15-21
- Chronaki, A. (1992). *The Epistemology of Constructivism in Mathematics Education*. Bath: University of Bath
- Clynes, M.E & Kline, N.S. (1960). Cyborgs and Space. *Astronautics*. September 26-27, 5-76
- Dafermos, M. (2005). *Marginalisation and Educational Inclusion*. Athens: Atrapos (in Greek).
- Du Gay, P. (1996). *Consumption and identity at work*. London: Sage.
- Dunne, M. (2008). Gendered docility: the power of technology and technologies of power for low-income countries. In A. Chronaki (ed.), *Mathematics, Technology, Education: The Gender Perspective* (45-57). Volos: University of Thessaly Press
- Dunne, M. & Sayed, Y. (2002). Transformation and Equity: Women and Higher Education in Sub-Saharan Africa. *International Studies in Educational Administration*, 30(1), 50-65
- Ellul, J. (1964). *The Technological Society*. New York: Alfred A. Knopf
- Fennema, E., & Leder, G. (Eds.). (1990). *Mathematics and Gender: Influences on Teachers and Students*. New York: Teachers College Press

- Foucault, M. (1991). Governmentality. In G. Bruchell, C. Gordon & P. Miller (Eds.), *The Foucault Effect: Studies in Governmentality* (87-104). Hemel Hempstead: Harvester Wheatsheaf
- Foucault, M. (1977/1980). *Discipline and Punish: The Birth of the Prison*. New York: Pantheon
- Foucault, M. (1972). *Archaeology of knowledge*. New York: Pantheon
- Frantzi, K. (2008). Greek Women Mathematicians in the 20<sup>th</sup> century: An overview. In A.Chronaki (Ed.), *Mathematics, Technology, Education: The Gender Perspective* (111-114). Volos: University of Thessaly Press
- Greeno, J. & The Middle School Mathematics Through Applications Project Group (1998). The situativity of knowing, learning, and research, *American Psychologist*, **53**(1), 5–26
- Hanna, G. (2003). Reaching Gender Equity in Mathematics Education. *The Educational Forum*, **67**, 204-214
- Haraway, D. (2006/2009). Feminist Epistemology and Philosophy of Science. Retrieved March 14, 2010 from <http://plato.stanford.edu/entries/feminism-epistemology/>
- Haraway, D. (2004). *The Companion Species Manifesto: Dogs, People and Significant Otherness*. Prickly Paradigm Press
- Haraway, D. (1997) *Modest\_Witness@Second\_Millennium. Female-Man©Meets\_OncoMouse™: Feminism and Technoscience*. New York: Routledge
- Harding, S. (2008). *Sciences from Below: Feminisms, Postcolonialities and Modernities*. Duke University Press
- Harding, S. (1998). *Is Science Multicultural? Postcolonialisms, Feminisms, and Epistemologies*. Bloomington: Indiana University Press
- Hershkowitz, R., Dreyfus, T., Schwarz, B., Ben-Zvi, D., Friedlander, A., Hadas, N., Resnick, T., & Tabach, M. (2002). Mathematics curriculum development for computerized environments: A designer-researcher-teacher-learner activity. In L. D. English (Ed.), *Handbook of international research in mathematics education* (657–694). London: Erlbaum
- Illich, I. (1972). *Tools for Conviviality*. New York:Harper & Row
- Jutting, J.P., Morrisson, C., Dayton-Johnson,J. & Drechsler, D. (2006). *Measuring Gender (In) Equality: Introducing the Gender, Institutions and Development*

- Kontopodis, M., Wulf, Ch. & Fichtner, B. (submitted). *Children, Culture and Education*, (series: International Perspectives on Early Childhood and Development). New York: Springer
- Lakoff, G. (1987). *Women, Fire, and Dangerous Things: What Categories Reveal About the Mind*. University of Chicago Press
- Lister, M., Dovey, J., Giddings, S., Grant, I., Kelly, K. (Eds.). (2003). *New Media: A Critical Introduction*. London: Routledge
- Marriotti, M. (2002). The influence of technological advances on students' mathematics learning. In L. English et al (Eds.), *Handbook of International Research in Mathematics Education* (695-723). Mahwah, NJ: Erlbaum
- Mendick, H. (2005). *Masculinities in Mathematics Education*. Maidenhead. Berkshire: Open University Press
- NCTM: National Council of Teachers of Mathematics. (2000). *NCTM Principles and standards for school mathematics*. Reston, VA: Author
- Rose, N. (1999). *Powers of Freedom: Reframing Political Thought*. Cambridge: Cambridge University Press
- Ruthven, K., Hennessy, S. & Brindley, S. (2004). Teacher perspectives on integrating ICT into subject teaching: Commitment, constraints, caution and change. *Journal of Curriculum Studies*, 37(2), 155-192
- Schneider, J. (2005). *Donna Haraway: Live Theory*. New York: Continuum
- Schoenfeld, A. (Ed.). (1994). *Mathematical Thinking and Problem Solving*. Hillsdale, NJ: Lawrence Erlbaum Associates
- Secada, W. (1995). Social and critical dimensions for equity in mathematics education. In W. Secada, E. Fenemma & L.B. Adajan (Eds.), *New directions for equity in mathematics education* (146-164). New York: Cambridge University Press
- Secada, W. (1992). Race, ethnicity, social class, language and achievement in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (623-660). New York: Macmillan
- Spivak, G. C. (1999). *A Critique of Postcolonial Reason: Towards a History of the Vanishing Present*. Cambridge, Massachusetts: Harvard University Press
- Spivak, G. C. (1992a): Interview with Gayatri Chakravorty Spivak: New Nation Writers Conference in South Africa (Interviewer Leon de Kock). *Ariel: A Review of International English Literature*, 23 (3), 29-47

- Spivak, G. C. (1992b). Can the Subaltern Speak? In P. Williams & L. Chrisman (Eds.), *Colonial Discourse and Post-Colonial Theory* (66-111). New York: Columbia University Press
- TIMMS. (2007). Trends in International Mathematics and Science Study: Results 2007. Retrieved from <http://nces.ed.gov/timss/results07.asp>
- Treffers, A. (1987). *Three Dimensions: A Model Goal and Theory Description in Mathematics Instruction - The Wiskobas Project*. Mathematics Education Library :D. Reitel Publishing Company
- Turkle, S. (2008). *Falling for Science: Objects in Mind*. Cambridge: The MIT press
- Wajcman, J. (2007). From Women and Technology to Gendered Technoscience. *Information, Communication & Society*, 10, 287-298
- Walkerline, V. (1993). Beyond Developmentalism? *Theory & Psychology*, 3(4), 451-469
- Walshaw, M. (2001). A Foucauldian Gaze on Gender Research: What Do you Do when confronted with the Tunnel at the End of the Light? *Journal of Research in Mathematics Education*, 32(5), 471-492
- Wertsch, J. V. (1991) *Voices of the mind: A sociocultural approach to mediated action*. Cambridge, MA: Harvard University Press
- Wesch, M. (2007). Creating 'Kantri' in Central New Guinea: Relational Ontology and the Categorical Logic of Statecraft. *Media/Culture*, 11(5). Retrieved from <http://journal.media-culture.org.au/index.php/mcjournal/article/viewArticle/67>
- Xin Ma (2008). Gender Differences in Mathematics Achievement: Evidence from Latest Regional and International Student Assessments. Paper presented at the ICME 11. Mexico. Topic Study Group 32: Gender and Mathematics Education

## Special Session

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### **Misconceptions and misunderstandings. Can statistics education improve democracy?**

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The development of technology and computers led to an outbreak of gathering information data and fed the need of the evaluation and use of the gathered data. Information as a property product and as a means of policy making replaced in a great degree the material products leading to a new form of concentrating power.

Statistics today, in the era of information outbreak can be generally defined as the  
*“Science that is concerned with the gathering, evaluation and processing of information”*.

The demand by the society for qualitatively controlled information absolved from “noises” which intentionally or without purpose are included, is especially obvious in a returning, from time to time discussion, about the control of the public measurements related to the Mass Media audience, political parties and persons, educational parameters, economical and social indexes and factors, etc.

The aims of the school curriculum in Primary and Secondary education are accordingly significant in relation to the teaching hours referred to them in the curriculum and are especially referred in the pupils’ accomplishment of the process and the meaning of organizing and conducting “researches” as well as the analysis of their data. Although the above are important for the making of a critically thinking civilian who can evaluate the information received, something that nowadays has a central role in education, it is limited by the curriculum in the least teaching hours (4 and 8) without

offering accordingly the necessary training to teachers'-trainers of Primary and Secondary education.

The small access of Statisticians in the educational planning and the confusion of researchers who use data and handle statistical methods, that they are Statisticians, and the limited use of official statistical sources in the educational process, have caused the stereotype that Statistics is only using statistical packages, tabling results and presenting data using graphs. So, it happens often, that the necessary requirements for the application of statistical techniques and methods are not kept or improper methods are used.

Statistics is not a total of rules and recipes for the analysis of data. It is not exhausted in the use of complicated computerized programs and nice graphs. It requires a good knowledge of the observed phenomenon, the organization of observation, good knowledge of gathered data, and description and examination of hypotheses for the parameters of the analyzed phenomenon.

In the level of Higher Education Statistics is taught as an obligatory subject in all university departments with a main goal the comprehension of students' for the use of statistical techniques orientated to the specific field of knowledge. Although students of so-called "*theoretical*" departments consider that they do not have a good relation with numbers, mathematics, statistics and computers, the need of experimental research leads them to the need of understanding and using not only of simple descriptive methods, but especially advanced statistical techniques which demand mainly "*mathematical thought*", that is the ability of constructing, using and explaining abstract models from daily examples.

In this paper we will refer more extensively in the problems of using statistical techniques in the political sciences which is connected to democracy.

Prior to electoral processes the civilians are bombarded with information, graphs, tables and findings of opinion polls which are referred in indexes, figures such as the expected share of parties participating in elections, the publicity of persons, evaluative classification of issues and politicians. After the elections the results are explained and analyzed with the use of advanced



statistical techniques aiming to support the pre-electoral statements and attitudes of political parties.

By following and commenting the phases of observation of electoral behavior we will refer to some problems.

### **1. Electoral behavior**

The methods of prediction and observation of electoral (and widely the political) behavior of the electorate have a long history of application and use in the countries of Western Europe and the U.S.A. and they decisively depend on the specification of each country (political history, electoral system, electoral framework, etc.), we can be divided in two sub-categories: Opinion Polls and PANEL researches.

In the first category, Opinion Polls, there is an effort to get results by the study of a sample of the electorate about behavior and the opinion of the total of voters often with repetitive polls on the same or similar sample. As it is in every opinion poll, the initial problem is the selection of the sample, which should be representative (of the population) so that the drawn results could be generalized by the use of the proper statistical techniques. At the same time, the tool that is used for the measurement of behavior is very important, that is the questionnaire and the method by which the measurements are conducted (by telephone, mail, and interview).

In the formation of the sample some demographical factors (such as sex, age, social and financial status), which influence electoral behavior, should also be considered as well as the previous electoral behavior of the electorate (results of previous elections, behavior of the sample in previous elections, etc).

In other countries the structure of areas with homogeneous social characteristics helps in the formation of the sample using geographical criteria, (blocks are usually chosen first with an attention to population quota of the specific area and the number of households of these blocks), though later on there is an effort to weight the sample according to the general demographic characteristics based on already known population parameters (e.g. sex, age).

In Greece the formation of mixed areas of residence (which are not defined by specific social groups), except for some exceptions, makes us to

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see skeptically geographical methods of selecting the sample, and we rather pay more attention to criteria of electoral behavior for the selection of the sample.

Of course, when the electoral behavior of the electorate is homogenous, (they all behave in almost the same way), whether we follow a geographical selection of sample or not, the result of the poll will be the same. Accordingly, our results cannot be especially analytical because there is no identification of the electorate with the resident-voters of the area.

The formation of the electoral registration leads to essential differentiation between voters and residents. It has been estimated that 50% of the residents (of the urban area of Thessaloniki) vote in their residence-voting area. In theoretical level political and (as an effect electoral) behavior is formed in four levels (central political stage, work, neighborhood and family). The inability of observing the two “*medium*” levels, because of the above, leads to results with a specific significance from the central political stage underestimating the local features.

As far as the method of research is concerned, due to the general behavior of the Greeks in opinion polls, interview (using a structured questionnaire) is considered a better method in comparison to others methods (internet, phone, by mail, etc.).

While publishing the results, many analyzers and journalists do not understand that the values that are counted for the figures (usually percentages or flows) are not accurate values but estimations. They do not understand that a confidence interval means that the real size (e.g. the percentage of a political party) is found with exactly the same probability between the upper and lower limit, considering that it is found nearer to the center of the confidence interval. Yet, the possibilities of getting results about subpopulations of the electorate are limited as they depend on the design of the sample.

All political parties are interested in the analysis of behavior of subpopulations of people. However, if the sample can be designed so that to represent the general population, this does not mean that each part of it represents the subpopulation from which it happened to come from. The

supporters of some party found in the sample are not a representative sample of the total supporters of the party, and the youth between 18 – 25 found in the sample are not necessarily a representative sample of all the youth, etc. The use of methods of analyzing two-way tables (such as correspondance analysis) can also lead to errors and misunderstandings.

Naturally, the study of behavior should cover a time period prior to the elections in order to examine the differentiation during the pre-electoral period so that the percentage of the indecisive voters -the distribution of which is important for the final results of the elections - to be reduced. So this observation is done through a series of polls (using the same or similar samples) that observe specifically the flows of voting intention from party to party (expressing negative attitude as “party”) and not the exact estimated percentages of voting intention.

The PANEL method (with replacement) gives the possibility of observing a “representative” sample of the population, but its success is doubtful in Greece due to problems of pre-selecting the sample (PANEL) and because of its relatively high costs. This method is applied for the monitoring of television viewing in Greece.

### **Sampling, choice of sample**

The selection of the size of sample in a poll is one of the most important and because of its costs (which depends on the selected sample) and because of the demanded accuracy of the results.

The formation of sample in all cases should be based on criteria of previous electoral behavior in specific areas based on the analysis, with advanced statistical techniques, of the results of the previous electoral data, (in the level of voting areas), so that the “*representation*” of each sample from the analysis and not from the random sampling in geographical levels and avoiding of testing.

In each sample region, the selection of sample should be done randomly securing dispersion in time and region. The analysis should be based in the estimation of the flows from state to state. Flow is the central size of political changes, according to the (international) relevant bibliography, and the specific sizes with the analysis of comparative data of the sample without the influence of weightings, which alter the first characteristics.

It is known that the size of the selected sample affects the basic results (e.g. for the percentages of the political parties we calculate 95% confidence interval with 2-4 percentage units of width). This means that with a probability of 95% the estimated parameters (e.g. percentages) will be within the limits of the confidence interval. If these values move off the limits of the confidence interval, irrelevant to the numerical difference, they are proved as wrong.

### **Questionnaire, a tool of measurement**

Questionnaires are often designed as simple lists of questions to be answered and not as cohesive total-tools. Especially when a multivariate analysis technique is going to be used (such as e.g. FACTOR or DISCRIMINANT) for the drawing of results, the above can lead in many misunderstandings and problems. Especially because in the teaching of methods we do not pay a special attention to the pre-requirements and their limits, but we pay attention mainly to the use of the technique.

### **Exit Polls**

Exit Polls were organized in the western European countries and in North America mainly in order to keep the interest of audience for the commented viewing of the elections results. As the sequence of the announcement of results does not represent the final result, due to some reasons for each country, the initial picture that is given from the official results is not offered for estimations, and the gap in timing that intervenes creates confusion in the flow of information. At the same time the unclear picture shown by the first results can lead to wrong estimations and disappointments to voters.

The need for a presentation of elaborated, trust-worthy and commented information brought in the previous elections in Greece the method of exit poll which is successfully used for the prediction of the final results especially in western European countries. This method consists of the selection of a “*representative*” sample on the elections day and of the results about the final outcome by the sample. At this point we should distinguish the method of exit poll from the methods of predictions based on real results from the ballot box.

As it becomes obvious, the exit poll carries all the characteristics of an opinion research with the decisive “*advantage*” that the percentage of indecisive voters is dramatically reduced (actually it is limited to those refusing expression of opinion). Here as well the sample has to be formed using the criteria of electoral behavior rather geographical ones. Also we should rather observe the voters’ flows than the absolute percentages.

Furthermore we face some difficulties such as: a part of the sample does not have a registered electoral behavior (new voters) and the questionnaire must be especially simple, clear and small since it should be completed by the person asked – if this is possible – in the absence of the interviewer.

In its first application in Greece this method had to face the special phenomenon of “non-resident” voters, i.e. a part of the electorate voting in their place of residence and not in the place of registration as voters. This fact created some difficulties in the prediction of flows of voters because of the increasing of that part of the sample, which did not have a previous recorded behavior. It is estimated that an important part of the “no-resident” voters voted differently in their place of residence than it would vote in the place of registration.

Exit polls give at the same time valuable data for the extract of a complete picture especially about the flows between political attitudes of elections that cannot be imprinted on the election results.

Political Party	Parliamentary Elections 2009	Parliamentary Elections 2012
Void	2,6	2,4
PASOK ( <i>Panhellenic Socialist Party</i> )	43,9	13,2
ND ( <i>New Democracy</i> ) right wing party	33,5	18,9
KKE ( <i>Communist Party of Greece</i> )	7,5	8,5
LAOS (Orthodox rally) right wing party	5,6	2,9
SYRIZA ( <i>Radical Left Party</i> )	4,6	16,8
DHMAR ( <i>Democratic Left</i> )	*	6,1
ANEL (Independent Greeks) right wing party	*	10,6
LS-XA (Golden dawn) ultra-right wing party	0,3	7,0
DHMSY (Democratic Alliance) liberals	*	2,6
Liberals	*	3,9

Other	4,6	9,5
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**Table 1. Results of Parliamentary Elections in 2009 and 2012**

The comparative observation of results of the two recent parliamentary elections (table 1) it is not enough to estimate these flows. For example, observing that the sum of percentages of ND, ANEL and DHMSY in the Parliamentary elections in 2012 equals to the percentage of ND in the previous elections we can assume that 30% of the ND 2009 voters have chosen ANEL, as it was already estimated by polls prior to the 2012 elections.

This result is contradicting the percentage of flow about 15% estimated by exit polls.

In previous parliamentary elections exit polls were based on selections of giant samples (10000-30000) citizens and the conducting of preliminary opinion polls of a wide range for the estimation of random local differentiation. To what extent the selection of **different** giant samples is generally approved and useful for the aim of fast information is still for discussion. (The question of “*why should we pay some hundreds thousands Euros to have the result 3-4 hours earlier*” makes sense), though it cannot certainly give, because of the geographical dispersion, the slight differences of “**local vote**” i.e. the specific flows in each voting region, that remain to be estimated with other methods by the final results, but, due to the design of the sample, gives the general characteristics of flows, that is an estimation of the average. In the election of 2009 and 2012 there was one exit poll (10000 sample) conducted by a consortium of 5 companies.

	2009	2012
PASOK	41,0 - 44,0 %	13,4
ND	34,3 - 37,3 %	19,7
KKE	7,3 - 8,3 %	8,8
LAOS	5,0 - 6,0 %	3,1
SYRIZA	3,9 - 4,9 %	16,4
ANEL		10,5
DHMAR		6,0
LS-XA		6,8
DHMSY		

GREENS	2,0 - 3,0 %	3,1
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**Table 2. Results of exit polls for Parliamentary Elections 2009 and 2012**

The estimation of the average gives accurately the general characteristics of election results in central figures but it is not easy to give the exact result, as it is shown from the following comparative table for exit polls for 1996 elections (table 3).

Estimations from Exit Polls

Parties	Organization/company conducted the poll						electoral RESULT
	PRC	OPINION BVA	ALCO	KAPA RESEARCH	EURIN COM	A.U.TH	
PA.SO.K.	42,1	42	41,7	41,8	41,7	41,64	41,51
N.D.	38,9	38	38,5	38,9	38,8	38,32	38,16
POL.AN.	3,3	3	3,3	3,3	3,2	3,11	2,94
K.K.E.	4,9	5,5	4,9	5,2	5,4	4,80	5,60
SYN	4,5	4,5	4,9	4,6	4,7	5,50	5,10
DI.K.KI.	4,0	4	3,9	4,1	3,9	4,61	4,43
OTHER	2,4	3	2,8	2,1	2,3	2,03	2,84

**Table 3. Exit Polls 1996 (Greece)**

The estimation of results for the country was conducted by the author (*A.U.TH*) in a representative sample (of 1161 citizens of the area A' in Thessaloniki), with a confidence interval  $\pm 2,5$  for the two first parties and  $\pm 1,5$  for the rest and around the above central values, though other estimations were giving an interval from  $\pm 1$  to  $\pm 0,5$  around the central values.

Especially for the A' voting area of Thessaloniki, in a different sample of 1464 citizens a poll was also conducted by the author in order to estimate the regional features which gave the following results (table 4):

Parties	CENTRAL VALUE	LIMITS	ELECTION RESULT
PA.SO.K.	40,65	$\pm 2,5$	38,95
N.D.	35,40	$\pm 2,5$	36,60
POL.AN.	2,39	$\pm 1,0$	2,65
K.K.E.	5,74	$\pm 1,2$	6,28
SYN	6,47	$\pm 1,3$	6,16

DI.K.KI.	4,49	±1,1	5,67
OTHER	4,75	±1,1	3,75

**Table 4 Estimated share and election results (A' Thessaloniki)**

As it was mentioned above, the central finding of exit polls are the table of flows between attitudes in consecutive elections especially concerning the total of the Country.

The table of flows computed for each voting area gives results about the structure of the supporters of the political parties, considering the local features, and the general table of flows gives results about the flows throughout the country. These figures (percentages of flows from attitude to attitude) are also estimated central values and for this reason they do not have an accurate numerical significance.

Nevertheless, it is generally an arbitrary result that the way voters behave in the total of the country they also behave in each voting area. In order to estimate whether these two “pictures” are identical, (i.e. whether the flows in each voting region are similar to the general flows for the country), electoral results should be used, in order to get estimations of the marginal flows in voting geographical units.

In order to realize the above, it is enough to consider the table of relative shares of the political parties in the larger voting areas (regions). (Table 5). In this table the relative share is given for each political party according to its power in the previous elections.

Especially for ANEL and DHMSY the relative share is computed considering the share of ND in 2009. From this table it is obvious that the flows from attitude to attitude are not similar in all voting regions, as the relevant share of each political party is not uniform. We see, for instance, that the relative share for ANEL is much higher than the average in Attica- (the area around Athens) and for DHMSY higher than the average in the region of Crete.



REGIONS	Parties					
	ND	ANEL	DHMSY	PASOK	KKE	
EASTERN MAC-THRACE	0.58	0.23	0.13	0.35	1.19	
ATTICA	0.47	0.40	0.07	0.25	0.92	
NORTH AEGEAN	0.55	0.27	0.04	0.31	1.11	
WESTERN GREECE	0.57	0.23	0.05	0.30	1.18	
WESTERN MACEDONIA	0.55	0.22	0.03	0.33	1.28	
EPIRUS	0.58	0.18	0.06	0.33	1.08	
THESSALY	0.55	0.22	0.04	0.31	1.07	
IONIAN ISLES	0.53	0.21	0.04	0.28	1.02	
CENTRAL MACEDONIA	0.52	0.28	0.04	0.34	1.05	
CRETE	0.35	0.38	0.35	0.30	1.21	
SOUTH AEGEAN	0.50	0.44	0.05	0.29	1.26	
PELOPONNESUS	0.62	0.17	0.03	0.33	1.12	
CONT. GREECE	0.47	0.31	0.07	0.27	1.13	
<b>TOTAL</b>	<b>0.52</b>	<b>0.29</b>	<b>0.07</b>	<b>0.30</b>	<b>1.04</b>	

**Table 5. Table of relative shares of political parties in the larger voting regions**

REGIONS	Parties					
	SYRIZA	DHMAR	LAOS	LS-XA	OIKOL	ALLO
EASTERN MAC-THRACE	3.08	1.24	0.50	1.08	1.25	3.59
ATTICA	3.07	0.90	0.40	1.10	0.83	2.25
NORTH AEGEAN	2.76	1.30	0.56	1.16	1.39	2.98
WESTERN GREECE	4.97	1.55	0.59	2.03	1.43	6.55
WESTERN MACEDONIA	3.64	1.57	0.60	1.54	1.38	3.58
EPIRUS	3.45	1.02	0.59	1.39	1.21	5.35
THESSALY	3.58	1.52	0.55	1.25	1.26	3.46
IONIAN ISLES	3.77	1.06	0.51	1.82	1.30	5.09
CENTRAL MACEDONIA	3.14	1.49	0.48	1.07	1.09	2.51
CRETE	3.65	1.83	0.68	1.37	1.55	4.70
SOUTH AEGEAN	3.54	1.84	0.50	1.46	1.50	3.78
PELOPONNESUS	3.46	1.27	0.48	1.82	1.21	3.88
CONT. GREECE	4.45	1.47	0.52	1.39	1.18	2.87
<b>TOTAL</b>	<b>3.36</b>	<b>1.22</b>	<b>0.48</b>	<b>1.24</b>	<b>1.07</b>	<b>3.03</b>

**Table 5 (cont.). Table of relative shares of political parties in the larger voting regions**

The following tables represent the flows from the conducted exit poll. The flow in each cell on the table represents the percentage of voters of the political party, which corresponds with the line on the table the party voted corresponding on the column of the table.

An additional indication about the difference of flows comes out from the comparative observation of the tables of flows which were estimated for the country and the A' region of Thessaloniki by VTR-algorithm<sup>1</sup> (Tables 6 & 7).

	PASOK	ND	KKE	LAOS	SYR	OIK	OTHER	VOID	NEW
PASOK	33.04%	1.06%	2.33%	0.78%	1.44%	3.14%	3.50%	1.89%	9.52%
ND	3.41%	48.70%	0.72%	5.49%	0.72%	3.77%	4.00%	0.94%	9.96%
KKE	4.94%	1.63%	65.95%	4.31%	2.87%	3.14%	4.00%	1.89%	7.36%
LAOS	1.97%	2.35%	0.18%	23.14%	1.44%		1.50%	0.94%	3.90%
SYRIZA	19.55%	7.16%	16.67%	6.27%	69.38%	15.72%	10.50%	18.87%	16.02%
OIK	3.09%	1.25%	0.72%	1.57%	2.39%	28.30%	2.50%	4.72%	1.30%
DHMAR	8.59%	1.44%	2.33%	1.96%	9.81%	8.81%	2.00%	2.83%	6.49%
ANEL	7.99%	15.71%	2.51%	22.75%	2.63%	6.29%	11.50%	9.43%	10.39%
DHMSY	1.36%	3.51%	0.72%	1.18%	0.72%	3.14%	0.50%	0.94%	1.30%
LS-XA	4.78%	8.31%	1.61%	24.71%	0.72%	1.89%	17.00%	9.43%	12.99%
LIBER	3.17%	4.23%	1.61%	3.14%	2.15%	11.95%	7.50%	7.55%	3.17%

**Table 6. Flows for the country from columns (B09) to lines (B12)**

	ND	PASOK		ND	PASOK	
PASOK	3.03%	26.96%		2.80%	27.36%	
ND	44.80%	1.31%		48.55%	2.39%	
KKE	0.45%	2.56%		0.88%	2.87%	
LAOS	8.55%	0.92%		6.28%	0.54%	
SYRIZA	3.84%	18.42%		3.80%	17.53%	
DHMAR	4.04%	8.95%		2.20%	7.64%	
ANEL	8.33%	13.09%		6.30%	12.28%	
filel	5.67%	10.87%		4.05%	9.91%	
LS-XA	7.95%	0.33%		9.52%	1.19%	
absent	7.67%	6.12%		11.53%	9.21%	

**Table 7. Flows for the A' region of Thessaloniki and Country from the columns (B09) to lines (B12)**

<sup>1</sup> A method for the estimation of voter transition rates, Andreadis, Ioannis and Chadjipadelis, Theodore, Journal of Elections, Public Opinion and Parties, 2009, 19(2):203-218

The picture of flows that comes out for the A' region of Thessaloniki is slightly different from the picture of the total country. The tables of flow are useful because they can give the **profile** of each attitude.

### **Polls prior to Elections**

Opinion polls nowadays tend to cover the observed lack of communication between political parties and social groups. In this way they replace the traditional form of political action through organizations and activists which were the “sensual nerves” of political parties in the society and were malfunctioning, transforming this interactive communication into “expression” of opinion from “representative samples” towards the political parties which, however, have already taken care of addressing to the civilians, mainly through declarations in the press and the mass media well in advance. Of course this fact is somehow dangerous for the functioning of the representative social structures, but this is not a responsibility of exit polls. **Furthermore**, they offer the possibility of examining hypotheses about the political phenomena. It is a fact that 50-60% of the studies published in scientific journals, in North America, Britain and Germany, contains diagrams, equations and tables. The according percentage for the South European countries and those of South America is about 30%, (*c. Riba, European Journal of Political Research Vol. 29 June 1996, pp. 477 – 508*).

Opinion polls, particularly prior to elections, should be used mainly towards two directions. In the first direction that of the percentage of citizens who do not answer (about intention of attitude) the differences between “**intentions of flow**” estimated in the tables of flow in opinion polls and the “**movement**” recorded in exit polls should be examined. At the same time, in pre-electoral polls, depending on the sample used, there is – to a smaller or larger degree- a **problem of denial of answer** (completing the questionnaire), which explains the underestimation or overestimation from attitude to attitude. The percentage of denial of answer, usually reaches 15% –20% and this is not counted in the analysis of the poll. This means that in order to complete 1000 questionnaires about 1200 citizens must be contacted out of whom the 200 do not accept to take part in the research, though the results usually announced concern the 1000 (completed questionnaires).

The main problem of the distribution of **“undecided”** voters, which leads to the inability of referring to the total of the electorate, gives a value to polls concerning the monitoring of attitudes and intentions of the time they are conducted and not a value of “prediction” of the elections result.

In the second direction, the pre-election period **between the time of voting and conducting the poll**, a time of political activation and intervention of political parties explains any distance between the poll results and the election results. In this respect, analyzing the differences of polls before and after elections can lead to the analysis of influence of the pre-elections period in the formation of the result.

It is rather arbitrary to assume that polls conducted 15-20 days before the elections can possibly reach in figures the elections final result, since in this case, what happened between the two measures (before and after) would not have any influence, a result that is arbitrary and wrong at any sense.

Pre-election polls should answer central, mainly qualitative, questions. The published pre-election polls (table 8) did not all record in the same degree the apparent qualitative trend of SYRIZA growth.

Depending on the size of the sample and the percentage of answers *“I did not decide, I do not answer”* the limits of confidence interval for central values are (+/- 2) to (+/-3) around them. So, estimated values of 5% power give estimations from 2% to 8%.

Naturally, the course of pre-election period will probably lead to the rearrangement of trends of the electorate. At any rate, the choices of a short pre-election period and the transfer of political activation, almost exclusively in the main political stage, can influence decisively the elections result leading a part of the electorate to different choices.

From the comparative observation of the election results (table 1) and the distance of tables of flows derived from exit polls or VTR we can realize the limits of polarization which is described in the differences of the two first columns and the two first lines of the tables of flows.

The following is a comparative table of polls during the pre-elections period for the country.

Parties	Organization/company conducted the poll						
	RASS	MRB	KAPPA	VPRC	PULSE	ALCO	MARC
PA.SO.K.	13,2	12,7	14,2	14,5	12	13,8	17,8
N.D.	18,6	20,5	19,0	22	17,5	18,5	22,3
KKE	6,7	7,9	7,9	11,5	8,5	7,0	9,7
LAOS	2,4	2,5	3,0	3,5	2,5	3,0	3,9
SYRIZA	8,0	8,6	6,8	13,0	9	7,9	9,8
OIKOL	2,9	2,9	2,7	2,5	3	2,5	3,1
DHMAR	6,5	7,0	4,0	10,0	7	6,6	8,6
ANEL	7,3	7,1	5,7	9,0	8	8,3	9,9
LS-XA	3,3	3,9	3,9	5,0	4,5	3,9	5,7
DHMSY	2,5	2,4	2,8	1,5	2,5	2,3	3,0
ALLO	5,7	4,1	4,4	6,5	6	7,1	6,2
NA	22,7	19,4	25,7	(24)	19,5	19,4	

**Table 8. Pre-elections polls for parliamentary elections (2012) (country).**

It is rather common sense to repeat that polls have a value for the political phenomena which they seem to describe, **(either as a chronically comparative presentation, or as an observation of their specific features)**, and that the figures they gave are estimations. In any case, findings of polls should be used having in mind the requirements without trying to exceed the limits of their measurements.

### **Some final comments**

Estimating and using transition probabilities is essential, especially in political and social sciences. Predicting the behaviour of a structure (i.e an educational system, the World Wide Web, a traffic network, an academic organisation) by using stochastic processes and operational research methods gives the opportunity to evaluate the effectiveness of the structure and to control its future evolution.

All of us (statisticians and researchers) try to understand and explain the world using theories and data. Statistics is just one among several methods dealing with the above issue. As statisticians, we use data to estimate and predict, and we assign probabilities to express a complex non-deterministic world.

In any case before reaching a conclusion you need a reasonable explanation for every model or outcome in the frame of the original

problem. As Kruskal (1979)<sup>2</sup> said,

*“A scientist confronted with empirical observations goes from them to some sort of inference, decision, action, or conclusion. The end point of this process may be the confirmation or denial of some complicated theory; it may be a decision about the next experiment to carry out. An end point is typically accompanied by a statement, or at least by a feeling, of how sure the scientist is of his new ground. These inferential leaps are, of course, never made only in the light of the immediate observations. There is always a body of background knowledge and intuition, in part explicit and in part tacit.”* (Kruskal, 1979, p. 84).

We need to use “statistics” and “probability” to understand and describe the “real world”. All of us agree that, statistics is not a set of rules and recipes for the analysis of data. It is not reduced to the use of complicated computer programs and nice graphs. It requires a good knowledge of the observed phenomenon, the planning of observation, good knowledge of the data gathered, and description and examination of the hypotheses about the parameters of the analysed phenomenon.

Generally we design courses that encourage students to think creatively and imaginatively about his or her scientific research problems and the role of modern theories of statistical data analysis and modelling, versus courses that usually describe the mathematical solutions of various routine statistical analysis problems without much proof, and are based on probability models for observed data whose validity is not usually checked.

But should we make (or try to make) a “statistician” out of a researcher? Or should we make researchers understand that they need a statistician as a helper, a colleague or even as a leader? Potential information can only be useful if it is generated. One reason why some Japanese industries achieve high levels of productivity is that employees are provided with statistical tools for generating, analysing, and acting on their own information. We should educate the statisticians in a more practical sense:

*“The brilliant minds of mathematical statistics would do well to leave the construction of abstract admissible decision functions, cease to ride*

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<sup>2</sup> Kruskal, W. H. (1979). Comments on Dr Braham’s article. *Journal of the American Statistical Association*, 74, 84-86.

*martingales into the teeth of zero-one laws and join the few of us who are attempting to stem the tide of confusion.*" (Hunter, 1981, p. 113)<sup>3</sup>

Hunter (1981) also criticised statistics education in Mathematics departments saying:

*"The statisticians' training, narrow and technical, is the orderly climb up a staircase of mathematical problems that each have only one right answer. Later steps rest on earlier ones. Progress is always up. Teachers watch the climbing techniques of the fledgling statisticians, and help them master the steps, one at a time. Statisticians' work, for which this training is supposed to equip them, is the disorderly climbing of rugged hills, outdoors, in fair weather and foul. The path is anything but clear. A promising path can get lost in tangled undergrowth or a patch of dense forest. Or else: a path branches in several directions and there is not enough time or money to explore all of them to determine which is the best to follow."* (Hunter, 1981, pp. 113-114 )

Statistical literacy and thinking is another issue. It concerns the general public in order to understand and criticise what is written in the press, what is seen on TV, what is presented by the authorities. We should educate the general public in order to become critical citizens. But this is the story of statistical education in compulsory education.

Let me tell you some conclusions I have drawn:

- Learn as much as you reasonably can about the general subject matter field and the specific environment in which the data were collected;
- As part of this effort, statisticians need to probe, be curious, and ask good "non-statistical" questions;
- Correlation measured from an observational study does not imply causation; Confusing correlation and causation is particularly troublesome in the social sciences;
- The real problem is often different from the one initially posed;
- An empirical approach is sometimes better than a theoretical one;
- Scientific logic is our business. Statisticians can often be most helpful

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<sup>3</sup>Hunter, W.G. (1981). Six statistical tales. *The Statistician*, 30, 107-117.

by getting perspective on all aspects of a particular problem and then contributing ideas related to scientific method (Hooke, 1980)<sup>4</sup>;

- Try to understand what is really going on;
- Valuable data are sometimes non-numerical;
- Scientific inference is broader than statistical inference.

And finally, let me tell you an old story about the practising statistician by Salsburg (1973)<sup>5</sup>. David Salsburg asked himself the question, what is it really like to be a practising statistician. Below we reproduce his answer:

*“The statistician is first called into consultation during the design of a scientific experiment. At this point, the texts tell us the statistician is supposed to estimate minimal sample sizes and prepare a BIBD that produces all kind of clever contrasts for testing.*

*I do not do this. Instead, I spend my time asking stupid questions. I know that when the experiment is finished I will have to analyse the data. I must protect myself from impending chaos. With such a fear behind me, I ask such questions as whether it is possible to observe something every 15 minutes or whether this thing they have given a name can, in fact, be observed at all.*

*I ask them what can be wrong. Frequently, I am the only one at the conference with a non-deterministic outlook. The others conceive only three or four clear-cut outcomes. I think about the in-between outcomes, the two correlated variables that happen to go different ways, the test tube someone is bound to drop, the patient who revives from death’s door on placebo.*

*I know that when it is all over with, the man who must make some kind of decision about the results will ask me to compare two means or to show him a linear regression. I try to make sure that the design will produce two comparable means regardless of how many test tubes are dropped and that, somewhere, there will be a somewhat controllable variable manipulating a somewhat responsible variable.*

*The bulk of my time, however, is spend in trying to make sense out of data... When I see data it is frequently because the results have not made sense to the client... I feel very uneasy with a client who nods blandly and takes back my numbers for his report. I feel better if he argues with me. After all, I do not know an isatine derivative from an isonitroso-acetyamine, but I hope to God he does...*

*I suspect that at least 50 per cent of all the data accumulated today never gets more than a cursory look, and I doubt if 5 per cent of it gets examined*

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<sup>4</sup> Hooke, R. (1980). Getting people to use statistics properly. *The American Statistician*, 34, 39-42.

<sup>5</sup> Salsburg, D. S. (1973). Sufficiency and the waste of information. *American Statistician*, 27, 152-154.



*effectively. All that money, all that anguish, all that pain will have gone for nothing and even be spent again and again in unknowing duplication unless these floods of data are converted to usable information. This is, par excellence, the place for the well-trained statistician...*" (Salsburg, 1973, p. 152).

## **WORKING GROUP 1:**

**Democracy in mathematics curriculum: How does school mathematics contribute to critical thinking and decision-making in the society?**

**GROUP DE TRAVAIL 1 : Curriculum de la démocratie en mathématiques: Comment les mathématiques à l'école contribuent-elles à la pensée critique et à la prise de décision en société?**

Hellenic Mathematical Society

International Journal for Mathematics in Education

### **On the role of Inconceivable Magnitude Estimation Problems to improve critical thinking**

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We introduce Inconceivable Magnitude Estimation problems as a subgroup of Fermi problems. The problems we use in our study require counting the amount of people in different situations. Based on the experience of a classroom activity carried out with 15-year-old students, we describe the process they followed to solve the problems, and discuss in which ways these problems provide knowledge to critically analyze the information that appears in the media.

#### **INTRODUCTION**

“Democracy in its purest or most ideal form would be a society in which all adult citizens have an equal say in the decisions that affect their lives (p. 168)” (Diamond & Plattner, 2006). In order to achieve their own opinion to make decisions, citizens might need to understand their environment and be able to interpret a huge range of information. Compulsory Secondary School (in Spain, the schooling between 12 and 16 years of age) could be a good period in which to introduce realistic context activities to make sense of the real world and use mathematics critically. We suggest Inconceivable Magnitude Estimation Problems (IMEP) as a means for improving critical thinking in secondary classrooms. IMEP present the student with a situation that requires to estimate the value of a considerably large real magnitude,

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well outside the range of their normal daily experience. These problems can be considered a subgroup of Fermi problems, and allow for different approaches to solving them.

In this article, we present a class experience introducing some IEMP to show the improvement of critical thinking in 15-year-old students. The aim of this experience is for the students to estimate the number of people in a demonstration by themselves and, thus, be able to judge the information given in the press.

According to Van Den Heuvel-Panhuizen (2005), presenting a real context for problems can make them more accessible and suggest different strategies to students. Problems which involve a realistic context help to begin teaching mathematics within the realm of the concrete and then move on to the more abstract. Chapman (2006) observes that many teachers present problems that are related to daily life in a closed way which does not allow for a proper discussion of the situations that the problems may present. According to Winter (1994), solving problems with a real context includes the mathematization of a non-mathematical situation, which involves the construction of a mathematical model, the calculation of the solution, and transferring the result back to the real situation.

Fermi problems are problems which, although being possibly difficult to solve, can actually be solved by being broken down into smaller parts that can be dealt with separately. They are named after the physicist Enrico Fermi (1901-1954), who often used such problems in his lectures. Ärlebäck (2009) defines Fermi problems as “open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations (p. 331).” Carlson (1997) describes the process of solving a Fermi problem as “the method of obtaining a quick approximation to a seemingly difficult mathematical process by using a series of educated guesses and rounded calculations” (p. 308) and asserts that they possess a clear potential to motivate students.

### **INCONCEIVABLE MAGNITUDE ESTIMATION PROBLEMS**

Our class experience focuses on problems based on magnitudes that we can not perceptually estimate without some training, as well as magnitudes which we can imagine, but for which it is difficult to interpret their value. If we think of magnitudes with which we are familiar and to which we have given meaning (the size of a table, the time that passes during a film, or the number of people in a classroom), we can metaphorically assert that they are

familiar and conceivable. Some examples of magnitudes which are inconceivable in this sense are the number of medical doctors in a state, the number of cars that pass by a certain point in a city, or the number of persons in a demonstration. Taking these ideas as a starting point, we define an inconceivable magnitude as a physical or abstract magnitude which is beyond our ability to interpret and for which we have not created any meaning. It must be emphasized that, according to this definition, the determination of whether a magnitude is conceivable or inconceivable varies from person to person, depending on their knowledge, abilities or experiences. Once we attempt to determine the value associated with an inconceivable magnitude, we must by definition work with approximate values. The most natural way of obtaining values for inconceivable magnitudes is to come to an estimation through reasoning. Our aim is that, as they solve these problems, the students see the necessity of focusing on the essential components of the given situation and create a meaning for it.

#### THE CLASS EXPERIENCE

This activity was carried out with 21 students in Secondary School, lasting several sessions. Firstly, they were invited to answer the following question: *How many people would there be room for in the high school playground if a concert was being held there?* Students were encouraged to make an individual resolution proposal. After that, the students were distributed into workgroups to elaborate a team resolution proposal. The next step was to carry out their proposals.

Three different resolution strategies were detected to estimate this quantity. The first one is the use of the idea of “density”, by calculating the playground area and checking, by actually doing it, how many students can fit in a square meter. The second one is the use of the idea of “a reference point”, by calculating the playground area and dividing it by the area occupied by one person. The third strategy is the model of “grid distribution”, which consists of estimating how many people fit in a rectangular grid by multiplying the number of persons that fit on each side.

The results obtained by the students for a rectangular playground are shown in Table 1:

	Strategy	Area of Playground	Result
Group 1	Reference point	353 m <sup>2</sup>	1179
Group 2	Density	348 m <sup>2</sup>	2175

Group 3	Grid distribution	—	2132
Group 4	Density	360 m <sup>2</sup>	2160
Group 5	Density	365 m <sup>2</sup>	1462
Group 6	Density	275 m <sup>2</sup>	1651

Table 1: Results for the number of people that fit in the playground

The next step was the discussion of these results in order to reach an agreement on the density of teenagers in a concert, which was 5 persons per square meter. Thus, the estimation of people that can fit in the playground was of 1.750 persons. This activity was useful to notice that, although this number might not be exact, it gives a reasonable value for this magnitude.

In a second activity, the students were asked to answer the following questions: *How many people can there be (standing up) in the Palau Sant Jordi (Barcelona, Spain) to attend a concert? How many people can there be in the city council square of Sabadell (Barcelona, Spain) to attend a demonstration? How many people can there be in Plaça Catalunya (Barcelona, Spain) to attend a demonstration? How many trees are there in Central Park (New York, USA)?*

These questions presented the students with some added difficulties. In Palau St. Jordi, people can sit down in some areas, Plaça Catalunya and the city council square do not have a rectangular shape and their area cannot be measured directly. The density of trees in Central Park should be estimated. Since students observed that they cannot make their own measurements directly to solve these problems, after a brainstorming, they suggested using Google Maps as a tool to obtain these measurements.

In the particular case of Palau Sant Jordi, the students got estimations between 15.000 and 24.000 persons. After that, they looked for a real value on the Internet. Some students found that the maximum capacity is 17.960 persons, but they also found that PSOE (a left wing political party) announced that more than 30.000 people attended a political meeting there in 2.008<sup>1</sup>. From their own results and their own mathematical productions, the students were able to decide on the reliability of some statements made by the press, using their critical thinking to accept or refuse them.

The last activity was focused on a real case. On July 11th, 2011, there was a very crowded demonstration in Barcelona. People concentrated in Plaça Catalunya and Passeig de Gràcia. The day after that, different media

<sup>1</sup> <http://www.elmundo.es/elmundo/2008/03/06/eleccionesgenerales/1204831825.html>

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sources provided different numbers of demonstrators. The organizers of the demonstration claimed that there were 1.500.0000 people, the local police declared that there were 1.100.000 people, and an independent company<sup>2</sup> estimated 62.000 people, by means of their own recounting methods.

This information was provided to the students and the question they had to solve was: *which of these numbers is the most reliable?* The students were encouraged to use the knowledge acquired in the former activities to answer this question. In summary, they used two main strategies. The first one consisted of estimating the area occupied by assistants and use the density to obtain an estimated number. The second strategy was to calculate the area needed to fit 1.500.000 people and try to imagine how long a street like Passeig de Gràcia should be. Due to the difficulty of the situation (people were moving, streets around the initial meeting point were occupied, street furniture) the students obtained a wide range of values (300.000 to 600.000 people). However, they were able to refuse all the figures given by the different statements.

### CONCLUSION

In this article, we introduce Inconceivable Magnitude Estimation Problems as realistic context problems that can be solved through the use of partial estimates made by the students. We have seen that students produced many different strategies that would result in successful solutions to these problems. By using IMEP, teachers have access to open problems which can be discussed in an open manner due to the existence of different approaches to their solutions and connections with real life (Chapman, 2006). At the same time, the students suggested to use an open acces tool like Google Maps to carry out their work when they needed it. These kind of proposals allow them to integrate school mathematics with real life tools to improve their skills for understanding their enviroment.

In addition, by means of these activities, students can produce their own knowledge to improve their critical thinking and become citizens with their own criteria.

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<sup>2</sup> [www.lynce.es](http://www.lynce.es)

**REFERENCES**

- Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast*, 6(3), 331-364.
- Carlson, J. E. (1997). Fermi problems on gasoline consumption. *The Physics Teacher*, 35(5), 308-309.
- Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics*, 62, 211-230.
- Diamond, L. J. & Plattner, M. F. (2006). *Electoral systems and democracy*. Johns Hopkins University Press.
- Van Den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics*, 25(2), 2-10.
- H. Winter. Modelle als Konstrukte zwischen lebensweltlichen Situationen und arithmetischen Begriffen. *Grundschule*, 26(3), 10-13.

## Participation in mathematics problem-solving through gestures and narration

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**Abstract.** *Recent results in neuroscience and communication bring to the fore the role of multimodal resources in cognitive and communicative processes. On the other hand, Bruner highlights the different role of narrative and logical thinking in human understanding. In this background, the background, we investigate the teaching and learning processes when children are introduced to the mathematical meanings by the use of narrative context and multimodality. In this paper, we focus on a narratively-based problem-solving activity in the first year of primary school. In particular, we analyse the teacher-students multimodal interaction and discuss the potential and limits of what we call the “semiotic game” between teacher and students. The importance of the teacher being conscious of the multimodal aspects of teaching and learning mathematics comes to the fore.*

### **Introduction and background**

The dynamics that occur in the mathematics classroom are a theme of great interest in mathematics education. In particular, means for promoting participation and inclusion are worth considering and discussing. In this perspective, in our research we focus on two specific issues, i.e. the role of narration and the use of multimodality, which are basic components for promoting interest and participation in mathematical problem-solving.

Our research hypotheses are grounded on Bruner’s ideas on the role of



*narrative thinking* in human cognition (1986), and the *multimodal perspective* in mathematics education (Arzarello et al., 2009).

With respect to the complex relationship between thinking and speaking, Bruner (1986) identifies two different cognitive styles: *narrative* thinking and *logical (or paradigmatic)* thinking. Narrative thinking focuses on intentions and actions, and is strongly anchored to experience in context, as the subjects perceive them. It constitutes the ordinary way through which human beings make sense to their experiences and to the world. In fact, we show an attitude to organize experience by giving it the form of the narratives we use in telling about it. In contrast, logical thinking is based on cause-effect reasoning and is guided by principles of coherence and non-contradiction, in consonance with the typical character of mathematical argumentations and proofs. Though irreducible, since based on different principles and procedures, narrative and logical thinking are complementary, rather than juxtaposed modes of thought. Indeed, Bruner claims that the human condition can only be understood by focussing on how human beings create their “possible worlds”, through the inclusion of both kinds of thinking (*ibid.*). This claim is very relevant for mathematics educators, who face the double challenge of introducing students to deductive ways of arguing, and of including all them in such a process (lower achievers not being excluded). In our view, the route leading to theoretical thinking in mathematics, which makes intense use of logical-scientific thinking, starts and deeply intertwines with the use of the narrative dimension. It is our conviction that this intertwining can be a fruitful premise for an inclusive teaching of mathematics at all levels.

In view of realising a synergy between narrative and logical thinking in mathematics activities, we think that it is fundamental to go beyond purely linguistic plane, and to include gestures and extra-linguistic modes of communication. This claim is in line with recent results in psychology and cognitive science about the role of the body in communication and in cognition. In particular, our research is informed by the so-called *embodied cognition* perspective, which assigns the body a crucial role in the constitution of mathematical ideas (Lakoff, & Núñez, 2000), and by the results on the role of *gestures* in communicating and thinking (McNeill, 1992, 2005; Goldin-Meadow, 2003).

The perspective of embodiment, criticizing the platonic idealism and the Cartesian mind-body dualism, advocates that mathematical ideas are

founded on our bodily experiences and develop through metaphorical mechanisms. Many limits have been recognized in this theory, in particular concerning the lack of social, historical, and cultural dimensions in the formation of mathematical concepts (e.g. Radford et al., 2005). However, it has the great merit of having focused the attention on the role of the body in knowledge formation. In accordance with historical-cultural perspectives, we assume that the embodied nature of thinking has to be related with the historically constituted cultural systems. In short, the two aspects (i.e. embodied and individual on the one side, and historic-cultural and social on the other) must be considered together.

Furthermore, recent results on *multimodality* in neuroscience and communication give us new suggestions when analysing classroom dynamics. Gallese and Lakoff (2005) use the notion of “multimodality” to highlight the role of the brain’s sensory-motor system in conceptual knowledge. Their claim is in particular based on the recent discovery of multimodal mirror neurons, which fire both when the subject performs an action and when observes it, as well as when imagines it. This model entails that there is not any central “brain engine” responsible to sense making, controlling the different brain areas devoted to different sensorial modalities (what would occur if the brain behaved in a modular manner). Instead, there are multiple modalities that work together in an integrated way, up to overlap with each other, like vision, touch, hearing, but also motor control and planning. On the other hand, in the field of communication design the term “multimodality” is used to refer to the multiple modes we have to communicate and express meanings to our interlocutors: e.g. words, sounds, figures, etc.

If we look at the mathematics teaching-learning processes in the multimodal perspective, we must enlarge the focus of attention and consider variables that are usually neglected, especially those of extra-linguistic or extra-symbolic origins. We mainly refer to gestures, gazes, and inscriptions (drawings and sketches). Keeping in mind the theory-practice dialectics (Bazzini, 2007) this approach suggests that classroom mathematical activities are planned and managed in order to foster the students’ multimodal participation to mathematical activities. An increasing participation will be obtained if children are left free to express their thinking through their body and non verbal languages, like gestures. It is of course important that the teacher fosters students’ participation in suitable activities and manages the interactions in order to support the students to

evolve from their personal meanings to the scientific ones (Vygotsky 1978).

These hypotheses are at the base of our on-going research study aiming at i) introducing children to the mathematical meanings by the use of narrative context and multimodality, and ii) studying the teaching and learning processes therein.

In this paper, we focus on the role of multimodal resources in a narratively-based problem-solving activity in primary school. In particular, we highlight the role of the teacher in the outlined background.

### **Teaching experiment**

We present and analyse a problem solving activity carried out in a class of 6 y.o. pupils (first grade of primary school). The teacher is a member of our research group and shares the theoretical background of the study. In view of pointing out the dynamics of interaction, the research methodology includes audio and videotaped recordings, beyond the collection of written productions by the students; the analysis of the processes is mainly based on the semiotic productions of students and teachers in a multimodal perspective (Arzarello et al., 2009).

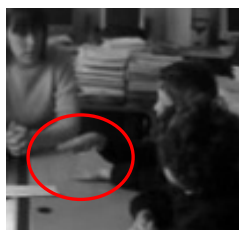
The activity is based on a story on the adventures of Iride, a fancy character that accompanies the children in their mathematics lesson. It is also linked to previous events lived by the children, i.e. the growth of small tomato plants in the school garden. The story is delivered to children by the teacher. Here the text:

*Iride likes our idea of having a vegetable garden. She plants tomato plants like we did and observes their growth. Close to a small tomato plant, she put a "Lego" tower (tower composed of small cubes). As soon as the plant grows, she adds cubes. From the beginning of the month, Iride has added 18 cubes. Today the plant is like a 26-cube tower. How long was the plant at the beginning of the month?*

This is a subtraction problem where the initial data is unknown. For children, it is the first time they face problems of this type. Under these conditions, the pupils cannot transform the initial data, as usual in previously encountered problems. Furthermore, the problem requires a temporal inversion in order to be fully understood. So, the problem could be difficult for the children. The teacher decides to organize the work in groups of four, under her presence. Children are around a table, where paper, coloured pencils and little cubes are at their disposal to solve the problem.

We focus on a medium level group, being Marco E., Zoe, Matteo, and Alessandro its members.

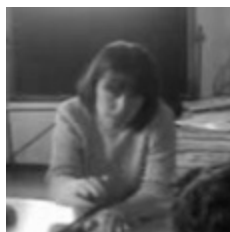
The children are soon engaged in the narrative side of the story, and Matteo provides immediately a hypothesis: “*In my view the plant is high like this*”, while showing the hight by means of his hands, one over the other (Fig. 1).



**Figure 1.** Matteo shows with gestures the initial height of the plant.

The child appears not to consider the information about measure provided by the story; he probably refers to experience and real data, i.e. the hight of the tomato plants which are actually present in the garden of the school. All the children declare to agree with Matteo, even if they are not able to justify their hypothesis, and the solution process seems blocked up. However, the teacher decides to profit from Matteo’s proposal and makes an intervention in order to highlight an important mathematical feature, i.e. the initial non nul hight of the plant. After the experiment, the teacher disclosed to us the purpose of her intervention: she feared that the children did not consider that the initial height of the plant could be different from zero, and wanted to make it clear for them. It is interesting to observe that the teacher’s intervention is made by words accompanied by a gesture:

*Teacher: When we located them in the garden, the plants were high more than zero (gesture shown in Fig 2).*



**Figure 2.** The teacher’s gesture, reproducing Matteo’s previous gesture.

In so doing, she reproduces the same gesture as Matteo, and accompanies it with suitable speech in order to help children in their progress. This

phenomenon is quite frequent also in previously observed teaching experiments: we have called it the “*semiotic gam*” (Arzarello et al., 2009). The semiotic game is a didactic phenomenon that can be grasped only if we consider the teaching-learning processes in a multimodal perspective. In fact, it typically involves embodied resources, such as gestures, and speech. A semiotic game may occur when the teacher interacts with the students, for example in classroom discussions or during the groups’ work. In a semiotic game, the teacher tunes with the students’ semiotic resources (e.g. words and gestures), and uses them to make the mathematical knowledge evolve towards scientifically shared meanings. More specifically, the teacher uses one kind of sign to tune with the students’ discourse (usually gestures), and another one to support the evolution of meanings (usually language). For instance, the teacher repeats a gesture that one student has just done, and accompanies it with appropriate linguistic expressions and explanations. The semiotic game can develop if the students produce something meaningful with respect to the problem at hand, using appropriate signs (words, gestures, drawings, etc.). The teacher can seize these moments to enact her/his semiotic game. Even a vague gesture can really indicate the level of comprehension by the student, who has not yet found the words to express it. In a Vygotskian perspective, the gesture can witness that the student is in the Zone of Proximal Development for a certain concept (Vygosky, 1978). As a consequence, the teacher has the chance to intervene properly, and encourage the student to develop his/her intuition.

Coming back to the Iride’s problem, the children meet a strong difficulty caused by the linguistic ambiguity between the two sentences: “the plant grows of 26 cubes” and “the plant grows up to 26 cubes”. Some children even say: “*the plant grows 26 cubes*”, and do not clarify further what they exactly mean. In order to face this point, the teacher proposes to simulate the plant’s growing with the Lego blocks. By building the Lego model, she persuades the children that the numer 26 refers to the final height of the plant (and not to its growth). However, there is still no agreement about the initial height of the plant: Alessandro is still convinced that the initial height is zero. Hence, the teacher guides the children in checking the proposal of Alessandro, by building a tower: 18 cubes are used (and added to zero) and eventually all children agree that, in this last case, the plant’s height would be of 18 cubes and not 26.

This episode shows that the previous semiotic game performed by the teacher was not grasped by all children. This sort of “failure” of the semiotic

game can be framed in a wider perspective that consider the teaching-learning processes as complex phenomenon, not reducible to simple cause-effect schemas. On the contrary, it is interesting to notice that the teacher does not continue in the same line (i.e. with another semiotic game, or recalling the previous one), but changes the strategy to tackle the problem. In fact, the teacher resorts to the Logo tower, in order to support the children in getting a visual image of the situation. She lifts the tower of 18 cubes up from the table (Fig. 3) asking: “*What do the 18 cubes represent?*”. In this way, she guides the children to realize that the 18 cubes have to be put on a pre-existing tower, which measured the initial height of the plant. Then, she points to the space at the top of the tower, to stimulate the children to visualize the cubes that are needed to pass from 18 to 26 (Fig. 4).



**Figure 3.** *The teacher lifts the tower up to 18 cubes.*



**Figure 4.** *The teacher points to the space above the tower.*

Alessandro, who is attentive to the teacher’s gesture, asks: “*How many cubes did we add to reach 26?*”

The children add cubes to the tower up to 26, while Alessandro looks at them and checks the counting in his mind. At the end he claims: “8!”. Then the teacher focuses the attention on what the numbers 18, 26 and 8 do represent with respect to the height of the plant.

### **Concluding remarks**

The analysis of the teaching experiment gives evidence of what we called the *multimodal* nature of the processes of teaching and learning. In order to face the problem, the children and the teacher used words, embodied signs (gestures), and tools (the Lego blocks) in synergic ways, confirming our hypothesis that learning occurs in a multimodal way. This synergy was exploited in particular by the teacher, especially when students met difficulties in modeling the situation, as in the case we have just described.

The narrative dimension was important for getting the students involved in

the problem-solving activity, and enhancing participation in the mathematical work. However, differently from other cases we have previously analysed (Bazzini, Sabena & Villa, 2009), it did not provide an essential support for the solution of the problem. Probably, the reference to the actual experience of the tomato plants in the garden of the school prevented the children to suitably imagine Iride's situation and activate narrative thinking to deal with it.

From a methodological point of view, it is worth noticing the synergy between teacher and researchers in analysing classroom episodes. It resulted very fruitful for both the teacher and the researchers: on the one hand, such an opportunity allowed the researchers a deeper insight and a more comprehensive analysis of the experiment. On the other hand, discussing the analysis of her own teaching activity, the teacher gained a great consciousness of many phenomena that did happen in her interaction with the pupils, but that she could not notice in the flow of the activity. For instance, the teacher herself highlighted the missed opportunities. As a result, the analysis methodology allowed the teacher to become more conscious of her own actions during her interactions with the children. With a changed perspective, the teacher acquires new possibilities of intervention by means of the semiotic game, to guide the students in their construction of mathematical meanings.

Finally, an important result concerns the possibility of widening the teacher's perspective to the multimodal aspects of the classroom processes by means of an attentive analysis of the classroom processes. To do so, the teacher has to learn how *to look* at the processes according to a multimodal perspective, therefore considering not only words and mathematical symbols but also other kinds of signs, such as gestures, gazes, and so on. Furthermore she has also to be sensitive to the relationships existing between the signs involved in the activity. We are aware that acquiring such abilities may be a long and hard process. However, we are convinced that their impact on the educational practice may significantly improve school mathematics. This will be our concern in future research, and our challenge as well.

#### **ACKNOWLEDGMENT**

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## References

- Arzarello, F., Paola, D. Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, 70(2), 97-109.
- Bazzini, L. (2007). The mutual influence of theory and practice in Mathematics Education: implications for research and teaching. *ZDM – The International Journal on Mathematics Education*, vol. 39, p. 119-125.
- Bazzini, L., Sabena, C. & Villa, B. (2009). Meaningful context in mathematical problem solving: a case study. *Proceedings of the 3rd International Conference on Science and Mathematics Education* (pp. 343-351). Penang, Malaysia: CoSMEd.
- Bruner, J. (1986). *Actual Minds, Possible Worlds*. Cambridge, MA/London, UK: Harvard Univ. Press.
- Gallese, V. and Lakoff, G. (2005): The brain's concepts: The role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology* 22, 455-479.
- Goldin-Meadow, S. (2003). *Hearing gesture. How our hands help us think*. The Belknap Press of Harvard University Press. Cambridge, Massachusetts, and London, England.
- Lakoff, G. & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.
- McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. Chicago: University of Chicago Press.
- McNeill, D. (2005). *Gesture and Thought*. Chicago, IL: University of Chicago Press.
- Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics*, 70(3), 111 – 126.
- Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.



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## RÉFORME CURRICULAIRE EN MATHÉMATIQUES AU QUEBEC SOUS L'ANGLE DU CONTRAT SOCIAL: UN EXEMPLE DE DÉMOCRATISATION DU DESIGN CURRICULAIRE

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**Résumé :** La récente modification du curriculum en mathématiques au Québec s'inscrit dans une réforme majeure pour l'école primaire et secondaire, touchant simultanément à l'ensemble des programmes d'études ainsi qu'à l'organisation scolaire. Dans cette communication, nous présentons l'analyse que nous avons réalisée du processus de conception, de mise en œuvre et de régulation de cette réforme curriculaire en mathématiques, telle qu'elle a été pensée et vécue par les acteurs qui ont participé à son élaboration. Il en ressort une vision organique, systémique, participative, ouverte et même aporétique du design curriculaire. Cette analyse nous permet non seulement de mieux comprendre le sens que prend la démocratisation de l'enseignement des mathématiques dans ce processus de réforme curriculaire ainsi que dans le curriculum qui en résulte, mais également d'apprécier de façon positive les nombreuses protestations soulevées par cette réforme.

### **Introduction**

Divers efforts ont été entrepris à travers le monde pour s'émanciper du modèle « top down » de conception curriculaire, démocratiser ce design, accorder plus de place et de responsabilités à ceux qui ont en charge la mise en œuvre de ces curricula (voir notamment Abrantes, 2004; Fiorentini, 2011). Cette réflexion sur le degré d'ouverture du design curriculaire nous semble centrale au regard des questions portant sur "l'enseignement des mathématiques et la démocratie".

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La réflexion que nous proposons ici trouve son ancrage chez Rousseau. Dans son célèbre ouvrage sur le Contrat social, il se donne comme ambition de « chercher si, dans l'ordre civil, il peut y avoir quelque règle d'administration légitime et sûre, en prenant les hommes tels qu'ils sont, et les lois telles qu'elles peuvent être ... [tentant] d'allier toujours, dans cette recherche, ce que le droit permet avec ce que l'intérêt prescrit, afin que la justice et l'utilité ne se trouvent point divisées. » (Livre I, Préambule).

Pour Rousseau, une telle règle suppose l'existence d'un « pacte social » au service de l'intérêt général, et dont le fondement est la souveraineté du peuple, qui elle-même n'est possible que si chacun accepte de renoncer à sa « liberté de nature » au profit de la « liberté civile » que lui assurerait la société. Un tel pacte s'offre comme une des bases de la démocratie. Étudier les évolutions curriculaires en mathématiques dans une perspective de contrat social suppose dès lors de se livrer à une enquête qui aille au-delà des textes et argumentations officiels pour reconstruire des dynamiques complexes auxquelles contribuent, de façon plus ou moins visible, une diversité d'acteurs et de communautés. Notre analyse de la réforme curriculaire récente en enseignement des mathématiques au Québec est une tentative en ce sens.

*Les objectifs poursuivis par notre étude visent* : d'une part, à retracer, au plan structurel, le processus de conception de la réforme du curriculum d'études en mathématiques, la manière dont a été pensée sa mise en œuvre et son accompagnement, ainsi que sa régulation et, d'autre part, à cerner comment celui-ci a été vécu par quelques-uns des acteurs impliqués.

### **1. Quelques repères méthodologiques**

L'analyse s'appuie sur un ensemble de données touchant à différents aspects du processus (conception, mise en œuvre, accompagnement, régulation). Une étude documentaire nous a permis de retracer le processus au fil du temps et ses éléments clés (Carpentier, 2010 ; Comité conseil sur les programmes d'études, 2007 ; Commission des programmes d'études, 1998-a,b, 1999, 2002, 2005-a, 2005-b, Inschauspé, 2007 ; Gosselin et Lessard, 2007 ; MEQ, 1996, 1997, MELS, 2008, 2010). Par ailleurs, des entrevues individuelles menées avec des enseignants ayant participé à la conception du programme de mathématiques au secondaire, nous ont permis de cerner ce processus de l'intérieur, du point de vue des acteurs y ayant participé.

Dans un second temps, cette analyse a servi de base à une réflexion autour du contrat social, sous le thème de la « réussite pour tous » en mathématiques : une démocratisation de la réussite qui constitue la visée

principale de cette réforme mais dont, nous allons le voir, la signification demeure ouverte.

## **2. Quelques balises pour situer cette réforme curriculaire.**

Le Québec a connu deux réformes majeures de son système éducatif, la première dans les années 60. Au fondement de cette réforme se trouvait l'idée d'une école accessible à tous. La seconde s'est mise en route suite à de nombreux constats « d'inégalités » dont cette l'école se faisait le lieu, sinon l'outil : décrochage, filières élitistes, etc. (Robert et Tondreau, 1997). Démarrant dès 1995, avec Les « États Généraux de l'Éducation », par une vaste consultation populaire sous le thème de « l'égalité des chances » et « de la réussite pour tous », elle donnera lieu à une ambitieuse réforme du système éducatif. Tous les programmes allaient être modifiés du primaire au secondaire, et ce dans tous les domaines. Nous rappelons ici quelques-unes des étapes importantes de ce processus:

- Mise en place, à la suite du rapport final de la Commission des États Généraux sur l'Éducation (1996), d'un groupe de travail sur la réforme du curriculum (Réaffirmer l'école, 1997);
- Création dès 1997 par la ministre de l'Éducation, dans la foulée de la recommandation de ces deux groupes, de la commission des programmes d'études qui jouera un rôle important dans le processus de design curriculaire;
- Conception des programmes prise en charge, dans chacune des disciplines, par un comité de rédaction. Cette conception, dans le cas des mathématiques, sera appuyée par de multiples consultations auprès d'autres enseignants, mais aussi de didacticiens, et autres intervenants.

La mise en œuvre de ces programmes en mathématiques s'étalera de 1999, pour le programme du primaire, à 2008, pour le programme du second cycle du secondaire, programme présentant, pour les deux dernières années de ce second cycle, un parcours de différenciation avec 3 séquences possibles (Culture technique société; Technico sciences; sciences naturelles).

Ce processus de design curriculaire, on l'entrevoit dans ce qui précède, est donc extrêmement complexe, puisqu'il renvoie à de multiples acteurs, à plusieurs ordres d'enseignement, à une réforme des programmes qui touche simultanément plusieurs disciplines, à une vision transversale de l'ensemble des programmes (ce que l'on appellera le « programme des programmes »), à une coordination à de multiples niveaux. C'est ce dont nous essaierons de rendre compte afin de comprendre les assises des évolutions curriculaires

actuelles en mathématiques et de dégager, dans ces nouvelles manières de penser le design curriculaire, des indicateurs de changement de « contrat social ».

### 3. Quelques éléments mis en évidence par l'analyse

Succinctement, l'analyse montre que le processus examiné s'installe sur une longue durée (1995-2008) avec des chevauchements entre les différents moments de conception, de mise en oeuvre, voire de régulation, et ce dans un esprit de *continuité*. Une *vision systémique et organique* s'impose tout au long du processus (qui échappe aux seules mathématiques) permettant d'assurer *une certaine* cohérence à l'ensemble de cette réforme. Cette cohérence suit le *fil directeur* de la « réussite pour tous », mais de manière non normative: une cohérence, continuité, qui semble porter sur *l'esprit* des grandes orientations émergeant des premiers cycles de consultation-synthèse. On aura ainsi à coeur d'assurer l'implication de multiples acteurs (d'horizons et milieux différents) dans une logique *participative* plutôt que simplement consultative: tous étant appelés à *contribuer* aux différents moments du processus à la formulation de ce que cette réussite peut signifier. Un rôle important est joué par les enseignants (on parle de leur expertise comme *professionnels* de terrain), en particulier dans l'écriture du programme de mathématiques, le texte étant par ailleurs soumis à de nombreux autres enseignants et conseillers pédagogiques au cours du processus de construction, mais aussi dans une consultation plus large au regard de sa faisabilité.

On note aussi la présence d'un processus d'accompagnement important, avec beaucoup de moyens, et conçu de manière à ce que les enseignants et les conseillers pédagogiques (aussi les directions d'école) *s'approprient* le curriculum mis par écrit, le réinventent, poursuivant jusque dans l'implantation ce qu'on pourrait qualifier de modèle hybride (Carpentier, 2010) entre l'approche « top down » (où un quelconque comité déciderait et imposerait « sa » réforme) et « bottom up » (dans laquelle les changements, venus des pratiques elles-mêmes, seraient inscrits dans les programmes). Cette ouverture par l'appropriation de ce programme et de ses fondements s'inscrit dans l'ensemble d'une démarche où la recherche d'un consensus ne s'est jamais présenté comme un idéal à atteindre.

Le curriculum ainsi construit se présente comme un objet frontière (Star et Griesemer, 1989), agissant comme interface entre différentes communautés, leur permettant de s'articuler, de se coordonner, sans pourtant faire consensus. L'objet frontière a cette particularité d'appartenir

(mais pas en propre) à chacun des groupes à travers lesquels il est transigé en même temps qu'il est un point d'intersection de ces différentes pratiques (Bowker and Star 1999): compréhensible par chacun sans devoir être entièrement compris et accepté par tous, objet de médiation et de négociation des intentions, des visées, ou des visions qui se rencontrent alors que différentes communautés l'utilisent (Corcoran 1992; Fujimura 1988). C'est ce rôle que semble bien jouer le programme de mathématiques en construction, dans le cas du processus de design curriculaire analysé. Le curriculum traverse en effet, tout au long du processus, les frontières de différentes communautés : enseignants des comités restreints de rédaction et des comités élargis, didacticiens des mathématiques et formateurs universitaires réunis lors des consultations, intervenants autres, membres de la Commission des programmes d'études. Il révèle, ce faisant, différentes manières de voir ce curriculum, d'en parler, d'interpréter son contenu, d'en faire sens.

#### **4. Quelques éléments de discussion en termes de contrat social**

Le changement dans les orientations de l'école québécoise d'une politique « d'accessibilité » à une mission de « réussite pour tous » n'est pas seulement énoncée tout au long du processus que nous avons étudié: *il est vécu par les acteurs*. Ainsi, cette démocratisation ne concerne pas que les élèves mais aussi les enseignants, et tous les acteurs impliqués. En effet, on va voir cette image de l'enseignant (du conseiller pédagogique, du directeur) expert mise à contribution dans le design du curriculum en mathématiques comme texte (Pinar, Reynolds, Slattery et Taubman, 1995), mais aussi dans la classe, ce que Aoki (1993) appelle le « curriculum vécu » (c'est ce que révèle l'analyse de sa mise en œuvre).

Le projet de réforme de l'enseignement que nous avons examiné se révèle en fait bien plus qu'un simple effort de refonte du curriculum en mathématiques. Il s'agit d'un projet de société qui, au delà même du programme, s'inscrit dans une entreprise collective de signification à propos « d'une école de la réussite ». Cette visée, clairement énoncée dès le début de ce grand mouvement comme étant l'un de ses fondements, n'est pas traitée autour d'une définition imposée. Elle sera au contraire construite et reconstruite au fur et à mesure que le processus progresse. Le processus est ainsi ouvert au débat, mettant à contribution différents acteurs (différentes visions, expériences, horizons), se donnant les moyens de faire signifier l'idée de réussite en mathématiques. Il ne s'agit donc pas du terme d'un « contrat social » établi, auquel tous sont appelés à souscrire, un contrat dont

la signature marque le moment d'une adhésion. C'est au contraire l'écriture elle-même qui devient la marque d'une contribution, d'une convocation, d'une construction de la société.

La métaphore de l'écriture, qui nous conduit sur les traces de Derrida (e.g. 1967), évoque ainsi le devoir d'écriture face aux différences, et l'aménagement d'un espace de liberté (une logique de réappropriation par les personnes ressources, les enseignants en mathématiques) et l'importance accordée au temps de l'action. On retrouve dans cette vision ce que nous avons appelé ailleurs le curriculum comme « prétexte » (plutôt que comme « texte »): l'occasion d'une *rencontre* avec l'autre (Maheux et al., sous presse).

### Références

- Abrantes, P. (2004). L'innovation curriculaire en éducation de base au Portugal. Dans P. Jonnaert, D. Masciotra (Eds.) *Constructivisme, choix contemporains*, (pp 37-49). Presses de l'Université du Québec.
- Aoki, T. (1993). Legitimizing lived curriculum: re-mapping the curricular landscape. *The Journal of Curriculum and Supervision*, 8(3), 255-268
- Bowker, G. C. & Star, S. L. (1999) *Sorting Things Out: Classification and Its Consequences*. MIT Press: Cambridge, MA.
- Carpentier, A. (2010). *Étude de la mise en œuvre du curriculum québécois du primaire, de 1997 à 2003*. Thèse de doctorat en éducation. Université de Montréal
- Maheux, J.F., Swanson, D. & Khan, S. (sous presse). From Text to Pretext: An Ethical Turn in Curriculum Work. In T. Mason & R.J. Helfenbein (Eds.), *Ethics and International Curriculum Work: The Challenges of Culture and Context*. Charlotte, NC: Information Age Publishing.
- Pinar, W. & Reynolds, W. (Eds.) (1992). *Understanding Curriculum as Phenomenological and Deconstructed Text*, New York: Teachers College Press,
- Robert, M., Tondreau, J. (1997). *L'école québécoise. Débats, enjeux et pratiques sociales. Une analyse sociale de l'éducation pour la formation des maîtres*. Montréal : CEC.

## Mathematical modelling and problem posing: how school mathematics can contribute to critical thinking in the society

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**Abstract.** In this contribution we will present a teaching experiment that is part of an ongoing research project based on a series of classroom activities in upper elementary school, using suitable artifacts and interactive teaching methods. The focus is on fostering a mindful approach toward mathematical modelling and problem solving, as well as a problem-posing attitude. In particular this teaching experiment wanted to investigate the potential that the combination of these processes have for identifying and stimulating critical thinking in mathematics.

An important aim for compulsory education should be to teach students to interpret critically the reality they live in and understand its codes and messages so as not to be excluded or misled (Bonotto, 2007). We argue for modelling as a means of recognizing the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, and society in general.

The term mathematical modelling is not only used to refer to a process whereby a situation has to be problematized and understood, translated into mathematics, worked out mathematically, translated back into the original (real-world) situation, evaluated and communicated. Besides this type of modelling, which requires that the student has already at his disposal at least some mathematical models and tools to mathematize, there is another kind of modelling, called ‘emergent modeling’ (Gravemeijer, 2007). Its focus is on long-term learning processes, in which a model develops from an informal, situated model (“a model of”), into a generalizable mathematical struc-

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ture (“a model for”). In this contribution the focus will be more addressed to the second aspect of mathematical modelling.

As regard the problem posing process it represents one of the forms of authentic mathematical inquiry and of creative activity that can operate within tasks involving to-be-structured rich situations (Freudenthal, 1991), including situations involving real-life artifacts and human interactions (English, 2009).

Problem posing has been defined by researchers from different perspectives (see Silver & Cai, 1996). In this paper we consider mathematical problem posing as “the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems” (Stoyanova & Ellerton, 1996). It, therefore, becomes an opportunity for interpretation and critical analysis of reality since: i) they have to discern significant data from immaterial data; ii) they must discover the relations between the data; iii) they must decide whether the information in their possession is sufficient to solve the problem; and iv) to investigate if numerical data involved is numerically and/or contextually coherent. These activities, quite absent from today’s Italian school context, are typical also of the modelling process and can help students to prepare to cope with natural situations they will have to face out of school, for examples in the work situations.

We deem that at school it is important to use different types of school activities in order to promote the different potentials of students and to promote greater mental flexibility. So problem posing is just one of the possible ways to achieve these broader goals. Furthermore, in accord with other researchers, we maintain that less structured and open-ended tasks could foster more flexible and critiquing thinking.

In this contribution we will present a teaching experiment that is part of an ongoing research project based on a series of classroom activities in upper elementary school, using suitable artifacts and interactive teaching methods, in order to create a substantially modified teaching/learning environment.

The focus is on fostering a mindful approach toward realistic mathematical modelling and problem solving, as well as a problem-posing attitude (Bonotto, 2009). In particular this study wanted to investigate the potential that the combination of these processes have for identifying and stimulating critical thinking in mathematics.

### **About artifacts**

Although mathematics learning and practice in and out of school differ



in significant ways (Resnick, 1987, Lave, 1988), we deem that those conditions that often make extra-school learning more effective (Nunes et al., 1993) can and must be re-created, at least partially, in classroom activities. That can be implemented in a classroom, for example, by encouraging the children to analyze some ‘mathematical facts’ that are embedded in opportune ‘cultural artifacts’ (Saxe, 1991). We are talking here of materials, real or reproduced, that have relevance for the children and that are meaningful because they are part of the children’s real life experience and refer to concrete situations.<sup>1</sup>

In this way we offer the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics, by presenting mathematics as a means of interpreting and understanding reality and by increasing the opportunities of observing mathematics outside the school context. The usefulness and pervasive character of mathematics are merely two of its many facets and cannot by themselves capture its very special character, relevance, and cultural value. Nonetheless, these two elements can be usefully exploited from a teaching point of view, by enabling students to become involved with mathematics, to break down their conceptions of a remote body of knowledge and to develop a positive attitude towards school mathematics.

The use of suitable artifacts can allow the teacher to propose many questions, remarks, socially and scientifically interesting inquiries. These artifacts contain different codes, percentages, numerical expressions, and different quantities with their related units of measure, and hence are connected with other mathematical concepts and also other disciplines (chemistry, biology, geography, astronomy, economy, etc.).

Furthermore by asking children i) to select other artifacts from their everyday life, ii) to identify the embedded mathematical facts, iii) to look for analogies and differences (e.g. different number representations), iv) to generate problems (e.g. discover relationships between quantities), the children should be encouraged to recognize a great variety of situations as mathematical situations, or more precisely mathematizable situations (Bonotto, 2007).

A “re-mathematization” process is thereby favoured, wherein students are invited to unpack from artifacts the mathematics that has been “hidden”

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<sup>1</sup> for example menu of restaurants and pizzerias, advertising leaflets containing discount coupons for supermarkets and stores, weekly TV guide, and so on (see e.g. Bonotto, 2005, 2006 and 2009).

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in them. In this way students should have the opportunity to think critically about world issue and their environment through mathematics.

Artifacts are important components of mathematical learning but do not directly determine ways of reasoning (Schliemann, 2002). The mathematical goals emerge for children not only in relation to artifacts but even in relation to structure of activity, social interaction and children's prior understanding. Therefore assume a particular importance the role of the teaching/learning environment designed and implemented in the classroom. In our studies it is characterized by the application of a variety of complementary, integrated and interactive instructional techniques (involving children's own written descriptions of the methods they use, in pair work, whole-class discussion, ...).

### **The study**

This exploratory study involved four fifth-grade classes (10-11 years old) from two primary schools: two classes of a school of Padua (one consisting of 14 students and the other of 16) and two classes of a school of the province of Vicenza (composed respectively of 20 and 21 pupils).

The artifact utilized was the page of a brochure explaining the special rates for groups visiting an Italian amusement park and wishing to benefit from different services. This artifact was chosen since it was found that all students were already familiar with such artifacts and had experience at an amusement park. This page is full of information, including prices (some expressed by decimal numbers), percentages, and constraints on eligibility for the various offers.

The study was structured in three phases: i) the presentation of the artifact used; ii) a problem posing activity; iii) a problem solving activity (in a couple or in groups of three students) combined with a collective discussion.

Data from the teaching experiment include the students' written work, fields' notes of classroom observations and audio recordings of the collective discussions.

All the problems created by the students were analyzed with respect to their quantity and quality.

### **Some results and discussion**

Despite the richness of variables and linguistic limits of the artifact (on the one hand it was particularly attractive inasmuch as it referred to an amusement park known to the children, while on the other hand it was also very dense with information) the majority of the pupils nevertheless succeeded in understanding it and in carrying out both the problem posing

and the problem solving activities. Moreover, the brochure's complexity favored the development of interesting discussions stimulating the students to ask themselves questions and to formulate hypotheses.

The problem solving phase combined with group discussions allowed students to reflect on different types of problems and explore new possibilities (e.g. by reflecting that math problems do not always require a numerical answer or one solution, and that there are problems which are not solvable).

Furthermore the results suggest that asking students to analyze the problems they have created facilitated their critical thinking because students were more free to discuss the validity of the problem, to consider different assumptions and to decide whether a problem had been solved or not. The results of this teaching experiment, as in other we conducted, show that, contrary to the practice of traditional word problem solving,<sup>2</sup> children do not ignore the relevant and plausible aspects of reality, nor did they exclude real-world knowledge from their observation and reasoning. They confronted with this kind of activity also show flexibility in their reasoning processes by exploring, comparing, and selecting among different strategies. These strategies are often sensitive to the context and number quantities involved, and closer to the procedures emerging from out-of-school mathematics practice; so mathematical reasoning needed in extra-scholastic contexts, for example in work places, is favoured.

Finally by presenting the students activities that are meaningful because they involve the use of material familiar to them, increased their motivation to learn even among the less able ones. For this reason, even children with learning difficulties related chiefly to linguistic problems (for example immigrant children) are helped.

### References

- Bonotto, C. (2005). How informal out-of-school mathematics can help students make sense of formal in-school mathematics: The case of multiplying by decimal numbers. *Mathematical Thinking and Learning*, 7(4), 313–344.
- Bonotto, C. (2006). Extending students' understanding of decimal numbers via realistic mathematical modeling and problem posing. In J. Novotná et al (Eds), *Proceedings of the 30<sup>th</sup> PME* (II, pp. 193-200)

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<sup>2</sup> While not denying the importance of the word problems in the curriculum “they do not address adequately the mathematical knowledge, processes, representational fluency, and communication skills that our students need for the 21<sup>st</sup> century” (English, 2009).

Prague: Prague Charles University.

- Bonotto, C. (2007). How to replace the word problems with activities of realistic mathematical modeling. In W. Blum, P. Galbraith, M. Niss, & H. W. Henn (Eds), *Modelling and applications in mathematics education* (pp. 185-192). New ICMI Studies Series 10. NJ: Springer.
- Bonotto, C. (2009). Artifacts: influencing practice and supporting problem posing in the mathematics classrooms. In M. Tzekaki, M. Kaldrimidou & C. Sakonidis (Eds) *Proceedings of the 33<sup>th</sup> PME* (II, 193-200), Thessaloniki, Greece: PME.
- English, L. D. (2009). The changing realities of classroom mathematical problem solving. In L. Verschaffel et al (Eds), *Words and worlds: Modelling verbal descriptions of situations* (pp. 351-362), Rotterdam: Sense Publishers.
- Freudenthal, H. (1991). *Revisiting mathematics education. China lectures*. Dordrecht: Kluwer.
- Gravemeijer, K. (2007). Emergent modeling as a precursor to mathematical modeling. In W. Blum et al (Eds) *Modelling and applications in mathematics education* (pp. 137-144). New ICMI Studies Series 10. NJ: Springer.
- Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge, MA: Cambridge University Press.
- Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Nunes, Mathematics in the Streets and in Schools*. Cambridge, U.K: Cambridge University Press.
- Resnick, L. B. (1987). Learning in school and out. *Educational Researcher*, 16(9), 13–20.
- Saxe, G.B. (1991). *Culture and cognitive development*. Mahwah, NJ: Erlbaum.
- Schliemann, A. D. (2002). Representational tools and mathematical understanding, *Journal of the Learning Sciences*, 11 (2/3), 301-316.
- Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education*, 27(5), 521-539.
- Stoyanova E., Ellerton N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. C. Clarkson (Ed), *Technology in mathematics education* (pp. 518-525), MERGA: Melbourne University.

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## **Mathematics and Music: a paradigmatic pair for basic learning**

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*To our Master Filippo,  
who was able to put  
our intuitions into harmony.*

**Summary.** *The aim of the trial path described in this article is to highlight the possibility to develop intuition, analysis and synthesis abilities, which are typical of the logical - deductive mathematical thought, in primary school children (2<sup>nd</sup> class), through a significant approach to musical education, meant as systematic analysis of the sound parameters, which characterize and qualify its language. The Vygotskijan theoretical framework, which leads and interprets the didactic engineering proposed in class, emphasizes how the Mathematics - Music pair has promoted the changeover among different semiotic registers (Duval 2002), referred to Natural, iconographic, musical, geometric and pre - algebraic Language. The didactic strategy, masterfully lead in class by the teacher through a polyphony of voices (Bartolini Bussi et al, 1995) and a democratic climate (Skovsmose & Valero, 2002), has fostered the acquisition of good skills of analysis and production of argumentations, typical of the musical and mathematical thought.*

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Key words: *Problem solving, Classification, Discrimination, Analysis and Synthesis, Objectification, Music, Orff.*

### **1. Musical and mathematical education: perceptive and objectification aspects**

The process leading to musical education and knowledge is not nearly simple and consists of different elements, apparently far from each other. First, it is appropriate to highlight the existence of a primitive component in music listening, which connects sensations and emotional reactions to specific timbre and tonal ranges. However, the inborn component does not provide an exhaustive explanation of the multidimensional responses provided, nor can explain the cultural differences among them. Therefore, the majority of our responses to music seems to be learnt, so linked to our experiential background, as well as product of the socio - cultural context in which the listener is integrated and that determines his emotional reaction (Sloboda, 1983).

#### *1.1 Ratio and delectatio*

Hence, music - as art and science at once- shows a prominently irrational soul off, which is fruit of intuition, unlikely classifiable or expressible through specific algorithms, carrying out processes of analysis and synthesis, which characterize the production and comprehension of music production and listening. In agreement with Zotto, it is possible to affirm that, in its "linguistic" articulation, Music has got a logical structure, referable to axiomatic, grammatical and algorithmic elements, which can be assimilated (considering the appropriate exceptions) to the logical - deductive ones of the mathematical thought. *Ratio* and *delectatio*, in this sense, qualify the musical universe and, in their dialectical point of contact, contribute to create new synaptic circuits with relevant implications. Therefore, mathematical and musical languages qualify each other, finding an important point of contact in their "communicative" structure and in the cognitive processes of coding and decoding, which make them readable and reproducible at the same time.

As far as it was said, it seems evident that the concept of musical learning is conceived as a creative and collective experience, which involves what is or can be inherent in Music: gesture, movement, dance, verbal articulation, vocality, instrumentation, dramatization and performance (Piazza, 1979). These elements structure the musical language, providing meaning and structure, and allow its rationalization through notation, as essential form of memorization and communication. In this perspective, the choice of the Orffian methodology, that we adopted in the trial path definition, arises from the need to promote a global education path, which entirely involves the pupil on emotional, cognitive and physical level. (Piazza, 1979)

The Orff - Schulwerk can be considered as a general pedagogic criterion applied to basic musical education, which gets the child to know Music, making him familiarizing with elementary sound structures presented to him as accessible and

concrete, and which embodies a simple educative idea: Music is learnt by making it and not by abstracting it, putting it into the concrete need to live it, investing in all the fields, from the tightly physiological and psychological to the emotional one, connected with the need for music for the pleasure of doing it (Piazza, 1979).

Thus, it is a valid pedagogic tool, fit for promoting an approach that favours creativity, through processes ranging from play to free exploration, structured improvisation and composition, in a continue democratic dialectics between pupil and teacher.

In this theoretical - practical frame, the original contribution of the most recent researches about Music cognitive psychology can be found: E. Gordon's Music Learning Theory (1971), which allows a significant reading key of musical alphabetization processes.

In the centre of this process of authentic education, the child is placed as active and motivated subject, who can express his own personal nature, in agreement with himself and the excellence that qualifies the human being as such, resorting to different semiotic registers.

### *1.2 Objectification and generalization*

There are different analytical theories, which have highlighted the importance of musical education for the child education - and not only for this - considering its different aspects, from the neurological, to the more tightly educative area. Among these theories, the one considered to deserve special attention is that of the Multiple Intelligences of H. Gardner, in which the presence of a musical intelligence is underlined. This intelligence expresses itself as ability not only in composing and executing, but also in listening and distinguishing musical pieces, with reference to pitch, rhythm and timbre.

According to the author, the development of every intelligence comes to life from symbolization processes, strongly determined from the cultural context and, therefore, from educative stimulations, which the person is exposed to. Hence, one goes from simpler symbolization channels to gradually more complex forms. This idea of "generalization" is at the basis of Radford's Objectification Theory (2000) and represents the central fulcrum of the trial process, oriented to the promotion of an evident maturing, that actualizes itself in the changeover from a generalization level to the following. By means of this new "lens", one proceeds to the analysis of mathematical learning didactic problems, which are specifically referred to intuition, analysis and synthesis abilities, typical of the logical - deductive thought (cognitive conflicts and spontaneous and scientific learning models).

In agreement with Radford (2000), the mathematical objects we presented to the children in the musical and the strictly mathematical spheres have been considered as symbols of cultural units emerging from a system of customs linked to mathematical activities, which have been realized in class, in different semiotic contexts interacting with each other (Cortesi, 2010). In this perspective, the reflec-

tion activity proposed in class during the trial has involved a consciousness raising of the mathematical object by the pupil, through a process of actual involvement, which needs to work very hard in mathematical and musical activities, not to "build the object", which is already in the culture, but "to give it a sense". Radford defines this active and dynamic process with the term "objectification", underlining how learning corresponds to objectifying the well-known reality (Radford, 2004).

The creation of this kind of appropriate learning condition, where all the students develop the ability to solve and understand increasingly challenging problems, was, according to us, an important opportunity, in which knowledge (music and mathematics knowledge) can be developed and evaluated critically (Jaworski, 2003).

Moreover, the democratic laboratorial setting used in mathematics class practices designated the possibility to consolidate democratic social relationships (Skovsmose & Valero, 2002), that are strongly encouraged at primary schools. According to the Bartolini Bussi research (1995), we tried to organize the mathematics class as micro-society, in which democratic relationships among their members can be established during their communication/discussion.

In particular, among the social interaction modalities in the mathematics laboratory, we have specifically considered the *mathematical discussion*, mediated by the teacher. It is defined as "*polyphony of voices on a mathematical object (a concept, a problem, a procedure, a structure, an idea or a behaviour regarding mathematics), which is one of the reasons of the teaching - learning activity*" (Bartolini Bussi, al., 1995). This is a key idea, reasserted also in the information about the laboratory for primary school.

## 2. The trial path

The entire trial path has been structured in agreement with the above mentioned framework and oriented to refine the didactic tools, so that they make the educational intervention incisive on the basis of the valorisation of the person and in respect of the personal excellence, which is a human feature (Garcia Hoz, 1981).

In this sense, thanks to its epistemological nature, the Mathematics – Music pair has been useful to broaden pupils' knowledge horizons, in a sort of new culture of the change, that, equipped with suitable scientific tools, has revealed step by step its complexity in a social and democratic sharing vision. (Skovsmose, & Valero, 2002)

As said before, the trial path's basic idea was to promote, in a laboratorial setting, an efficient musical training, able to direct the learner for a knowledge "re-building" (Freudenthal, 1991) on more complex cognitive levels, which allow a conscious learning also in other disciplinary areas, particularly Mathematics.

Essential condition for this to happen is (Freudenthal, 1991) that Mathematics has to be seen in every phase as an activity in which everyone is protagonist and where knowledge is the product of personal achievements, that have to put them-



selves in line (Guidoni, 1985) with the natural learning process, which is active in everyone since the first years of life and which goes hand in hand with language acquisition.

For this purpose, we mostly pointed out the elaboration processes of musical and mathematical language coding and decoding, mainly through practical activities, proposed in recreational form, in respect of times and learning modalities, typical of this development stage.

In agreement with Quartuccio (2009), we wondered about the proposed path's generalization levels and we are conscious that the examined trial group cannot allow immediate generalization inferences concerning the results obtained on larger samples, though it is considered a significant subset of primary school students (2<sup>nd</sup> class), as heterogeneous in socio-cultural class context, start cognitive level of the involved persons and teachers' educative styles. However, starting from pre-existing studies on the same topic, the trial path can promote new reflection ideas, that could generate an innovation atmosphere in the panorama of basic musical and mathematical didactics in primary school. These, read in parallel to the experimentally obtained results, can foster the formulation of new research questions and hypothesis.

### *2.1 Research experimental hypothesis*

In the planning stage the relation between the proposed training and the kind of expected behaviour (or the response) has been highlighted more than once. This has led to the definition of a research hypothesis, which has been modified and refined in progress.

H1, "*IF there are correlations between the competences acquired through the structural - rhythmical musical education and those of intuition, analysis and synthesis, typical of the relational mathematical thought THEN appropriate laboratorial interdisciplinary learning situations between Mathematics and Music can facilitate and improve some processes of basic mathematical logical - deductive learning in primary school children*".

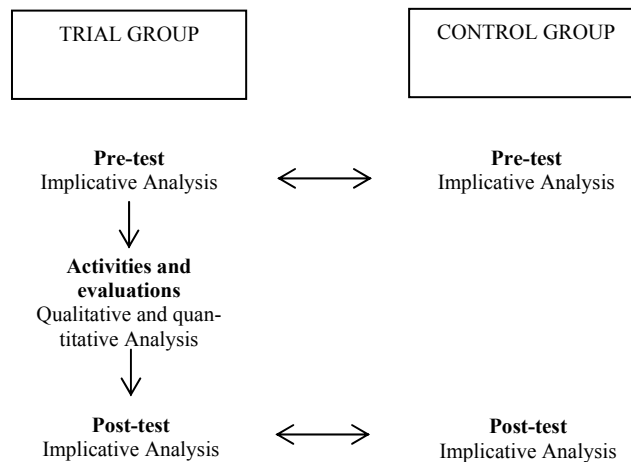
In order to falsify the start hypothesis, we have used tests elaborated according to Ragusa's work (2004). These tests have been administered to three trial groups and three control groups, before and after the training (Levorato, 2002).

The proposed items allowed to check pupils' ability to make structural - rhythmic transpositions and to detect the relation among different elements, (through the continuation or the completion of repeating forms); rhythmic and logical abilities to work with numbers; the ability of repeating a structure in cyclic form and therefore to reproduce a rhythm both from a graphic and a numeric point of view. At the end of each exercise we asked the pupils to motivate their answer, in order to trace and reconstruct the logical paths which lead to the solution (exact or not) of the

proposed problem-situations, by means of the detection of the suitable semantic indicators.

*2.2 Trial plan*

The trial plan is presented hereafter in schematic form, in order to make its reading more immediate and as synthesis of a path, developed during more than 6 months (from October 2010 to the end of April 2011):



Scheme 1. Trial plan

The trial groups have been chosen inside three different socio - cultural Sicilian realities, coherently with the aims definition. In order to remove the doubt that possible improvements acquired with the trial are connected to the natural maturing occurred from the pre- to the post-test and not to the proposed educative intervention's efficiency, a methodological precaution has been taken, that is, we referred to control groups equivalent in composition, teacher, socio - cultural level and verified competences.

The choice of the control groups has been made rigorously, in order to work with groups that were actually superimposable. The didactic problem's complexity required the recourse to different qualitative and quantitative analysis tools (through the resort to inferential statistics with the use of the software C.H.I.C.) (Gras, et al, 2008) to foster an as objective as possible observation of the examined phenomena and an adequate measurement of them<sup>2</sup>. Therefore, we chose to videotape the whole didactic action and plan an a priori analysis of the expected behaviours (Brousseau, 1997), which proved to be particularly significant in the last part of the activity that concerned the definition and the carrying out of two a-didactic

<sup>2</sup> We will show the result during the conference's talk.

situations, created ad hoc, in agreement with Brousseau's TDS framework (Brousseau, 1997).

For the sake of brevity, hereafter we present a table which summarizes the project's stages, and a brief contextualized description of the proposed activities, summarized next to the presented stage. We point out that the operative path proposed in class to primary school pupils has been referred to the theoretical/experimental frame of the Orfian Movement and in particular to the educational paths proposed in the studies of Bonfanti and Re (2009), Paduano (1998), Piazza (1979), Spaccazocchi and Perini (2003), Dani and Di Natale (2011). We decided to divide the trial stage into three cycles structured on the basis of ascending skills and abilities:

Preliminary Survey	Pre-test <i>Prerequisites evaluation through a set of three games</i>
<b>Trial stage: didactic situations</b>	<u>First cycle</u> ➤ Noise or not? ➤ Princess Scacciachiasso <sup>3</sup> ➤ Different cries ➤ <i>Let's play on it</i> – We are like... ➤ Let's draw the cries ➤ <i>Let's play on it</i> – In un coppo ➤ Verbal rhythmic articulation ➤ Let's stress us! ➤ <i>Let's play and sing on it</i> – Tick & Tock <u>Second cycle</u> ➤ Watch the ear! ➤ A sounds seesaw ➤ <i>Let's play and sing on it</i> – Ala Mala ➤ The dancing bears ➤ Let's give a timbre to the sounds ➤ <i>Let's play and sing on it</i> – Clap your hands, beat your chest, feet ➤ The body that plays ➤ One timbre, one guarantee <u>Third cycle</u> ➤ Music dictation ➤ Tell me your form and I'll tell you who you are ➤ <i>Let's play on it</i> – The canon
	<b>A-didactic situation</b> (Brousseau, 1997) ➤ Tale in music ➤ The MusiMatheMagic World
	<b>Final stage</b>

Tab.1 Trial path synthesis

In brief, the path's first stage has been divided into two essential moments:

<sup>3</sup> *Scacciachiasso* means "noise-banishing".

1. The distribution of a pre-test, appropriately structured in 6 classes, 3 of them experimental and 3 of control, with consequent data-gathering and analysis by means of the software of inferential statistics C.H.I.C. (Gras, Spagnolo et al, 2008).
2. The evaluation of prerequisites in the trial contexts to outline an exhaustive picture of the start situation.

### 3. Conclusions and open-ended problems

From the obtained data that we will deeply show during the conference's talk, it has been possible to gather that the experimental training administration produced positive effects, which characterize a mature relational thought, that materializes in a good argumentative ability, manifested also in the analysis of the related semantic indicators presented in the protocols. The improvement of the abilities of analysis, synthesis, transposition, discrimination and classification is evident in the ability to consciously use different semiotic registers and in the good use of the technical language referred to the two considered disciplines. In addition, the path has introduced trial groups to the use of an increasingly articulated symbology, whose application has been developed according to increasing complexity levels: we have changed over from the use of symbols referred to one-to-one relations, still closely tied to the *embodied* level (Lakoff and Nunez, 2000), to the use of symbols, that entail a cognitive breaking from the *embodied* level to the *disembodied* one. Such changeover can be seen also through a comparison between the ongoing proposed evaluation reports and the research protocols. It is believed that the path has produced also significant effects in the socialization context and in that of the relationship between peers and with the teacher, made evident by a relaxed cooperative atmosphere and the active participation of the whole class to the proposed activities (Skovsmose & Valero, 2002). Finally, in the perspective of the improvement of the experience which has been already carried out, it is important to highlight the existence of some open-ended issues, that can serve as a basis for the hypothesis formulation for any subsequent research. Among these, we wonder about the possibility of planning new musical paths, which facilitate the acquisition of additional abilities in the musical and logical-mathematical area, according to a perspective of growth and deep cognitive reorganization, and also considering the possibility of extending the research to a significant population sample, that could validate the initial hypothesis.

### Essential references

- Brousseau G.: 1997, Theory of didactical situations in mathematics, Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Bartolini Bussi, M.; Boni M. & Ferri M.: 1995, Costruzione sociale del sapere matematico: discussione matematica e rappresentazione dello spazio, Modena: Centro Documentazione Educativa).

- Duval, R.: 2002, Representation, vision and visualization: cognitive functions in mathematical thinking. Basic issues for learning. In F. Hitt, (ed.), Representations and Mathematics Visualization (pp. 311-335). North American Chapter of PME: Cinvestav-IPN, México.
- Freudenthal, H.: 1991, Revisiting Mathematics Education. China Lectures, Kluwer, Dordrecht.
- Gras R., Suzuki E., Guillet F, Spagnolo F.: 2008, Statistical Implicative Analysis: theory and applications, Springer, pp. 1-513.
- Lakoff, G., & Núñez, R. E.: 2000, Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being, Ed. Italiana: Da dove viene la matematica: come la mente embodied dà origine alla matematica. Bollati Boringhieri.
- Jaworski, B.: 1993, Research practice into influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partenships. Educational Studies in Mathematics, 54, 249-282.
- Paduano C.: 1998, Dispensa “Educazione al suono e alla Musica” – Published on La vita scolastica year 53 n°4
- Piazza G.:1979, Orff Schulwerk, Manuale – Edizioni Suvini Zerboni, Milano.
- Radford, L.: 2000, ‘Signs and meanings in students’ emergent algebraic thinking: A semiotic analysis’, Educational Studies in Mathematics 42(3), 237–268.
- Radford, L.: 2004, Cose sensibili, essenze, oggetti matematici ed altre ambiguità, La Matematica e la sua didattica, No. 1, 4,23.
- Skovsmose, O. & Valero, P.: 2002, Democratic access to powerful mathematical ideas. In L. English (Ed.), International Research in Mathematics Education (pp. 383-408). LEA.
- Spaccazocchi M.: 2003, Crescere con il canto 1, Ed. Progetti sonori.
- Spaccazocchi M., Perini L.: 2011, Noi e la Musica, Ed Progetti Sonori
- Zotto G.: 1993, Lo stereotipo in Musica, in Dallo stereotipo alla creatività, by Pino Parini, Trento, Publiprint, p. 163-167.

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## **Introducing CLIL methodology for mathematics in Italy**

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*This contribution analyses the Italian context with regard to the impending introduction of CLIL (Content and Language Integrated Learning) methodology in classrooms in Italy. Up to now, as far as the CLIL teaching of mathematics using CLIL is concerned, only a few studies have been reported upon. As a consequence, it is difficult both to identify the necessary competences for mathematics CLIL teachers, and to state how these competences can be attained. The framework of reference for our remarks on the use of CLIL for mathematics teaching is in part based on some theoretical guidelines developed within two European projects devoted to CLIL. Selected quotations from answers to a questionnaire distributed to a selection of secondary school Mathematics teachers are used to support our research hypotheses.*

### **Introduction**

Promoting the learning of languages is increasingly one of the main educational objectives of the European Union. Documents and consequent actions carried out by both the European Commission<sup>1</sup> and individual coun-

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<sup>1</sup> [http://ec.europa.eu/education/policies/2010/doc/jir\\_council\\_it.pdf](http://ec.europa.eu/education/policies/2010/doc/jir_council_it.pdf)

tries emphasize the importance of making students more competent in foreign languages. In Europe, many issues related to language teaching and learning are inspired by the Common European Framework of Reference (CEFR)<sup>2</sup>, with as a principal idea the development of language competences in meaningful contexts, which in themselves constitute learning goals. In other words, the perspective adopted in CEFR is consistent with teaching/learning situations which integrate ‘language’ and ‘content’ in a synergic way.

In Italy, only in recent years, has adequate attention been paid to the issues mentioned in the paragraph above. One of the main initiatives is the introduction of CLIL (Content and Language Integrated Learning) methodology in a large number of schools recommended in recent upper secondary school reform proposals

CLIL is a teaching/learning methodology which aims at helping students to develop both ‘language’ skills and ‘content’ awareness at the same time.

It encompasses many different forms of learning context in which a language carries a special role alongside the learning of any specific subject or content. It refers to any dual-focused educational context in which an additional language, thus not usually the first language of the learners involved, is used as a medium in the teaching and learning of non-language content. (Langé, 2002, p. 11).

Generally speaking, as far as many scientific subjects are concerned, little academic literature on this topic is available. As far as combined CLIL and mathematics teaching is concerned, apart from a few papers (e.g. Hofmannová & al., 2004), certain studies about particular teaching contexts are available, such as Bilingual Schools (e.g. Barton & Neville-Barton, 2003; Barwell, 2002; Clarkson, 1992) and Multicultural Classrooms (e.g. Barton & al. 2007; Setati. & Adler, 2001). In these contexts pupils can actually be taught in a second language, but the exact educational aim might vary and sometimes might not fully embrace CLIL methodology.

### **CLIL areas of education and competence**

If sufficient theoretical references are not abundant, what training frameworks for training CLIL mathematics teachers? It is possible to identify and utilize the theoretical framework developed within the *CLIL across*

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<sup>2</sup> [http://www.coe.int/T/DG4/Linguistic/CADRE\\_EN.asp](http://www.coe.int/T/DG4/Linguistic/CADRE_EN.asp)

*context* project<sup>3</sup> as a useful tool to come up to some answers to this question. In fact, a main project output consists of the identification, definition and description of the following ‘areas of education and competence’ which are considered necessary for CLIL teachers to be aware of, and this applies to most subject (mathematics included).

- *Learners’ Needs*
- *Planning*
- *Multimodal Teaching and Learning*
- *Interaction*
- *Subject Literacy*
- *Evaluation/Assessment*
- *Cooperation and Reflection*
- *Context and Culture*

Focusing on ‘areas of education and competence’ may represent an effective methodological tool to highlight issues which arise when designing CLIL teachers training courses, but it is possible an adaptation for different types of schools might need to be made.

### **The Italian context**

Up to now, in Italy, very few CLIL programs are publicly reported, mostly on school websites and/or internal reports. The very large majority of these programs are carried out sporadically, they are not part of a structured educational plan. Sometimes, CLIL based activities are an optional, additional educational opportunities. The choice of both the (second) language of instruction and the content subject mostly depends on incidental factors, such as the presence of mother tongue teachers who volunteer to teach their subject in their first language (L1), which is a second language (L2) for the students.

CLIL methodology is still very rarely used to teach mathematics, and due to the specificity of the subject, it is used by those mathematics teachers who happen to be fluent in an L2 (a second language also for students), either because it is their mother tongue or they learnt it for some other reason (e.g. studies or stays abroad). No reports are available about foreign lan-

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<sup>3</sup> *CLIL Across Contexts: A scaffolding framework for CLIL teachers education* is a three-year (2006-2009) SOCRATES-COMENIUS 2.1 project, EC co-funded. The first author was a partner in the project. [<http://clil.uni.lu>]



guage teachers who can also teach mathematics, without the support of additional subject teachers in the classroom. In fact, this situation is rare because the Italian teacher training system does not encourage to qualify for teaching both a foreign language and a scientific subject. The newness of the official, sudden introduction of CLIL in the Italian school system from 2012, without previous systematic piloting and, above all, teacher training in this methodology, has required investigations into maths teachers' current views on CLIL methodology in order plan training program.

### **The Italian mathematics teachers' perspective**

To collect teachers' views, a questionnaire was introduced<sup>4</sup> not only to mathematics teachers, but also to foreign language teachers in lower and upper secondary schools in Italy.

The main aim of the questionnaire was to collect information about the respondents' opinions about future teaching of mathematics in a foreign language. Since in this paper we are interested in the CLIL methodology 'applied' to teaching/ learning Mathematics, we take into account, in the following sections, answers given by only Maths teachers. The questionnaire comprises 24 questions and was answered by 126 mathematics teachers.

The questions were grouped as follows:

- *About you*
- *Prior experiences*
- *Present opinions*
- *Expectations*
- *Professional development*

The eight areas of competence for CLIL teachers, introduced in the first part of this paper, are interestingly, implicitly or explicitly, also referred to by the Italian mathematics teachers, whose answers to the questionnaire were analysed.

In this section our perspective on the areas of education and competence for mathematics CLIL teachers in Italy is presented. Our opinions are sup-

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<sup>4</sup> The questionnaire is part of the activities in the *Making Mathematics Teachers Mobile*, SOCRATES-COMENIUS 2.1 project (2006-2009), EC co-funded and coordinated by the first author. [<http://mta.dm.unipi.it>]

ported by a few quotations from teachers' answers to some of the questionnaire items in sections *Present opinions*, *Expectations* and *Professional development*.<sup>5</sup>

While, as expected, *Learner Needs*, *Interaction* and *Subject Literacy* are the competences most referred to by the respondents, it is surprising that *Evaluation / Assessment* is not such an area of great concern for them. On the contrary, the reported poor co-operation of mathematics and foreign language teachers and the lack of prior CLIL teaching programs are perhaps the main reasons why *Planning*, *Multimodal Teaching and Learning*, *Cooperation and Reflection*, *Context and Culture* are rated as less of a priority.

Apart from a positive comment from one teacher "*CLIL teaching is challenging for pupils*", *Learner Needs* are in the main identified by respondents who do not see learning mathematics as a foreign language as of great value: "*it would be confusing for many students*" and "*learning requires concentration and more concentration will be devoted to understand the language and not mathematics*" are different approaches to what the respondents most serious concern: "*Teaching maths is difficult even in the native Italian language...*" This last comment reveals a well-known conceals issues regarding effectiveness of their teaching and could also be interpreted as reluctance to embrace any methodological change..

More honestly, one respondent stated that *Learners Needs* are totally connected to the skills needed by CLIL teachers: "*the ability to communicate by means of terms understandable by pupils*".

This skill is, clearly, also connected to another area of education and competence: *Interaction*. As far as *Interaction* is concerned, teaching mathematics in a foreign language could possibly support mathematics learning as "*Teachers and students need to take greater care in the use of the language, also including a more rigorous and synthetic use of the mathematical language*" and "*Teachers develop a deeper concern about the difficulty of mathematics and then about its teaching*". But it is also cited as a potential obstacle: "*The language could represent a barrier despite the universal symbolism of mathematics*".

As far as the skills needed, *Interaction* in a CLIL classroom requires a great ability to use standard mathematical terminology, but it also requires

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<sup>5</sup> Further details on the questionnaire structure and analysis are available on the relevant webpage of the *Making Mathematics Teachers Mobile* project: <http://mta.dm.unipi.it>

non-formal language knowledge, which allows teachers to interact in a more relaxed way during classroom activities (e.g. providing additional explanations and examples).

**Subject Literacy** is, certainly, the CLIL competence area of biggest interest for mathematics teachers, as their comments clearly show. The importance of knowing a foreign language and using it to teach mathematics could even make teachers reflect on *“what teaching mathematics means”*. In particular, the language knowledge *“increases ability to teach and explain, using simple but rigorous words thanks to the particularly limited and specific mathematics vocabulary”*. Subject literacy is also important to make it possible to teach mathematics in a foreign language, thanks to *“its rules that one can understand also without speaking”* and *“the specificity of its language, and the use of symbols”*. CLIL teaching methodology is also considered as a possible support for mathematics learning as it represents *“a proof of the universality of mathematics and its independence from the language”* and requires *“greater reflection on the concepts to be taught”*. As to the skillsets needed, teachers said that **Subject Literacy** includes both *“good knowledge of the to-be-taught topic”* and *“the mastery of the language at least at scientific level”*.

### Conclusions

Since CLIL teaching is a methodology which has been sporadically included in a few pedagogical program in Italy only in the last few years, we are still far away from a situation where teachers are ready to use it, and its implementation can reach the objectives established by the CLIL methodology as it has been defined.

As the difficulty of mathematics learning is acknowledged worldwide, it is possible to expect that concerns and incentives of maths teachers are possibly greater than those of other subject teachers. In CLIL classrooms, teachers need to be able to find a good balance between the different languages, and the pertinent codes: the mother tongue, the foreign language, the mathematics language and, sometimes, the ‘language’ of the ICT tool (such as a computer algebra system or a dynamic geometry software).

However, teachers should be aware that using the CLIL methodology involves a long term learning process. As far as the *Assessment* is concerned, they need to be a better way of assessing both their own and the learners’ level of language competence, and the learners’ knowledge of the subject

specific content. Teachers also need to be able to identify possible conflicts between the mathematical language and the language of instruction, to distinguish learners' difficulties related to the mathematical content as opposed to language difficulties in their own right. Furthermore, CLIL teachers also need to appreciate learners' ability to look for and make use of communication strategies in solving problems and reporting solutions.

The issues commented on above are not exhaustive and further research into the relationship between CLIL and mathematics need to be carried out before any formal combined teacher training programme is introduced, set up, and run successfully.

### References

- Barton, B., Barwell, R. & Setati, M. (Eds.) (2007) Multilingualism in mathematics education. *Special Issue of Educational Studies in Mathematics*, Vol. 64, no. 2.
- Barton, B. & Neville-Barton, P. (2003). Investigating the relationship between English language and mathematical learning. In M.A Mariotti, (Ed.), *Proceedings of CERME-3*, CD, Pisa: PLUS.
- Barwell, R. (2002). Understanding EAL issues in mathematics. In Leung, C. (Ed.) *Language and Second/Additional Language Issues for School Education: A Reader for Teachers*, (pp. 69-80). Watford: NALDIC Publications Group.
- Clarkson, P.C. (1992), Language and mathematics: A comparison of bilingual and monolingual students of mathematics, *Educational Studies in Mathematics* 23(4), 417-429.
- Davies, S. (2003). Content Based Instruction in EFL Contexts. *The Internet TESL Journal*, 9(2), 24–28.
- Hofmannová, M., Novotná, J. & Pípalová, R. (2004). Assessment instruments for classes integrating mathematics and foreign language teaching. [http://www.icme-organisers.dk/tsg27/papers/16\_Hofmannova\_et\_al\_fullpaper.pdf]
- Langé, G. (ed.) (2002). *TIE-CLIL Professional Development Course*. MIUR Direzione Regionale della Lombardia, on behalf of the TIE-CLIL Project.

- Marsh, D. (ed.) (2002). *CLIL/ EMILE – The European Dimension: Actions, Trends and Foresight Potential*. Public services Contract DG EAC. Strasbourg: European Commission. [<http://europa.eu.int/comm/education/languages/index.html>]
- Oxford, R. (2001). Integrated Skills in the ESL/EFL Classroom. *ESL Magazine*. 6(1), 1-7
- Setati, M. & Adler, J. (2001). Between Languages and Discourses: Language Practices in Primary Multilingual Mathematics Classrooms in South Africa, *Educational Studies in Mathematics*, 43(3), 243-269.

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**Democratizing ‘big ideas’ of mathematics through  
multimodality: Using gesture, movement, sound and  
narrative as non-algebraic modalities for learning  
about functions**

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**Abstract:**

The acquisition of algebraic fluency takes years, and many learners find algebra to be insurmountably difficult – or not worth the effort. However many of the most interesting, useful and empowering mathematical ideas have traditionally been introduced via algebra. This practice acts as a de facto barrier to many people’s opportunities to learn the ‘big ideas’ of mathematics, and creates a fundamentality anti-democratic elitist mystique around these ideas. If we are serious about making mathematics accessible to critical thought in a democratic society, we must develop alternative ‘ways in’ to the big ideas that can supplement and/or precede the learning of algebra. The author has been successful in introducing mathematical functions to younger learners, aged 11-14 years (including resistant, dyslexic, gifted, and now visually impaired learners) via gesture and movement, vocal sounds and narrative. This paper describes a multi-year project in innovative, embodied, non-algebraic approaches to functions and examines implications for accessibility and democratic participation.

**Algebra as gatekeeper**

The role of algebra as a ‘gatekeeper subject’ has been noted by many researchers, particularly in the United States (Rech & Harrington, 2000; Moses & Cobb, 2001). Algebra also functions in a more subtle way in blocking many people from grasping the ‘big ideas’ of mathematics, ideas which underlie the design of many of the institutions of our mathematized society.

I will present a snapshot of a research program aimed at developing other ‘ways in’ to the big ideas of mathematics: performative and embodied pedagogies that might function prior to and alongside algebra to allow broader access to the meanings of mathematical ideas.

**Offering access to ‘big mathematical ideas’ without relying on algebraic fluency**

The abstract, technical nature of algebra and the need to master its complicated algorithmic rules can be discouraging for students. The acquisition of algebraic fluency takes years, and many learners find algebra to be insurmountably difficult – or sometimes not worth the effort.

There is often a period of several years (typically the early years of secondary school) during which students are primarily learning the *how to* of algebra. During this time, other big ideas in mathematics are deferred until students have sufficient algebraic competency.

Unfortunately, huge numbers of students become alienated and disenfranchised through this process. The practice of holding off conceptual teaching until algebra is learned acts as a *de facto* barrier to many people’s opportunities to learn the big ideas of mathematics, and creates a fundamentality anti-democratic elitist mystique around these ideas (Stinson 2004). If we are serious about making mathematics accessible to critical thought in a democratic society, promoting reflective or critical competency in mathematics (Skovsmose 1994), we must develop alternative approaches that can supplement and/or precede the learning of algebra.

Mathematical ideas are accessible through a variety of possible approaches, and mathematical meaning-making and pattern-noticing are initially more easily accessed through embodied, performative, narrative means than via algebra (Gerofsky, 2009 ; Swanson, 2009; Gadanidis & Borba, 2008). Embodied, performative and narrative modes of knowing and learn-

ing are not structured to defer meaning as algebra does, and by working in these modes, learners may have a better sense of following the thread of an argument or grasping a pattern without getting lost in labyrinthine, de-signified processes.

### **The Graphs & Gestures project: Methodology**

Since 2006, I have engaged in a research project aimed at theorizing, developing and testing multimodal, non-algebraic pedagogical approaches to big mathematical ideas about functions, including performative approaches to 'reading a graph with the body' and vocalizations.

This research is based in methodologies involving the qualitative, naturalistic study of learners in small group and classroom settings (Denzin & Lincoln, 2005; Patton, 2002), grounded in the research literature on multisensory, multimodal mathematics learning (for example, Arzarello & Edwards, 2005; Arzarello & Robutti, 2008; Fernandes & Healy, 2010; Nemirovsky & Ferrara, 2009; Radford, 2009; Roth, 2010) and methods from performance ethnography (Conquergood, 1998, 2002; Conrad, 2004; Denzin, 2006). A recursive, emergent model is used to inform the design cycle involving both pedagogy and new technologies (Smith & Maclean, 2006).

### **Graphs & Gestures project: Findings and future directions**

In the pilot study, students aged 13 and 16 from three different schools were asked to gesture the shapes of the graphs of functions in front of a video camera. A week later, students were interviewed while watching their videos. Teachers were elicited for holistic assessments of participant students' attitudes and abilities as demonstrated in mathematics class.

Three gesture types were identified, and these correlated strongly with three distinct learner approaches, as identified by learners in their interviews and by teachers' holistic comments (Gerofsky, 2008; Gerofsky, Savage & Maclean, 2009):

- i. **"Seeing the graph"**: Gestures with the  $x$ -axis placed high against the body, with eye tracking and no acceleration, little or no engagement of the spine, gestured distally (at arm's length), using only the finger, wrist and forearm.
- ii. **"Being the graph"**: Gestures with the  $x$ -axis place low against the body, with no eye tracking, marked acceleration, a high degree of engage-



ment of the spine, gestured proximally (close-in), using full arms and the whole body.

iii. **“Not knowing what was salient in the graph”**: Hesitance and tentativeness in commencing any kind of gesture; large parts of the graph not gestured at all; reliance on a verbalized approximating metaphor (i.e.: “it looks kind of like a W”) as a way to gesture the shape of the graph.

Further theorizing work using sources in gesture theory showed a correlation between (ii) and CVP (character viewpoint), and between (i) and OVP (observer viewpoint) (Gerofsky, 2010 a&b).

Based on these findings, the author and collaborator Kathryn Ricketts designed teaching experiments that aimed at introducing advanced mathematical ideas about functions to younger students via engaged, whole-body movement much like the gestural movements described in (ii) – “being the graph”. The idea was a fundamentally democratic one: that performative, embodied modes of engagement with mathematical ideas at an early age might make a robust understanding of those ideas available for all learners, rather than only for an elite.

An experiment with vocal sounds to represent mathematically salient features of functions (roots, extrema, slope) turned out to be one of the most memorable and effective ways for students to interpret functions, as evidenced by the students’ own comments and by their performance on a group think-aloud post-test.

In general, post-tests (even when conducted a full year after instruction) showed that most students were able to recall, interpret and reconstruct knowledge of advanced mathematical ideas by using gesture and sound as cognitive resources. Students learned about big mathematical ideas through performative, non-algebraic means and used embodied means to make sense of what they had learned long afterwards. Over 80% of students in the ‘average’, gifted and dyslexic groups showed consistent progress in understanding of the big mathematical ideas introduced.

The Graphs & Gestures project will move to the development of further multisensory pedagogies using sonification and whole-body movement, working with both visually impaired and sighted students and a computer interface based on Kinect movement-sensing technology. The aim is to extend

multimodal, multisensory mathematics pedagogy in ways that might benefit both special needs students and, ultimately, all students.

### References

- Arzarello, F. & Edwards, L. (2005). Gesture and the construction of mathematical meaning. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, 123-154. Melbourne: PME.
- Arzarello, F. & Robutti, O. (2008). Framing the embodied mind approach within a multimodal paradigm. In English, L. D. (Ed.), *Handbook of international research in mathematics education, second edition*. New York: Routledge. 716-745.
- Conquergood, D. (1998). Beyond the text: Toward a performative cultural politics. In Dailey, S. (Ed.) *The future of performance studies: Visions and revisions*. 25-36.
- Conquergood, D. (2002). Interventions and radical research. *The Drama Review* 46(2), 145-156.
- Conrad, D. (2004). Exploring risky youth experiences: Popular theatre as a participatory, performative research method. *International Journal of Qualitative Methods*, 3(1), 2-24.
- Denzin, N. (2006). Pedagogy, performance, and autoethnography. *Text and Performance Quarterly* 26(4), 333-338.
- Denzin, N. & Lincoln, Y. S. (2005). *The SAGE handbook of qualitative research*. Thousand Oaks, CA: SAGE.
- Fernandes, S.H.A.A and Healy, L. (2010). Embodied mathematics: Relationships between doing and imagining in the activities of a blind learner. In M.M.F Pinto, and T.F. Kawasaki (Eds.) *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* Vol.2, 345-352. Belo Horizonte, Brazil: PME.
- Gadanidis, G. & Borba, M. (2008). Our lives as performance mathematicians. *For the learning of mathematics* 28(1), 44-51.
- Gerofsky, S. (2008). Gesture as diagnosis and intervention in the pedagogy of graphing: Pilot studies & next steps. In In Figueras, O. & Sepúlveda, A. (Eds.). *Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education* Morelia, México: PME.
- Gerofsky, S. (2009). Performance mathematics and democracy. *Educational insights* 13(1). <http://www.educationalinsights.ca/>

- Gerofsky, S. (2010a). Teaching polynomial functions through gesture, movement and sound. In M.M.F Pinto & T.F. Kawasaki (Eds.) *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*. Belo Horizonte, Brazil: PME.
- Gerofsky, S. (2010b). Mathematical learning and gesture: Character viewpoint and observer viewpoint in students' gestured graphs of functions. *Gesture* 10 (2-3), 321-343.
- Gerofsky, S., Savage, M. & Maclean, K. (2009). 'Being the graph': Using haptic and kinesthetic interfaces to engage students learning about functions. In Bardini, C. & Fortin, P. (Eds.), *Proceedings of ICTMT 9: The Ninth International Conference on Technology in Mathematics Teaching*, Université de Metz, Metz, France.
- Moses, R. P. & Cobb, C. E. Jr. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Boston: Beacon Press.
- Nemirovsky, R. & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics* 70(2), 159-174.
- Patton, M. (2002). *Qualitative research and evaluation methods*. Thousand Oaks, CA: SAGE.
- Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics* 70(2), 11-126.
- Rech, J. F. & Harrington, J. (2000). Algebra as gatekeeper: A descriptive study at an urban university. *Journal of African American men* 4(4), 63-72.
- Roth, W.-M. (2010). Incarnation: Radicalizing the embodiment of mathematics. *For the Learning of Mathematics* 30(2), 8-17.
- Skovsmose, O. (1994). Towards a critical mathematics education. *Educational studies in mathematics*, 27, 35-57.
- Smith, J. & Maclean, K. (2006). Communicating emotion through a haptic link: Design space and methodology. *International Journal of Human-Computer Studies*.
- Stinson, D. W. (2004). Mathematics as "gate-keeper"(?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The mathematics educator* 14(1), 8-18.
- Swanson, D. (Ed.) (2009). Performing the sign: Mathematics, democracy and the arts. Special issue of *Educational Insights* 13(1). <<http://www.educationalinsights.ca/>>

**Hellenic Mathematical Society**  
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**DOES SCHOOL MATHEMATICS CONTRIBUTE  
TO CRITICAL THINKING AND DECISION-  
MAKING?  
AN EXAMPLE WITH THE TEACHING OF REAL  
NUMBERS IN BRAZIL AND IN CANADA**

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**INTRODUCTION**

One of the questions to discuss in this CIEAEM conference concerns whether school mathematics contributes to critical thinking and decision-making in the society. It is stated that many topics are taught across the world without taking into account socio-cultural contexts (Bishop, 2009). However, students are expected to link mathematical knowledge to their everyday lives, in a world which becomes more complex and more demanding. Do mathematics curricula broaden their goals to include concepts and processes that will maximize all students' opportunities for success in society?

We have decided to address these questions analyzing the introduction of a topic which appears in the secondary curricula of many countries and which played a key role in the development of mathematics: the concept of real number. The learning of this concept becomes one of the first encounters of the students with an abstract mathematical concept with no immediate everyday applications. However, the construction of many other concepts (mainly those of the Calculus) depends strongly on many of the properties of real numbers. We are mainly interested in analyzing whether the teaching of this key notion tries to develop critical thinking in the

students, providing the students with adequate bridges between intuition and rigor, and with useful ways of reasoning that may become important in subsequent stages of the learning of Calculus.

In analyzing the introduction of real numbers in two different countries (Brazil and Canada), we also aim at addressing Bishop's (2009) statement, in the sense that many notions are taught in the world in very similar ways. In this sense, our research tries to overcome national boundaries, to see whether socio-cultural contexts are taken into account, and to start some reflection about the teaching of this notion and the possible consequences of this teaching.

Our previous research (González-Martín, Giraldo & Souto, 2011, submitted) already showed a lack of results on the teaching and learning of real numbers. Most of the available research focuses the learning of this notion, generally following cognitive approaches to identify the main difficulties to learn and to understand this notion (see, for instance, Bergé, 2004; Dias, 2002; Fischbein, Jehiam & Cohen, 1995; Margolinas, 1988; Robinet, 1986; Schwarzenberger & Tall, 1978; Sirotic & Zazkis, 2007). For this reason, we are interested in analyzing how the teaching of this notion is developed (analyzing whether this teaching tries to develop some critical thinking and to take into account socio-cultural differences), and whether this teaching takes into account the difficulties already identified by research.

### **THEORETICAL FRAMEWORK**

As we are interested in analyzing the teaching of real numbers, we believed that it was necessary to consider a theoretical framework which takes into account institutional issues. In our case, interested in analyzing the choices made by secondary curricula to introduce real numbers, and the possible consequences for students' understanding of this notion, we decided to follow Chevallard's (1999) anthropological theory of didactics (ATD).

This theory offers specific tools to analyze the introduction of a mathematical notion made by an institution in terms of didactic organizations, and also to analyze the tasks demanding the use of this notion and the types of knowledge which are into play in the completion of these tasks. This analysis allows not only to identify the knowledge, and its structure, which is actually being mobilized in the teaching of one notion, but also to make predictions about the understanding that students might build.

### **METHODOLOGY**

Given the lack of research results concerning the teaching of real numbers, we decided to only consider, at this stage of our research, one of the main actors of the process of teaching: the textbook (Love & Pimm, 1996).

For our analyses, we chose a sample of 14 secondary textbooks approved by the Ministry of Education of Brazil (see González-Martín, Giraldo & Souto, 2011, submitted). In Canada, we are currently developing our analyses and we have analyzed 2 secondary textbooks approved by the Ministry of Education of Québec. We intend to analyze more textbooks in Québec, but for this presentation we will compare the tendencies found in the sample from Brazil with the sample from Canada. These two countries have been chosen mainly by opportunistic reasons; however, being in the same continent, they have very different socio-cultural characteristics, although the list of mathematical concepts to be taught in compulsory education is similar in both countries.

We constructed an analysis grid taking into account three main dimensions:

- The organization of the contents introduced to teach real numbers.
- The tasks offered to the students, and the methods to solve them.
- The semiotic activities develop by the textbooks to improve understanding.

In this presentation, we will only study the two first ones, which are directly linked to the use of Chevallard's (1999) approach. Regarding these two dimensions, our analysis grid included mainly the following items:

- Types of definitions.
- Statement of properties.
- Use of these properties.
- Use of examples.
- Tasks privileged to teach real numbers.
- Techniques used to solve these tasks.
- Knowledge required to justify these techniques (technology, in Chevallard's terms), and whether this knowledge is explicit or explicit.

The analysis of these elements (in particular the last two ones) allowed

us to make a critique about the type of thinking which is developed in these approaches. The use of our grid also allowed us to establish a portrait of the textbooks of our sample, the way in which they introduce real numbers (in particular, the development of critical thinking and whether socio-cultural elements are taken into account) and to identify some tendencies.

### **DATA ANALYSIS**

Our data analysis allows us to see strong tendencies in the two samples of textbooks, both in Brazil and in Canada. These are:

- The existence of real numbers (and of irrational numbers) is usually taken for granted, but no motivation for the need of these numbers is generally given.
- The decimal expansions of many numbers (like  $\pi$ ) are taken for granted, so there is not a real discussion about whether these numbers are irrational or not.
- Many properties are stated and followed by a number of examples. Usually, there is not any form of reasoning or argumentation to discuss why these properties are true. This organization might lead students to believe that to prove a statement, a number of examples suffices.
- The majority of tasks are articulated around the application of properties (whose validity has not been previously discussed). In this sense, it seems that teaching is focused on the learning of properties (without justification) and on the identification of cases to apply these properties.

Our presentation will discuss these elements more in depth, comparing the sets of textbooks from Brazil and from Canada. This discussion will be followed by the statement of some possible consequences of this presentation to students' understanding. We will finish with a general discussion about the teaching of key mathematical notions in many countries and about whether this teaching considers the contexts in which students are embedded and whether this presentation leads to critical thinking and to the development of decision-making skills. Does the teaching of real numbers maximize students' opportunities for success in society?

## REFERENCES

- Bergé, A. (2004). [*A study about the evolution of mathematical thinking : the example of the conceptualisation of the set of real numbers and the notion of completeness at university level*]. PhD Thesis, Universidad de Buenos Aires, Argentina.
- Bishop, A. (2009). Researching family mathematical practices: background research and future challenges. *3<sup>rd</sup> Conference of CARME*, 43-53, Rhodes.
- Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221-266.
- Dias, M.S. (2002). *Reta Real: Conceito imagen e conceito definição*. Master Thesis, São Paulo: PUC/SP.
- Fischbein, E., Jehiam, R. & Cohen, D. (1995). The concept of irrational number in high-school students and prospective teachers. *Educational Studies in Mathematics*, 29 (1), 29-44.
- González-Martín, A.S., Giraldo, V., & Souto, A. (submitted). The introduction of real numbers in secondary education. An example with Brazilian textbooks. *Research in Mathematics Education*.
- González-Martín, A.S., Giraldo, V. & Souto, A. (2011). Representations and tasks involving real numbers in school textbooks. *Proceedings of the 35<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 449-456). Ankara, Turkey: PME.
- Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A.J. Bishop (Ed.), *International handbook of mathematics education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer.
- Margolinas, C. (1988). Une étude sur les difficultés d'enseignement des nombres réels. *Petit x*, 16, 51-66.
- Robinet, J. (1986). Les réels: quels modèles en ont les élèves? *Educational Studies in Mathematics*, 17, 359-386.
- Schwarzenberger, R.L.E. & Tall, D. (1978). Conflicts in the learning of real numbers and limits. *Mathematical Teaching*, 82, 44-49.
- Sirotic, N. & Zazkis, R. (2007). Irrational numbers: the gap between formal and intuitive knowledge. *Educational Studies in Mathematics*, 65 (1), 49-76.



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## **Understanding the concept of measure on primary school level**

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Acknowledgement. The paper was supported by the research project No. P407/11/1740 Critical areas of primary school mathematics – analysis of teachers' didactic practices.

**Abstract.** The paper focuses on problems understanding geometrical concepts of area and perimeter, which are very common in Czech schools, and tries to pinpoint the sources of these problems. The findings from the below described experiment indicate the effects of scheme-oriented teaching on understanding the aforementioned concepts.

### **Present state**

2D and 3D geometry in Czech primary school curricula are regarded as one of the four areas of mathematics education. The geometrical subject matter is based on two pillars. The first one is learning 2D and 3D geometrical figures. Discovery of relationships between attributes of the different figures, which belongs to the second and third levels of understanding geometrical objects according to Van Hiele (1986), is not stressed. And it does not differ in the school practice where mainly the knowledge of terminology is stressed.

The second pillar of primary school geometry is the area of measure, namely the length of line segment, perimeter and area of a figure. The problem is that curricular documents and traditionally structured textbooks pay no attention to propaedeutic of these concepts. Usually these concepts are introduced in connection to formulas, both in textbooks and school practice.

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For example in the nowadays most widely-used Czech textbook of mathematics with many frames headed “Remember!” and with various visually emphasised formulas we can find instructions accompanying the problems aimed at computation of area or perimeter such as: “First write down the formula, and then substitute into it.”

This is undoubtedly one of the sources of common primary school pupils’ misconceptions regarding measure. This unfortunately affects their achievement in higher grades. To illustrate this point, let us present here some sample pupil solutions to a problem from TIMSS survey for 8<sup>th</sup> grade pupils, i.e. aged 14-15. The problem is the following:

*There is a shaded triangle inside a square in the picture. What is the area of the shaded triangle?*

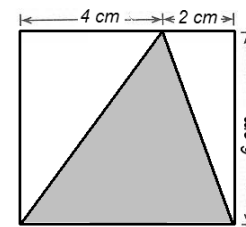


Fig. 1

This problem can be solved quite simply in several ways without having to use any formula. However, Czech 8<sup>th</sup> graders’ achievement in this problem was strongly below average both in comparison to the results from other countries and within the Czech Republic. The following four solutions illustrate the situation.

$$\begin{aligned} S &= a \cdot b \cdot c \\ S &= 6,3 \cdot 22 \cdot 6 \\ S &= 272,16 \text{ cm}^2 \end{aligned}$$

Solution 1

$$\begin{aligned} S &= a + b + c \\ S &= 6 + 6 + 6 \\ S &= \underline{18 \text{ cm}^2} \end{aligned}$$

Solution 2

$$\begin{aligned} S &= a^2 \cdot b^2 \\ S &= 6^2 \cdot 2^2 \\ S &= 36 \cdot 4 \\ S &= \underline{144} \end{aligned}$$

Solution 3

$$\underline{S = a \cdot na = 6 \cdot 6 = 36 \text{ cm}^2}$$

Solution 4

Another obstacle to good understanding of the concept of area and perimeter is language. This phenomenon is described for example by Hansen (2006). He speaks of the word “face” which is an everyday English word and also a geometrical term. The meaning of this word in everyday language is completely different than in geometry. Similarly the Czech word “obsah” refers to “content” – content of a pocket, content of a bottle, content of a book in everyday language and to the geometrical term “area”. Moreover, we do not use the word “obsah” in everyday language to refer to area and

use the word “plocha” (surface) or “výměra” (expanse). Also the word “obvod” (perimeter) is used in everyday language but has a different meaning than in geometry – circuit or waist length.

Another phenomenon that is the source of misconceptions is the relationship between area and perimeter. The concepts are introduced at the same time and initially for square. Pupils often solve problems in which the perimeter is given and the task is to calculate the area and vice versa. This means that pupils are consistently taught that a change in the area results in a change in perimeter and vice versa. Pupils then transfer this relation analogically to rectangles etc., and later also to the relation between surface and volume in 3D. Problems that thwart the idea of such relationship between area and perimeter in irregular polygons are very rare.

The notion of measure is also present when working with units of measurement and conversion. As pupils often have no idea about or understanding of prefixes mili-, kilo-, centi-, ..., they usually try to learn by heart when to multiply and when to divide and by which power of ten. Obviously they often make mistakes and the topic is perceived as difficult and tedious both by pupils and teachers.

### **Experiment**

The experiment was carried out by the second author of this paper. She is a 4<sup>th</sup> grade teacher at primary school. She tries to conceive her mathematics teaching as action research in the sense described in (Janík, 2004). She reflects on her teaching carefully, has her lessons videotaped, collects other materials that are subject first to individual and later also collective reflection on classroom interaction, which is carried out with her colleagues. Some of the recordings and written materials are also subject to reflection lead by the supervisor. She keeps incorporating experiments into her teaching that focus on some previously selected phenomenon. If necessary these experiments are appended by clinical experiments. She tries to be constructivist, which means for example that every pupil is given the chance to express their opinion and choose their solving procedure. Ideas and suggestions are discussed by the whole class and assessed by the pupils. Thus the pupils learn to listen and interpret their classmates' ideas. Pupils learn to cooperate in groups where they help each other and learn to realize their contribution to collective work. The atmosphere in the classroom can be described as favourable and very democratic.

The second author studies efficacy of her approach to teaching mathematics, she adopts the tools for getting to know her pupils and for improvement of her own teaching and gains very valuable material for her planned Phd thesis.

The experiment was held in the 4<sup>th</sup> grade on the 16<sup>th</sup> December 2011. There were 14 pupils, 4 girls and 10 boys. The second author has been teaching this class since the summer term of the 2<sup>nd</sup> grade. The pupils have been taught maths from the textbooks by Hejný et al. (2007-2011) since the 1<sup>st</sup> grade. These textbooks differ radically from all other textbooks on the Czech market, not only by their content elaborated for scheme-oriented education, but also by the demands on implementation.

The goal of the experiment was to map the level of pupils' understanding of the concepts of area and perimeter and the connection between area and perimeter.

The tool of the experiment was the following problem.

*Fill in the missing lengths of sides. You know that the perimeter of the rectangle is 64. Find the perimeter and area of each of the rectangles.*

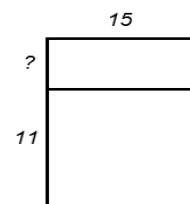


Fig. 2

This assignment was accompanied by the picture (Fig. 2). The problem was handed out to the pupils on worksheets and analysed verbally. The pupils' written production was analysed in several perspectives: the solving strategies used, organization of data on the worksheet, presentation of results, tools of computation, understanding the concepts of area and perimeter, geometrical terminology used and idiosyncrasies. After the first analysis, some of the pupils were invited to an interview in which they were asked to explain things that were unclear. Unfortunately, this interview took place two weeks later and some of the children were not able to recall many things.

### Discussion

This was the pupils' first encounter with a problem of this type. Never before had they come across a mere draft of the situation that was not accurate and could not be measured using a ruler. Up to this stage the pupils only had been looking for the area of a grid-figure or of a figure in which they could measure whatever they needed. They had not yet worked with length given by a number without any unit. On the other hand they had often met problems of the type: How many rectangles are there in the figure?

The concepts of area and perimeter are in the aforementioned textbooks carefully prepared in several learning environments (Wittmann, 1984), e.g. Paper folding, Tessellation, Wooden sticks and Grid paper. When trying to determine the length of the unknown side two pupils tried to measure it using a ruler. They stated that the length was 0.5 mm (Fig. 3) and replaced all the other given measures by lengths measured using the ruler in millimetres. When trying to tell the area of the rectangle, they used a square grid (Fig. 4). This means that their notion of rectangle and its perimeter is good but is still bound to concrete objects. According to Hejný (2011) it is on the level of isolated models, which is the first stage of the cognitive process. All the other pupils were able to work with the draft and numbers that do not correspond to real measures. According to Hejný's Theory of generic models it can be assumed that these pupils had already undergone abstractive lift and their understanding is on the level of generic models.

Another interesting phenomenon is the appearance of yet untaught mathematical knowledge. Namely decimal numbers 0.5 and 5.5 (Fig. 3, 5), which had not yet been introduced in mathematics lessons. And the algorithm of written multiplication. Computation of the area of the two rectangles required multiplication of  $11 \times 15$  and  $17 \times 15$ , i.e. of two two-digit numbers, which was also new to the pupils. We came across three good methods of calculating the product of two two-digit numbers: 1) repeated addition, 2) Indian multiplication using a table, 3) common algorithm of written multiplication.

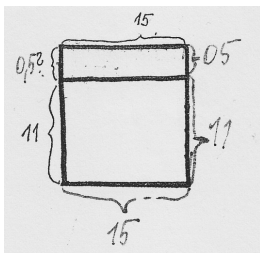


Fig. 3

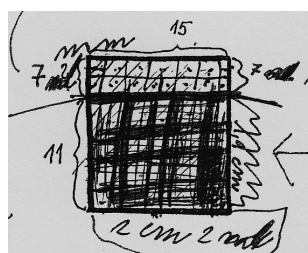


Fig. 4

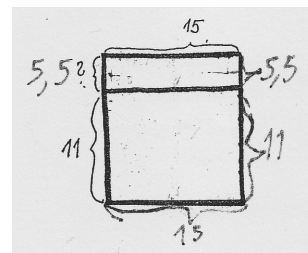


Fig. 5

It is also noteworthy that none of the pupils used any formula.

We are convinced that this variety of pupils' approaches to a brand new problem and their effort to solve new things signal that the teacher's constructivist approach to teaching bears fruit. Hejný (2006) carried out an ex-

tensive research in this area and, briefly said, assessed the teachers' teaching styles according to the variety of pupils' solutions.

When solving the given problem, the pupils must carry out six operations - they must compute the length of the side with the question mark, perimeters of two and areas of three rectangles. This means they are asked to handle a large quantity of data. The majority of the pupils tried to organize the chaotic jumble of calculations using colours. The calculation and the corresponding geometrical figure were marked by the same colour. This phenomenon does not belong to the area of geometry and measurements but is still worth our attention as it also confirms the successful constructivist work in the class.

Other interesting phenomena discovered in the analyses will be presented in our oral presentation.

### References

- HANSEN, A. et al. *Children's Errors in Mathematics. Understanding Common Misconceptions in Primary Schools*. Exeter: Learning Matters Ltd., 2006
- HEJNÝ, M. Diversity of students' solutions of a word problem and the teacher's educational style. In SIMPSON, A. *Retirement as process and concept*. Prague: UK v Praze, 2006
- HEJNÝ, M. et al. *Matematika pro 1.–5. roč. ZŠ*, Plzeň: Fraus, 2007 – 2011
- HEJNÝ, M. Scheme oriented educational strategy in mathematics. In MAJ, B., PYTLAK, M., SWOBODA, E. *Proceedings of CME 08*, Rzeszow: Nowa Era, 2008
- HEJNÝ, M. The process of discovery in teaching focusing on building schemes. In NOVOTNÁ, J., MORAOVÁ, H., *SEMT'11 Proceedings*. Praha: UK v Praze, PedF, 2011, p. 150-157
- JANÍK, T. Akční výzkum jako cesta ke zkvalitňování pedagogické praxe. In Maňák, J., Švec, V. *Cesty pedagogického výzkumu*. Brno: Paido, 2004, p. 51-8
- VAN HIELE, P.M. *Structure and Insight. A Theory of Mathematics Education*. Academic Press, Inc., London, 1986
- WITTMANN, E. Teaching Units as the Integrating Core of Mathematics Education. *Educational Studies in mathematics*. 1984, 15, p. 25-36

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## **Non-verbal communication in primary school mathematics: a case study focusing on eye movement**

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**Abstract** In this study, we investigate the role of non-verbal communication, in the sense of eye movements, when the students think about mathematics. Twenty Grade 3 students were verbally presented with three simple addition or subtraction problems, expressed in different levels of information load. The findings of this study suggest that most of the eye movements were towards the right (up or down) direction, suggesting left hemisphere activity. Moreover, the information load appeared to affect the students' reasoning time in ways that cannot be sufficiently explained only by the mathematical content of the task. Pedagogical implications are discussed.

### **Neuroscience, mathematics and eye movement**

The importance of left hemisphere in logico-mathematical reasoning and problem-solving has been acknowledged within cognitive psychology (Bear, Connors & Paradiso, 2007). Furthermore, left hemisphere activity has been linked with time perception, language-related functions (including speech, reading, writing), logical reasoning, processing of acoustic stimuli, and abstract information and others (Gazzaniga, Ivry & Mangun, 2009). Moreover, in Greece, the current Unified Interdisciplinary Curriculum Framework –according to which the curriculum of each school course is designed– describes the purpose of school mathematics with cognitive functions that have been linked with left hemisphere activity, thus, indirectly adopting the aforementioned findings from cognitive psychology.

Though non-verbal communication is considered to be primarily controlled by the right hemisphere (Joseph, 2011), eye movements have been linked with both left and right hemisphere brain activity (Garrett, 2008; Smith & Kosslyn, 2007). Argyle (1998) notes that in a question and answer situation, the respondent –during the process of thinking about the answer– usually avoids eye-contact with the questioner, whereas the respondents’ looking up and right may indicate intense or deep thinking. Moreover, the respondent usually looks straight to the eyes of the questioner, when reaching to a conclusion and uttering the response (ibid).

Within this framework, we argue that the study of eye movement during the students’ dealing with mathematical tasks may help in gaining deeper understanding of the students’ thinking, as it may reveal aspects of the brain activity, including the cerebral lateralisation of the student (Gluck, Mercado & Myers, 2007). Such knowledge is especially useful for an effective teaching process (Stamatis, 2011), as it allows the teachers to utilise non-verbal communication in the synchronous assessment of their teaching, thus enabling them to offer *an appropriately differentiated pedagogy* in terms of, for example, task presentation, allowed response time etc. Hence, we posit that the teachers’ awareness of the links between eye movement (and non-verbal communication in general) and mathematical thinking *by allowing such a student-differentiated pedagogy substantially contributes towards a pedagogy that enables the ‘democratic access’ to mathematical ideas by the learners.*

Consequently, in this study, we investigate the role of eye movement when students think about mathematical tasks.

#### Methods and procedures

For the purposes of this study, we drew upon the methodology suggested by Babad (2005). For each participant a three-phase process was followed: *listening, reasoning* and *response*. In the first phase, *listening*, the participants listened to one of the researcher’s uttering a task. Each task was uttered in a clear steady voice, medium speed and good enunciation, with the purpose to investigate the students’ reasoning and responses when a task is spoken only once. We focussed on the students’ responses when the task is uttered only once, because, on the one hand, we were interested in the students’ reasoning unaffected by cognitive process that may be related to the repetition of the utterance of a task and, on the other hand, this information is crucial for this study since we focus concentrate on identifying ways that may help the teacher to appropriately differentiate their pedagogy. The second phase, *reasoning*, refers to the participants’



thinking about the answer of the task, while the third phase, *response*, consists of the participants' uttering the response.

During these phases, the other two members of the research team were keeping notes regarding respectively two different aspects of the whole process: *time information* and *eye movement information*. One member collected time information, in terms of *reasoning time* (referring to the time that each participant spent to think about the answer), as well as *response time* (referring to the time that the participants spent to utter the answer). The third researcher focussed on the direction of the participants' eye movements differentiating amongst *right*, *left*, *up*, *down* and their *combinations*. Note that eye movement information was recorded in all three phases of each task. Furthermore, both time and eye movement information was noted in a log especially designed for the purposes of this study.

The presented mathematics tasks varied in their complexity, referring to the amount of information provided in the expression of each task. For each of the mathematical contents that we considered, three types of tasks were constructed: *symbolic task* (employing mathematical symbolic language), *natural language task* (employing natural language expressing the same information as the symbolic task) and *information overloaded task* (an expression employing natural language *and* more information than the information required for dealing with the task). Following this rationale and the fact that our interest lies in Grade 3 primary school students (8-9 years old), we focus on simple addition and subtraction problems. In this paper, we consider the following three tasks that the students were asked to verbally answer: i) "*How much is twelve plus nine?*" ('12+9' task; symbolic task), ii) "*George had twenty five stickers. He gave to Helen twelve stickers. How many stickers does he have?*" ('stickers' task; natural language task), and iii) "*Ann is a Grade 3 student. She likes painting, while listening to soft music. She uses twenty four markers of a variety of colours. Her father gave her a box containing twelve markers. How many markers does Anna have in order to paint more colourful pictures, while listening to soft music?*" ('markers' task; information overloaded task).

Moreover, we gathered information regarding the participants' handedness, since it is linked with cerebral lateralisation (Haken, 2008). Furthermore, we looked into their mathematics attainment (as identified by the teacher) and perceived attainment (as identified by the participants), as well as their affective disposition towards mathematics as factors that may

affect the participant’s performance. Finally, care was taken so that there is no gender bias in our sample.

Preliminary results

In this paper, we consider twenty Grade 3 (8-9 years old) students (N=20). The analysis of the data was quantitative. Though both descriptive and inferential statistical analyses were conducted, in this paper we discuss only descriptive statistical analyses (outlined in Table 1). Our study will be completed with the investigation of a wider sample with a greater variety of tasks (including inferential statistics) that we hope to present in the CIEAEM’s 64<sup>th</sup> conference in July.

		Eye Movements											
		S-A <sup>1</sup>	U	D	L	R	U-D	L-R	U-R	U-L	D-R	D-L	
Tasks	‘12+9’	LI <sup>2</sup>	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		RE	25%	10%	15%	0%	15	20%	30%	45%	30%	30%	5%
	‘stickers’	LI	100%	5%	0%	0%	0%	0%	0%	10%	5%	0%	0%
		RE	35%	10%	35%	5%	45	10%	40%	60%	45%	65%	25%
	‘markers’	LI	100%	0%	25%	0%	10%	5%	5%	10%	5%	20%	0%
		RE	35%	20%	45%	15%	45%	35%	35%	50%	40%	60%	35%
		<sup>1</sup> : Straight-Ahead, Up, Down, Left, Right, Up-Down, Left-Right, Up-Right, Down-Right, Down-Left											
		<sup>2</sup> : LIstening Phase, REasoning Phase											

Table 1: The participant eye movements during ‘listening’ and ‘reasoning’.

Regarding the listening phase of the ‘12+9’ task, all the participants were looking straight at the questioner’s eyes. During the reasoning phase, most of the participants (62%) looked right. The mean reasoning time for this task was 16 seconds, while the response time was only 1 second. Furthermore, in the response phase, all the participants were looking straight at the questioner’s eyes, while uttering the response. It should be stressed that the same response time and pattern of behaviour was noticed in all three tasks.

Considering the ‘stickers’ task, the vast majority of the students (90%) were looking straight ahead during the listening phase, while during the

reasoning phase there was low eye movement spread with the majority (57%) of the students looking right (up or down). Moreover, the mean reasoning time was 46 seconds.

In the listening phase of the ‘markers’ task, the direction of the eye movements was predominantly straight ahead (60%), while the participants’ gaze moved towards other directions (mostly down-right). During the reasoning phase there was a higher distribution in the direction of the eye movements, most of which were right (23%) or down-right (42%).

Notwithstanding the limitations of the sample of this study and the limited variety of the presented tasks, we argue that are clear discernible trends in these preliminary findings. First, during the listening phase, the participants, in their effort to understand better the task, they concentrate their gaze on the questioner. That is, they gather their attention to the questioner, as he is the one providing the data of the task. In cases with information overload (such as in the ‘markers’ task), as the data increase, the students turn their gaze to various directions, which may be interpreted as an effort to process the information (data and questions), in line with the listening process.

Regarding the reasoning phase, it appears that the higher the information load that the task contained the greater variety of eye movements was noted. Notably, most of these movements were right (up or down). Bearing in mind that the vast majority of the participants are right-handed (95%), these findings are in line with existing studies linking the left hemisphere activity with right body activity (Glannon, 2011). Moreover, the higher distribution of the eye movements identified in the information overloaded task (the ‘markers’ task) corroborates with the findings of studies that link the complexity of the information provided in a task with the activity of both hemispheres (Haken, 2008).

Considering information load and reasoning time, it seems that the complexity of the task affects the students’ reasoning time in ways that cannot be sufficiently justified only by the mathematical content of the tasks. Simple computations –such as  $12+9$ ,  $25-12$  and  $24+12$ – for Grade 3 high-attaining students (90% of the participants were characterised this way by their teacher), seem to require the students allocating notably different amounts of time (respectively 16, 46 and 57 seconds), in order to perform them.

Furthermore, there are clear *pedagogical implications* of these results, as the students provided an answer in less than a minute, which asks for the teachers’ patience to allow for the students to deal with the task (especially

in more information overloaded tasks). Importantly, the eye movements towards the right direction suggest the students' reasoning about the task, which may help the teacher in deciding whether or not the student should be allowed a few more seconds to answer.

Consequently, we posit that the preliminary findings presented in this paper highlight the importance of non-verbal communication and especially eye movement in teaching mathematics, as it may provide crucial information regarding the students' thinking processes while listening to or dealing with a task.

### References

- Argyle, M. (1988). *Bodily communication*. NY: Methuen & Co.
- Babad, E. (2005). Nonverbal behavior in education. In J. A. Harrigan, R. Rosenthal, & K. R. Scherer (Eds.), *The new handbook of methods in nonverbal behavior research*. NY: Oxford University Press.
- Bear, M. F., Connors, B. W., & Paradiso, M. A. (2007). *Neuroscience: Exploring the Brain*. PA: Lippincott Williams & Wilkins.
- Garrett, B. L. (2008). *Brain & Behavior: An introduction to biological psychology*. NY: Sage.
- Gazzaniga, M. S., Ivry, R. B., & Mangun, G. R. (2009). *Cognitive neuroscience: The biology of the mind*. NY: W.W. Norton & Company.
- Glannon, W. (2011). *Brain, body, and mind: Neuroethics with a human face*. NY: Oxford University Press.
- Gluck, M. A., Mercado, E., & Myers, C. E. (2007). *Learning and Memory: From brain to behavior*. NY: Worth Publishers.
- Haken, H. (2008). *Brain Dynamics. An introduction to models and simulations*. Berlin: Springer.
- Joseph, R. (2011). *Right hemisphere, left hemisphere, consciousness & the unconscious, brain and mind*. NY: University Press.
- Smith, E. E., & Kosslyn, S. M. (2007). *Cognitive Psychology: Mind and Brain*. NJ: Prentice Hall.
- Stamatis, P. J. (2011). Non verbal communication in classroom interactions: A pedagogical perspective of touch. *Electronic journal of research in educational psychology*, 9(3), 1427-1442. Retrieved January 29, 2012 from <http://investigacion-psicopedagogica.org/revista/new/english/ContadorArticulo.php?617>.

## From the “campanine” to Leonardo: a socio-political story

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### Abstract

*Our presentation starts from the construction of “campanine”, that makes recourse to a rule derived from the Euler-Bernoulli equation. Two reflections ensue, one about the centrality of mathematical model in the globalized society, the second one about the risk of “depriving the democratic process of substance and meaning”, a loss derived by an uncritical dependence on these models. In the following part, the Leonardo’s “folio 84” is at the center. It contains the material version of the Euler-Bernoulli formal model. The comparison between these material and the formal models, has the didactical aim of constructing a critical attitude about modeling. The conclusion is that, in the present globalized society, a particular evolution of the meaning of the term “democracy” realizes itself in a cooperative working into the articulate structure that forms the mathematical model. In the last part of our presentation, we reflect about the Leonardo’s restlessness as regards geometry; a restlessness characterized by the absolute dynamism of the line of his drawings, that was defined “interior geometry”. A restless attitude as to mathematical models could be fundamental to form a critical living.*

### Résumé

*Notre présentation débute par la construction des “campanine”, qui fait appel à une règle dérivée par l’équation de Euler-Bernoulli. De cela deux réflexions jaillissent : une sur la centralité du model mathématique dans la société globalisée, l’autre sur le risque de «priver le procès démocratique*

*de substance et de signification», une perte dérivée d'une dépendance non critique de ces modèles. Ensuite, le « folio 84 » de Leonardo devient le centre de la présentation. Il renferme la version matérielle du model formel de Euler-Bernoulli. La comparaison entre ces deux modèles, matériel et formel, a l'objectif didactique de construire une attitude critique à l'égard de la modélisation. Notre conclusion est que, dans la présente société globalisée, la démocratie se réalise dans une œuvre coopérative dans la structure articulée qui forme le model mathématique. Dans la dernière part de notre présentation nous réfléchissons sur la « irrequietudine » [inquiétude créative] de Leonardo à l'égard de la géométrie ; une « irrequietudine » qui est caractérisée par l'absolue dynamisme de la ligne dans ses dessins et qui a été définie « géométrie intérieure » . Une attitude d'« irrequietudine » à l'égard des modèles mathématiques pourrait être fondamentale pour la formation d'une citoyenneté critique.*

### Introduction

At Athens, fatherland of the concept, the term democracy meant the people that jointly acts; a direct democracy that called citizens to public decisions. Afterwards, the philosophical-political core of democracy has evolved regarding both the right definition and the concrete search of the material conditions that allow to make rational the decisions and the political productivity of single systems. The text of the Italian Constitution sums up the concept of democracy in a distinguished way binding it indissolubly to the concepts of equity, participation and recall to solidarity, in order to put each citizen in conditions of a critical participation to democratic process. This specific set-up marks our following reflections. In particular we highlight a characteristic contrast of the modern globalized society, that directly involves questions about democracy: the central role of mathematical modeling in the planning and the decision making processes, enjoins a choice between dominating the concrete democratic processes and being dominated by formal, not immediately intelligible, structures.

Our reflections about the relation between mathematics teaching and democracy started by the considerations presented to CIEAEM 54 [Longoni et al., 2002] and have been put into action in some didactic, practical activities. The first one, “Leonardo and the interior geometry” has allowed us to put in evidence the risk that reducing the mathematics to modeling could bring to “deprive the democratic process of substance and meaning” [Lon-

goni et al., 2010a]. The second one is an activity of introduction to the high school mathematics, introduction that contrasts the one that is widely used in Italy, at the present time; instead of elements of exclusion that characterize this latter, our proposal tries to favor a wider process of inclusion [Pulinetti et al., 2009]. The third activity stresses the importance of quality in teaching mathematics: the quality of presentation of some basic elements of mathematics may become a fundamental element of empowerment [Longoni et al., 2010b]. Time limits impel us to the first of these activities. We will give prominence to the context where the activity has been practiced and to the cultural motivations that have suggested to advance it; we will emphasize the elements that, in our opinion, mirror the conditions of democracy.

### **The local: “*Le campanine*”**

The “*Campanine*” are a simple musical instrument, used by bell-ringers in Bergamo and Brescia’s Italian provinces in order to practice. They look like a xylophone and are constituted by a sound box and by glass bars of different length. The bell-ringers, that in the occasion of particular festivities and church services played by a special keyboard sited upon the bell tower, trained in sacristy just by means of the “*Campanine*”. Today the “*Campanine*” are a curiosity, that have had a revival thanks to several searches in local history. [Campanine, 2011]

The construction of the “*Campanine*” has constituted the starting point of a reflection that later on has widened to the questions of modeling and, lastly, to the concern to ethics and democracy.

The class activity of construction of the “*Campanine*” started as a workshop in order to develop the theme “The sound”. The seventeen years old students of a third class of the “Liceo delle Scienze sociali” were involved in the school year 2001- 02 at the beginning. The last class activity has been done during the current school year, with sixteen and eighteen –year-old students of a scientific lyceum.

A musical instruments maker, Michele Sangineto, provided us with the suitable sound box. The specific problem of class activity consisted in cutting the glass bars. Today some websites describe materials, techniques and lengths exactly [Campanine, 2011]. But we believed more didactically meaningful to proceed in the construction by general rules. These rules are just contained in the Euler-Bernoulli beam equation [Suits, 2001].

**The model: Euler-Bernoulli equation**

Given the parameters of the bar:

L = length (x), w = width (z), h = thickness (y),

the Euler-Bernoulli equation is the following:

$$\frac{\partial^2 y}{\partial t^2} + \frac{E I}{\rho S} \frac{\partial^4 y}{\partial x^4} = 0$$

The following frequencies are the solutions of this equation:

$$\omega_n \approx \frac{\pi^2 \sqrt{E I}}{4L^2 \sqrt{\rho S}} \cdot \frac{1}{(2n + 3)^2}$$

L = length of the bar, E = Young’s elastic modulus, I = moment of Inertia , S = w·h = area of cross section, ρ = density of the bar, n = harmonic.

Equation and solution are not easy comprehensible to high school students; they seem nearly as a set of little meaningful signs.

The upper complex structures give rise to this following relation:

$$\omega'_n / \omega_n = L^2 / L' :$$

the ratio between the frequencies emitted by two bars of the same type, are in inverse proportion to the ratio between the squares of the lengths of the two bars.

Thanks to this rule, by knowing the relations between frequency and height of a sound, it’s possible to proceed in the construction: given a bar of a certain length, it is easy to determinate the length of the bar that emits a note of a double frequency (an higher octave), such as the length of the bar that emits a sound higher of one halfnote (in well temperament scale); and so on.



### **Mathematical modeling in the globalized society**

The construction of the “*Campanine*” allowed to introduce a reflection about mathematical modeling: the Euler-Bernoulli equation, inasmuch mathematical model, constitutes, from the didactic point of view, a metaphor of the valence of mathematical models in the present globalized society. Two the principal issues: the importance of mathematical models in mastering processes and the not easy comprehensibility of most mathematical models.

*The importance of mathematical models in mastering the real processes.* The Euler-Bernoulli model “represents something not yet realized” [Skovsmose, 2004] and identifies a technological path to attain to the realization not only of the specific instrument, the “*Campanine*”, but also of a wide range of analogous instruments. This predictive function of Euler Bernoulli model introduces us to the huge value of mathematical modeling in the globalized society. Skovsmose summarizes and exemplifies the centrality of the mathematical modeling in an exhaustive way: mathematical modeling establishes the basis for the planning and decision making processes in most of human activities.

*The not easy comprehensibility of the most of mathematical models.* The complexity of Euler-Bernoulli equation mirrors the not easy comprehensibility of the most of the models handled by the globalized society: each mathematical model is comprehensible and managed by few people. This fact, along with centrality of mathematical modeling, rise heavy questions of democracy: few people are able to handle the development of the socio-political system. The question about the relevance of an uncritical use of models in the democratic process, is very present: “... the current crisis is ascribable to an uncritical use of models in economy ... If a theory results true not because it rightly describes a reality independent from it, but rather because the involved actors behave as it were true, and they create so a reality that is completely dependent by the modeled representation, it is very probable that, when the actors stop to believe to reliability of the theory, the reality created by this theory goes to pieces. ... This system should have been able to deprive the democratic process of substance and meaning” [Gallino, 2011].

How these striking questions could affect the education of mathematics? Introducing the education of modeling in the mathematical classes, is now becoming relevant also in Italian high school [Boero et al., 2011]. The debate about how this introduction is proceeding, is well summarized in

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García [García et al. 2006]. We suggest further unusual possibilities . To this we have recourse to Leonardo da Vinci.

#### Folio 84

In the folio 84 of Codex Madrid I “Leonardo da Vinci established all of the essential features of the strain distribution in a rod while is subjected to the deformation of springs ...” [Ballarini, 2003]. Here the translation of the text.

“Of bending of the springs: If a straight spring is bent, it is necessary that its convex part become thinner and its concave part, thicker. This modification is pyramidal, and consequently, there will never be a change in the middle of the spring. You shall discover, if you consider all of the aforementioned modifications, that by taking part ‘ab’ in the middle of its length and then bending the spring in a way that the two parallel lines, ‘a’ and ‘b’ touch the bottom, the distance between the parallel lines has grown as much at the top as it has diminished at the bottom. Therefore, the center of its height has become much like a balance for the sides. And the ends of those lines draw as close at the bottom as much as they draw away at the top. From this you will understand why the center of the height of the parallels never increases in ‘ab’ nor diminishes in the bent spring at ‘co’.”

We don’t face the question about the Leonardo’s contribution to beam equation. From the didactic point of view the most important fact is that the Leonardo’s text, a mathematical text written in a natural language, can today be seen as a paraphrase of the Euler-Bernoulli model: it constitutes a material model that is faced to the formal model and contributes to its comprehension, as we verified in our class activity.

The comparison between the Leonardo’s material model and the Euler-Bernoulli’s formal model brings attention upon the fact that mathematical models are articulated structures in which more elements complement each other: observation, metaphor, mathematics, language .... Models didactics ought to act in the space between the material and formal models, into the articulated structure that forms the models, in order to paraphrase a mathematical model. While the mathematical models that are used in handling socio-political processes, often result in a watering down of responsibility, as Skovsmose asserts, the articulated paraphrase of the formal models, could put in evidence a “cooperatively” acting. Just the cooperative handling of responsibility should have an evolution of meaning of term “democracy” . In this way the development of a critical teaching and learning could be fostered.

### **Leonardo's interior geometry**

We have seen how the teaching of mathematical modeling has had to face two opposite moments: formal models that are associated with material models by means of principles and laws of physics and formal models associated with “realities” constructed by the development of the model itself. In the first case, our suggestion is well signified by the Leonardo's material model as paraphrase of Euler-Bernoulli formal model. The attitude toward the other types of modeling constitutes a continuous challenge. In order to answer this challenge, we take Leonardo's another indication as a starting point, a prompt that we believe essential in the formation of a critical citizenship.

Leonardo is a brilliant deviser of models. His drawings, even if sketchy, “are now interpreted as equivalent models, by means of which he is able to represent the machines not only with their own fundamental mechanisms, but also with their schemes of functioning.” But Leonardo's geometric ratio goes beyond the static, Euclidean geometry and the linear perspective. At the last years of his life he arrives to “an endless manipulation of geometrical forms with a bearing on his concept of life as energy, and therefore as a process of everlasting transmutation.” Throughout this evolution, Leonardo's geometry acquires an “interior” role”. The sheet Tema is meaningful in this respect: “The geometrical figures that stay behind the profile of an old man, are curiously embodied in the texture of the mantle and spring out on the left, in order to bring the same impulse of the pen in sketching the formation of the rocks. In the “Sala delle Asse” the endless weaving of branches and the bursting out power of the roots that break the rocks are not a simple decoration, but they are the geometric expression of a complete dynamism of Nature.” [Pedretti, 1982]

The restlessness that goes with his interior geometry moves Leonardo away from the trust that characterizes the mechanism in Modern Science, and puts him close to the doubts of the emphasized proceeding of the present science. The present modeling is the last step in the path that begins by the modern absolute trust and passes through the enthusiasm for technological achievements. The socio-political troubles as regards mathematical modeling, echo the Leonardo's restlessness and focus on the need of a critical attitude.

Also in teaching and learning mathematical models, restlessness ought to produce an ethical attitude that, beyond respecting rules, must be under-

stood in his “originary” meaning of “self staying”. To reveal the power of mathematic models and their limits and fallibility, is a first step to form critical subjects that don’t inhabit an empty democracy.

### References

- Ballarini R. (2003) *The Da Vinci-Euler-Bernoulli Beam Theory?*  
<http://www.memagazine.org/contents/current/webonly/webex418.html>
- Boero P, Douek N., Garuti R. (2011). *Contributi sulla costruzione dei concetti e sull'approccio al pensiero teorico in matematica*. XXVIII seminario nazionale di ricerca in didattica della matematica “Giovanni Prodi”. Rimini.
- Campanine: <http://www.baghet.it/campanine.html>
- Gallino L. (2011). *Finanzcapitalismo*. Einaudi, Torino.
- García F. J., Gascón J., Higuera L. R., Bosch M. (2006) *Mathematical modelling as a tool for the connection of school mathematics*, ZDM Vol. 38 (3).
- Longoni P., Riva G., Rottoli E. (2002) *Globalisation, Democracy and Mathematics*. CIEAEM 54, Vilanova i la Gertrú. Edited by Gimenez J, FitzSimons G.E., Hahn Corinne. Editorial GRAÓ, Barcelona.
- Longoni P., Riva G., Rottoli E. (2010a). *La modellizzazione nell'insegnamento/ apprendimento della matematica: potenzialità e rischi*. Sfida 33, Genova.
- Longoni P., Riva G., Rottoli E. (2010b). *Alla ricerca della qualità*. Incontri con la matematica n°24. Castel San Pietro. Pitagora Editrice, Bologna.
- Pedretti C. (1982) *Leonardo da Vinci. Nature Studies from the Royal Library at Windsor Castle*. Giunti Barbera Editore, Firenze.
- Pulinetti L., Milicia C., Sanso M., Rottoli E., Saviano E. (2009) *Le difficoltà in matematica nel passaggio dalla scuola media alla scuola superiore: strategie di intervento*. Grimed XVI. Pitagora Editrice, Bologna
- Skovsmose O. (2004) *Critical mathematics education for the future*. ICME 10.  
[http://www.icme10.dk/proceedings/pages/regular\\_pdf/RL\\_Ole\\_Skovsmose.pdf](http://www.icme10.dk/proceedings/pages/regular_pdf/RL_Ole_Skovsmose.pdf)
- Suits B.H. (2001). *Basic physics of xylophone and marimba bars*. Am. J. Phys. 69(7) (2001) 743.

## **Fairness through mathematical problem solving in preschool education**

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Democracy learning in the preschool context appears, directly or indirectly, through many levels and forms. Fairness and the notions of sharing, owing and giving characterize children's cognitive and social/moral development during the preschool ages. The aim of this study is to investigate whether young children ( $N=40$ ), aged 4-6, express fairness through problem solving situations. Storytelling was used in 2 Conditions with 3 identical mathematical problems engaging processes of correspondence, division/distribution and switching; in the 1<sup>st</sup> Condition children were addressed in the 1<sup>st</sup> person, whereas in the 2<sup>nd</sup> Condition they were asked to assist a neutral hero. Results imply that children at this age may accomplish division and correspondence in a fair way independent of the person who is directly concerned. Concerning the swapping process, children would propose an identical item to be considered as fair in the process of giving and taking, mainly when they were personally involved instead of a neutral hero. Such findings indicate educational and methodological aspects in developing fairness through mathematical problem solving in preschool education.

### **Theoretical background**

Values, norms and concepts of democracy underlie everyday activities and engagements. Amongst them participation, equity, negotiation, play,

fairness, morality, laws and rules, expression of creativity, justice as well as the defense and respect of children's rights consist an integral part of preschool education. In the classroom, children participate in a shared reality, while they develop prosocial behaviors and moral reasoning (Miller, 2006). Fairness and the ideas of sharing, giving and owing as democratic notions undergo multiple situations, activities and realities. As young children engage in such situations they learn to make their own choices, take initiatives, solve problems, cooperate and take risks and overall, develop democratic attitudes, skills and strategies (Emilson, 2011). Under these lines, children aged 4-6 learn to adapt and adopt in terms of 'being' and 'doing' democracy.

From a developmental point of view, children undergoing the pre-operational stage have difficulty in viewing the world and diverse situations through the point of view or the feelings of others; their thinking is characterized as egocentric (Piaget, 1932). Accordingly, at this age they reveal self-interest and are about to gradually construe the mental states of others, by moving from the false belief state of mind (Callaghan et al, 2005). It is the point where children start to differentiate their thinking in order to split the first-person and third-person angle (Rochat et al, 2009). In this sense, an important aspect in perceiving and in turn claiming for subjective and objective fairness relates to whether it relates or not to personal interests, experiences and needs.

Under this perspective, previous research on fairness has shown that children at the ages 5-7 demonstrate sensitivity to fairness either the recipients are themselves or someone else (Fehr, Bernhard, & Rockenbach, 2008; Gummerum et al., 2010; Moore, 2009). Harbaugh et al (2003) underlined that with age, bargaining behaviors change towards greater fairness in distributive justice and participants overcome self-interest after the age of 7. In a cross-cultural study carried out by Rochat et al (2009) 202 children demonstrated fairness in sharing even at the age of 3, through both differentiation and strategic coordination of first-person and third-person perspectives. In the same direction, Moore (2009) found that among 66 preschoolers the resource-allocation decisions depend on the recipient with a strong preference to themselves, after their friends and finally the non-friends.

Within the classroom fairness and the notion of sharing are enhanced through diverse occurrences; free play or thematic learning activities; digital games or storytelling; peer interaction or personal reflection; mathematics or social sciences and so on. In particular, storytelling is a pedagogical tool that encourages imagination and learning through the articulation on powerful abstract concepts that children already understand and empathize, and through oppositions, conflicts and meanings (Egan, 1988). Through various scenarios, various didactic tools, through enquiry children learn to make connections between different experiences in learning and existing knowledge (Bruner, 1977) and in turn may be encouraged to develop critical thinking, fair decision-making and distributive justice.

The aim of this study is to examine how preschoolers respond to mathematical problem solving contexts that imply fairness and the claim of sharing and ownership. Do they express this democratic value while participating in storytelling problem solving situations? Do they link fairness to the mathematical procedures of correspondence, division and switching diverse items? Do they show the same strategy when they are personally involved as when they participate through the perspective of others?




### **Materials and methods**

The research was conducted at the University kindergarten at Ioannina, during 2012. Forty children, aged 4-6 years old, participated in the within-subject test, in groups of 5. Collaborative practices were selected and children, mixed girls and boys, took part in the experiment in a separate classroom within their school. Storytelling and problem solving situations were used in order to engage children in reasoning and thinking practices concerning fairness.

In the beginning children were tested on their prior knowledge on the meaning of 'fairness'. They were asked to express, according to their opinion, a 'fair' or an 'unfair' example. Subsequently, participants listened to a narrative accompanied with relative pictures and were asked to attribute and reason fairness and unfairness to three diverse situations (Table 1). There were two Conditions; in Condition 1 questions and dilemmas were personally addressed to participants as if they were the protagonists (1<sup>st</sup> person)

and in Condition 2 the respective dilemmas and questions were referred to a neutral hero (3<sup>rd</sup> person).

**Table 1:** The methodology of the experiment

	Condition 1 (1 <sup>st</sup> person)	Condition 2 (3 <sup>rd</sup> per- son)
1 <sup>st</sup> problem (correspondence)		
2 <sup>nd</sup> problem (division)		
3 <sup>rd</sup> problem (swap)		

In the 1<sup>st</sup> problem children were asked to attribute fairly 4 items to 2 heroes (correspondence among two uneven sets; corns and chicken). In the 2<sup>nd</sup> problem they were asked to divide fairly one object in 4 parts (division; a pie in 4 equal parts). In the 3<sup>rd</sup> problem they were asked to fairly switch an item (a bicycle) with another item of their preference considered as ‘fair’ (swap). In the first two problems children were given the available data to be attributed as fairly thus in the last problem children had to complete the image of justice by proposing what they subjectively considered to be fair (Table 1).

Children had the opportunity to discuss and exchange ideas in order to attribute a fair solution to the three diverse situations and in the end they recorded their answers individually on specially designed sheets. Children’s personal recordings were used for further analysis.

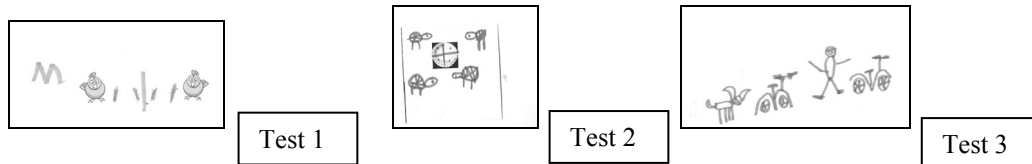
**Results**

Concerning their prior knowledge on fairness all children admitted having heard and knowing the meaning of the term. However, only the 60% of the children could express and link fairness to an example. Indicatively, “*fairness is when we don’t cheat*” (child A), or “*fairness is when you can’t draw something and the others help you*” (child D), or “*it is not fair to play*



*a game with cards and get few cards” (child K).*

**Figure 1:** Examples of the ‘fair’ responses



In the 1<sup>st</sup> problem the 85% of the children achieved correspondence among two uneven sets of objects correctly (i.e. 2 corns per chicken). Independent of the perspective (1<sup>st</sup> or 3<sup>rd</sup>), preschoolers managed to attribute fairly the items to the recipients;  $t(38)=0,618, p>0.05$ . It is worth mentioning that the proposal to give 1 corn per chicken and keep the rest for the next day was considered amongst the ‘fair’ solutions.

In the 2<sup>nd</sup> problem the 70% of the children recorded and explained how the division would be fair. In this case, there were children able to explain orally the procedure but had difficulty in expressing such procedures graphically. Again the perspective, Condition 1 vs Condition 2, didn’t play an important role;  $t(33)=-1.08, p>0.05$ .

In the 3<sup>rd</sup> problem, the answers were more subjective as the option of swapping one item with a non-presupposed item was based on personal beliefs. The 60% of the responses would indicate an identical item with the one to be exchanged in the first place; a bicycle. Here, participants in Condition 1 showed a significant difference in selecting the ‘identical other’ compared to participants in Condition 2;  $t(33)=-2.56, p<0.05$ .

### Discussion

Findings demonstrated that preschoolers can identify fairness to problem solving situations involving correspondence and division processes, independent of the perspective of the actor. These results support previous research (Fehr, Bernhard, & Rockenbach, 2008; Gummerum et al., 2010; Moore, 2009; Callaghan et al, 2005; Rochat et al, 2009) implying that at the age of 4 children are cognitively and socially capable to overcome self-interest, understand fairness and discriminate their state of mind from others’ while distributing justice.

In the first test preschoolers managed to make fair correspondence between two uneven sets of items and in the second test they showed the ability to divide fairly an object in equal parts no matter if they were personally addressed or they had to assist a neutral hero. In the last test where the attributed value to be considered as fair was subjective most children selected an identical item to the one being switched. They considered fair the exchange of a bicycle with a bicycle, even though they could select anything else. This sense of fairness under the particular strategy was more significant in Condition 1, where the 1<sup>st</sup> person perspective was addressed.

While working on fairness, a democratic notion of great importance in everyday life, aspects such as age (Harbaugh et al, 2003), socio-cultural influences (Rochat et al, 2009), the recipient (Moore, 2009), the context, the content and other instructional factors, children's prior knowledge and intuitive thinking (Bruner, 1977), and not only should be taken into account. In order to create meaningful learning experiences preschoolers should confront problem solving situations in accordance to their interests, experiences and needs. In the current study children participated in groups; they had the opportunity to exchange ideas and discover problem solving techniques through storytelling and graphical representations. In further research more factors could be studied on grounds of educational practice such as the use of New Technologies, the use of more scenarios linked to other mathematical notions, methodologically different approaches. Additionally, further research could include greater sample and more dilemmas implying mathematical, emotional and ethical mechanisms.

Results may be constructive in an educational, didactic direction. Based on their cognitive and social development preschoolers at the age of 4 may be inserted to the meaning of fairness through logico-mathematical activities. By entering formal education, children begin to develop reasoning and understandings in a more organized and goal-oriented way. Meanings, values and practices of democracy are crucial to exist in the preschool classroom. As children overcome their egocentrism and sense of ownership and self-interest they should be given the opportunities to learn to share, give, distribute, interpret, decide, assert, negotiate and last but not least be fair.

**References**

- Bruner, J. S. (1977). *The Process of Education*. London: Harvard University Press.
- Callaghan, T., Rochat, P., Lillard, A., Claux, M. L., Odden, H., Itakura, S., et al. (2005). Synchrony in the onset of mental-state reasoning: Evidence from five cultures. *Psychological Science*, 16(5): 378-384.
- Egan, K. (1988). *Teaching as story telling*. London: Routledge.
- Emilson, A. (2011). Democracy Learning in a Preschool Context. *International perspectives on early childhood education and development*, 4: 157-171
- Fehr, E., Bernhard, H., & Rockenbach, B. (2008). Egalitarianism in young children. *Nature*, 454:1079-1083.
- Gummerum, M., Hanoch, Y, Keller, M., Parsons, K. & Hummel, A. (2010). Preschoolers' allocations in the dictator game: The role of moral emotions. *Journal of Economic Psychology*, 31: 25-34.
- Harbaugh, W. T., Krause, K., Liday, S. G., & Vesterlund, L. (2003). Trust and reciprocity: Interdisciplinary lessons from experimental research. In E. Ostrom & J. Walker (Eds.), *Trust in children* (pp. 302-322). New York: Russell Sage.
- Miller, J. G. (2006). Insight into moral development from cultural psychology. In M. Killen & J. Smetana (Eds.), *Handbook of moral development* (pp. 375-398). Mahwah, NJ: Lawrence Erlbaum.
- Moore C (2009). Fairness in Children's Resource Allocation Depends on the Recipient. *Psychological Science*, 20(8): 944-948
- Moore, C. (2009). Fairness in children's resource allocation depends on the recipient. *Psychological Science*, 20: 944- 948.
- Piaget, J. (1932). *The moral judgment of the child*. London: Routledge Kegan Paul.
- Rochat, P., Dias, M., Liping, G., Broesch, T., Passos-Ferreira, C, Winning, A and Berg B. (2009). Fairness in Distributive Justice by 3- and 5-Year-Olds Across Seven Cultures. *Journal of Cross-Cultural Psychology* 40: 416-442.
- Sloane S, Baillargeon R, Premack D. (2012). *Do Infants Have a Sense of Fairness?* *Psychological Science*, 23(2): 196-204.

## **Democratic game play: Is it a matter of rules?**

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### **Résumé**

Un jeu démocratique requiert la compréhension des règles par tous les joueurs. La connaissance des règles et leur explication crée des relations autoritaires. On suppose que la construction par les élèves et leur instituteur de nouvelles règles pour un jeu inconnu peut créer les conditions pour un jeu démocratique. Dans cet article, nous avons examiné les pratiques communicatives que l'instituteur et les élèves utilisent afin de construire les règles du jeu de table présenté. Nous avons également examiné si les règles construites ont comme résultat un jeu démocratique pendant sa séance. Les résultats ont démontré que bien que la communication ait eu lieu dans une ambiance démocratique et que les règles aient été construites, le jeu démocratique n'est pas uniquement une question des règles.

### **Abstract:**

A democratic game requires the understanding of the rules by all the players. The knowledge of the rules and their explanation creates authoritarian relationships. We assume that the construction of new rules in an uncommon game by students and their teacher might create the conditions for a democratic game. In this article we investigated the communicative practices that teacher and students used to construct the rules of the board game presented. We also investigated if the constructed rules resulted to a democratic game during its playing. The results showed us that although the communication took place in a democratic environment and the rules were constructed the democratic game play is not only a matter of rules.

### **Introduction**

An eminently communicative and social interacting context for

teaching/learning mathematics is board games (Skoumpourdi, 2010). Games usually include a relevant set of social practices that could contribute to the emergence of democratic practices. Playing board games is a kind of mathematics classroom practice which could enact particular values such as social justice, respect and dignity, as well as particular ethical principles such as patience and fair play.

The rules are one of the key features of a board game. The designer of a game forms the experience of it with the rules. Rules help students understand the meaning, the purpose and the function of a game as well as communicate accurately with each other based on them (Bruce, 1991). Before the game can be played the rules must first be read or/and explained. It must be ensured that all players know the rules fully before the game starts. The best chosen game can prove to be an unpleasant experience due to a poor understanding of the rules. But what happens if the rules of the game are too difficult or unjust for some students? It leads to an undemocratic game. It is not democratic to play a game when not all of the players have fully understood its rules. It is sometimes necessary to transform the rules of a game in such a way so as to meet the needs of the players.

The explanation of the rules, for example, by the teacher creates authoritarian relationships between the teacher and the students in the classroom (Kamii & De Clark, 1985). The teachers often impose contractual pre-constructed rules that they consider right and this imposition increases their power. The way the rules are treated by an adult—teacher or parent— influences young players' experiences when playing the same board game (Skoumpourdi, 2011). Setting rules by students reduces the power of teachers and transfers the responsibility for resolving conflict situations concerning those rules to children (Kamii & De Clark, 1985).

In this article, taking into consideration the above views, we assumed that if the students could construct their own rules, they would understand them easily, think about what is fair and possibly create the conditions for a democratic game play. When the children make the rules themselves they may feel more familiar with the game and play it with greater comfort than learning the rules developed by someone else. For this reason we investigated: (a) what communicative practices teacher and students used to construct the rules of an uncommon board game, and (b) if the constructed rules lead to a democratic game play.

### **Methodology**

The research was conducted in the first grade (5 girls and 5 boys), of a

public primary school, after the school program. A designed board game the “Shapes” (photo 1) was used and children were asked to construct the rules in order to play the game. The “Shapes” consists of a board with a perimeter path with squares that have shapes on them. In the middle of the board there are 24 big empty squares. The game also contains cards in several shapes—same with the shapes on the board—several solid shapes as well as four markers. The solid shapes are used for constructing buildings on the empty squares (photo 2).



Photo 1

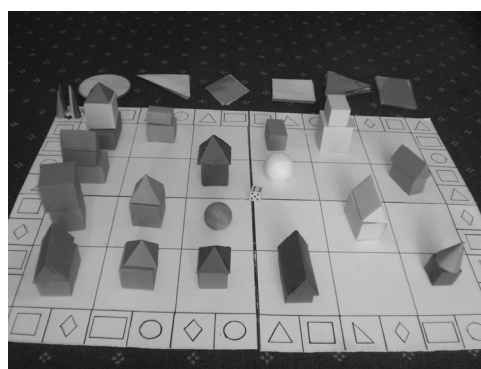


Photo 2

Children worked all together, in order to develop rules for the game. We did not ask the students to work in teams of twos or threes because all of them have to agree with each constructed rule. Students were informed that they could start playing the game after they would have constructed the rules.

### Results

The students used several communicative practices for developing rules.

(I) A student suggested a rule and others agreed. The rule was recorded.

(C: Child, T: teacher, ...: many children):

[1] C2: *The first rule should be to start all markers from the beginning.*

[2] T1: *Everyone agrees with this rule?*

[3] ...: *Yes.*

[4] C4: *and if we fall into this (shows a rectangular shape on the path) to get this card (showing a rectangular card)*

[5] T1: *Could this be the second rule?*

[6] ...: *Yes.*

[7] T1: *Who wants to write the rules?*

(Each rule was repeated by the teacher to enable the child to write it).

[10] C1: *Whether throw 3, three moves will be made. We have to be accurate.*

[28] C7: *Another rule should be to read what the card says, to do it and show it to the others to see if we did it properly.*

[29] T1: *How would I choose the card?*

[30] C8: *If I am in a circle I will be getting a round card*

[35] T1: *What else the game is containing?*

[36]...: *Solid shapes.*

[37] T1: *How can we use them?*

[38] C6: *Lets start playing and we will see*

(II) Students disagreed with a proposed rule justifying their views, interacted and reached a commonly accepted rule. The rule was recorded.

For example when the question about what dice to use arose, children disagreed:

[15] T1: *What other rules should we write?*

[16] C1: *To play with the dice that has dots.*

[17] C3: *This is not fair. I say that we can play with the dice that has numbers because it is easier. With the first dice we must count whereas with the other we can immediately see the number.*

[18] T1: *What do you say? Which dice to use?*

[19]...: *The dice that has numbers*

[20]...: *Both.*

[21] C5: *Both because they will take us very far. We can move six places and six more.*

[22] T1: *What do you say to do with the dice?*

[23]...: *To use them both.*

(III) Students, during the playing of the game, tried to solve the arising issues.

(a) They realized that important rules were missing:

[39] C9: *We must decide who will play first.*

[93] C9: *We must decide who will be the winner.*

(b) They tried to create the rules that were missing. When children come to a disagreement over a rule, they feel a responsibility to resolve it themselves:

[40] C1: *First will play the one who put his marker first at the start.*

[41] C2: *This is not fair, because we did not know it from the beginning of the game.*

[42] C4: *This is not fair, because we will be confused.*

[43] C6: *This is not fair, first will play whoever throws the biggest number on the dice.*

(c) They made the rules more precise.

[64] C10: *We will put the cards that we have used under the pile.*

(d) They discovered a rule for the solid shapes.

[73] C3: *The card says to construct a building with two solids and to name them ... in this way we will use the solids.*

(IV) The teacher used several communicative practices for developing rules. She was asking the students every time if they agree with each rule that was proposed and repeated the agreed rule to facilitate the student to write it [2], [7]. She posed clarification questions [5], questions that challenged the participation of the students [15], [18], [22], [35], [37], as well as questions that crosschecked a former rule [29].

In general, the students, during the game, did not always have patience to wait for their turn or to wait for their playmate to complete his/her move. But the students generally threw the dice properly and followed the indication of the dice consistently.

### **Conclusions**

The stages that were followed to construct the rules of the game were similar to those of Alro' and Skovsmose' (1996) communicative model, in a democratic concern. Students got in contact with the game, discovered it and identified its contents. They negotiated between themselves and with the teacher to create fair rules for the game, they thought aloud about the rules, challenging each other. Finally, they evaluated the rules during their play and they reformulated them.

Teacher and students worked collaboratively to construct the rules of the board game. Many communicative practices which contributed to the emergence of democratic practices were observed between teacher and students when they constructed the rules. A rule was proposed from a student and was accepted or not accepted by other students. Proposals were in general justified. During the game the arisen issues were solved by the



students' interactions. The teacher secured the agreement of all the students, for a rule and helped a student to write it by dictating it. She posed questions clarifying and crosschecking the rules, as well as challenging students' participation. The way the teacher and the students communicated was in accordance with Valero's view of communication in a democratic environment (1999) which "allows the establishment of a shared language for which all participants are responsible and in whose creation and modification all participants have a role to play" (pp. 22).

It did not seem to be very difficult procedure for students to propose rules for a game. But the rules were not always clear and concise for all of them. Each student's rules differed because of his/her different needs, preferences and experiences. The rules, that students have set, concerned in general how the game starts and how the game continues. Important questions such as how the game ends, how it evolves its elements, which are the rights and the obligations of the players, who is the winner, what choices will the player have to make in a typical round, the scoring objectives or winning conditions did not all find answers by the developed rules but during the game play.

A democratic game is guaranteed not only by the appropriation of the rules but also by the adoption of ethical principles that are not taken for granted for young children.

#### **References:**

- Alro, H. & Skovsmose, O. (1996). On the right track. *For the Learning of Mathematics*, 16(1), 2-8.
- Bruce, T. (1991). *Time to play in early childhood education*. Hodder & Stoughton Educational, (Ch. 6: 98-118). Great Britain.
- Kamii, C. & DeClark, G. (1985). *Young children reinvent arithmetic: implications of Piaget's theory*. Teachers College Press.
- Skoumpourdi, C. (2010). Play as context for approaching early childhood mathematics: Designing board games. *Journal of Contemporary Education*, 162, 139-156. (GR)
- Skoumpourdi, C. (2011). Playing board games inside and outside the classroom. *Proceedings of CIEAEM 63, Facilitating access and participation: mathematical practices inside and outside the classroom*, Barcelona. (CD-ROM)
- Valero, P. (1999). Deliberative Mathematics Education for Social Democratization in Latin America. *ZDM*, 31(1), 20-26.

**Hellenic Mathematical Society**  
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## **MATHEMATICAL JUSTIFICATION IN CZECH MIDDLE-SCHOOL TEXTBOOKS**

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**Abstract:** This paper summarizes the study conducted on eight series of middle-school mathematical textbooks in the Czech Republic. The aim of the text analysis was to find to what degree the authors of analyzed textbooks support teachers in the classroom practices that are based on the problem-solving nature of mathematics, mathematical reasoning and inter-relatedness of mathematical constructs. The term *justification* is defined to frame the analysis, and applied to the treatment of five distinct topics. The results yielded strong support of the need to justify, minimal support of student involvement and more complex results in making connections between mathematical constructs.

### **Introduction**

The role of the problem-solving element in mathematics, the role of mathematical reasoning and the level of inter-relatedness of mathematical constructs are strongly correlated with an educator's specific beliefs about the nature of mathematics. It is the aim of this study to explore how authors of selected mathematical textbooks in the Czech Republic support middle-school teachers in implementing teaching practices based on the dynamic, problem-solving view. The three characteristics of mathematics mentioned above are investigated through the quantitative and qualitative analysis of *justification* (as defined below), against the background of five selected topics.

The term *justification* will be employed here rather than that of *explanation* (which can involve "how") or *proof* (which implies a deductive quality) and will denote all evidence of communication with the purpose of clarifying, stating and/or explaining *why* (this definition will be applied

strictly to the selected mathematical constructs). It is chosen to better fit the social environment of mathematics learning in a classroom where justifying carries the element of acknowledging that the truth is not possessed by the teacher or textbook author.

The choice of concepts to frame our analysis is based on a) results from previous research and b) the aims of this study. The following topics were chosen because they all contain an element of “rule” or “fact” treatment that gives potential ground to the absolutist representation of mathematics. At the same time, they differ in the nature of the reasons for their existence:

1) *Division by zero* is not defined in order to maintain consistency in the system of operations

2) *Zero power* is defined as 1 (except for the case of  $0^0$ ) to maintain the relationship between division of expressions and subtraction of powers consistent.

3) *Square roots of non-positive numbers* follow directly from the definition of square root as the operation reverse to second power.

4) *Multiplication of two negative integers* results in a positive integer if we extend the system of operations from natural numbers to integers

5) *Division of two fractions*: follows from an application of operations;

Topics 1 through 3 are examples of a kind of mathematical convention. In topics 4 and 5 the procedural competence reportedly outweighs conceptual understanding (Fuller 1996, Quinn, Lamberg& Perrin 2008, Ma 2010, Son& Senk 2010, Streefland 1996).

Numerous studies point out a wide range of beliefs and levels of competence when teachers (Quinn, Lamberg& Perrin 2008) or students (Tsamir&Tirosh 2002) are asked to apply and explain division by zero.

Literature about the importance and forms of explanation and proof in mathematical classroom is vast. Namely, the models of Sierpinska (1994), Harel&Sowder (2007) and Stacey&Vincent (2009) were used in this study.

The following four questions guided our investigation:

1) How consistent are middle-school textbooks in providing justification and explanations of mathematical truths, including conventions such as division by zero?

- 2) Are conventions treated differently than other concepts?
- 3) What role are students (explicitly) given in the process of justification?
- 4) What forms of reasoning and what variety of models is utilized in justifications?

### **Methodology**

Text content analysis has been used in this study for eight major Czech middle-school mathematics textbook publishers. The middle-school grades in the Czech Republic are grades 6 to 9, and the sequence of mathematical topics covered by all textbooks is relatively similar. All four grades textbooks were submitted to the analysis. Out of the eight textbook series only two are supported by teacher's books which were also made part of the content analysis. Students' were left out of the study.

The text was searched for the topics outlined above, and relevant to guiding questions. Most relevant text was found in introductory and/or explanatory parts of chapters and subchapters, but individual practice and application problems (without a given explanation) were also reviewed as they contained some points related to our investigation (particularly, the concept of division by zero). These selected text chunks were further coded in terms of

- a) presence or absence of justification,
- b) (the variety of) models used in justification,
- c) pupils' involvement (explicit) in the establishing of rule/agreement

In addition, the categorization of the nature of reasoning used in justification was conducted to assess the authors' suggested treatment of informal proof in classroom. Further, the various occasions when zero division was applied (explicitly or implicitly) across the curriculum were noted and assessed in order to gain insight into the experience the authors offer pupils (and teachers) with the concept of division by zero. These additional results are beyond the scopes of the paper.

### **Some of the results and interpretations**

#### *Justification*

The analysis has shown that across the eight textbook series, justification of both mathematical concepts and conventions is strongly

avored. The topics that lacked justification in some texts (total of 8 out of 40 instances) were division by zero (three books), square root of non-positive integers (two books and one avoiding the non-positive domain altogether), multiplication of two integers (two books) and the zero power (one book). These no-justification cases were yielded from five of the eight series, none of them exceeding two instances.

The lack of justification of division by zero aligns with previous research (Quinn, Lamberg & Perrin 2008, Tsamir & Tirosh 2002). We cannot infer that the authors deny the importance of justification; it can be argued, though, that the rules found in the textbooks, such as “division by zero is meaningless”, “we cannot divide by zero” etc. are viewed as engrained enough to revisit and justify, especially as it was (hypothetically) already done in the third or fourth grade. The opportunity to make connections to new concepts and the reinforcement of understanding (reconnecting the separate fact to the body of new knowledge) at this developmental stage is missed.

No evidence of distinction between the two instances of mathematical convention (division by zero and zero power) and the other three topics. This non-distinction can be accounted for by the nature of *didactic explanation* at the developmental stage and the (didactically justifiable) absence of formal proof.

#### *Student involvement*

After preliminary scanning of the data it became apparent that student involvement in justification is not explicitly treated in majority of the series books. It may simply follow from the intended reference role (e.g. referential) of the particular textbook in a classroom and, lacking any other information on this issue, no conclusions can be made regarding the lack of explicit student involvement other than that it was consistent throughout all topics. One series of books explicitly involved pupils in justification of mathematical truths by providing a problem-solving context and supporting (via teacher's book) the teacher as a mediator. The concepts that were thus actively discovered (justified) were the case of division by zero, multiplication of two negative integers and division of fractions.

These results show a clear indication that the degree of student involvement in justification relies heavily on the teacher's pedagogical

competence and beliefs. The implicitly referential role of a textbook can allow teachers a higher degree of freedom in their instructional decisions.

*Variety of models used in justification*

The number of justification (deduction and semi-deduction) models in the analysed texts varied depending on the topic and its nature (e.g. the square root of non-positive rationals being linked directly to definition), and the models for justifying were fairly standardized across all textbooks. For example, there were two models used for justification of division by zero: the model of partition – dividing the whole into parts, and the model of relating division and multiplication. The model of dividing by smaller and smaller numbers was avoided. Only one textbook showed both models of justification (of the zero division) at various points in the curriculum.

**Conclusions**

The presented analysis of eight middle-school textbook series yielded some insight into the support teachers are given from textbook authors in their classroom practice. Before concluding this paper, it is crucial to point out again that the study does not attempt to make inferences about teaching practices in the Czech Republic.

With a few exceptions, the authors support teachers in the practice that follows from perceiving mathematics as based on reasoning and not on authority. The lack of acknowledgement of convention and “convenient” agreement may suggest a Platonist (as defined by Ernest 1989) position, understating the sociocultural aspect of mathematics. Further study would be needed to assess the treatment of both formal and informal proof at later stages of the mathematical curriculum in the Czech Republic.

Apart from one textbook series, involving students in mathematical justification (and discovery) is not explicitly suggested in the analyzed textbook material. The absence of teachers’ books (in six out of eight series) and methodological guides to accompany majority texts at the middle-school level suggests low support of this aspect of practice in classrooms. Further analysis of other topics is needed to gauge the extent to which the authors support teachers by providing a variety of models and multiple approaches to justification.

The comprehensive system of categories yielded by coding for student involvement and justification, the catalogue of models used in studied cases of justifications, the application of Stacey&Vincent's (2009) modes of reasoning, and the tracking of opportunities to offer further understanding of division by zero are beyond the scope of this paper.

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**References:**

- Ernest, P. (1989). The Impact of Beliefs on the Teaching of Mathematics, London, 1989: 249-254. In P. E. Ernest, *Mathematics Teaching: The State of the Art* (pp. : 249-254). London: Falmer Press.
- Fuller, R. (1996). *Elementary Teacher's Pedagogical Content Knowledge of Mathematics*. Retrieved January 20, 2012, from ERIC: <http://eric.ed.gov/PDFS/ED401270.pdf>
- Harel, G., & Sowder, L. (2007). Towards comprehensive perspectives on the learning and teaching of proof. In F. Lester (Ed.). Reston, VA: National Council of Teachers of Mathematics.
- Ma, L. (2010). *Knowing And Teaching Elementary Mathematics: Teachers' Understanding Of Fundamental Mathematics In China And The U.S.* Routledge.
- Quinn, R. J., Lamborg, T. D., & Perrin, J. R. (2008). *Teacher perceptions of division by zero*. Retrieved January 2012, from EBSCOHost Database.
- Sierpinska, A. (1994). *Understanding in mathematics*. London: Falmer.
- Son, J.& Senk, S. (2010). *How reform curricula in the USA and Korea present multiplication and division of fractions*. Retrieved January 2012, from SpringerLink database: [dx.doi.org/10.1007/s10649-010-9229-6](http://dx.doi.org/10.1007/s10649-010-9229-6)
- Stacey, K & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics* , 72, 271–288.
- Streefland, L. (1996). Negative Numbers: Reflections of a Learning Researcher. *Journal of Mathematics Behavior* , 15, 57-77.
- Tsamir P., Tirosh, D. (2002). *Intuitive beliefs, formal definitions and undefined operations: Cases of division by zero*. Retrieved November 2011, from ebrary: [site.ebrary.com/lib/cuni/Doc?id=10067541&ppg=352](http://site.ebrary.com/lib/cuni/Doc?id=10067541&ppg=352)

## **WORKING GROUP 2:**

**Democracy in mathematics classroom practices: What kind of mathematics classroom practices would enact a particular set of humanitarian values (e.g. social justice, respect and dignity)?**

**GROUP DE TRAVAIL 2 :Démocratie dans les pratiques mathématiques en classes: quelles pratiques permettraient de vivre un ensemble particulier de valeurs humanitaires (ie justice sociale, respect et dignité)?**

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## **Providing students with mild mental retardation the opportunity to solve Division Problems related to real life**

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### **Abstract**

We know that teaching children with mental retardation is subject to some restrictions. This research focuses on demonstrating that students can acquire problem-solving strategies when they find a suitable learning environment. The software "Hidden Treasure" offers students an appropriate learning environment that allows a meaningful conceptual understanding of the concept of division. Through this software (Serious Game) students can understand both quotative - measurement divisions and the partitive - distribution divisions. Each division has its own representation. Each representation is intended to reveal the concept that corresponds. Through active action and narrative, the student appears to be driven to learning how to solve problems which involve division. The context of the game seems to be crucial because even when the student is out of the game's context he continues to use the game's representations to solve the division problems that are given to him.



### **Theoretical Background**

The full social acceptance of citizens with disabilities must be a major concern in any democratic society. After all, democracy is founded on equal interaction between different people, working together to develop the total, while maintaining full respect towards the special status of each member of this set (Farrelly 2005). Within such a democratic framework, citizens with disabilities have the opportunity to co-create without needing to give up his identity. Instead, given the possibility to self-determined. In a democratic society, citizens with disabilities are treated as "citizens in action." When the organization of society based on democratic principles, policies and practices adopted, seek to remove all forms of exclusion towards people with disabilities. In such a democratic context the challenge is to implement a differentiated instruction which refers to students with mental retardation. For mathematics instruction to contribute to the building of a socially just and diverse democracy will require more than care with curriculum and teaching. It will also require more than committed teachers, sensitive to and skillful in working toward these aims (Ladson-Billings, 2001). Accomplishing this would require significant change in teachers' education and professional development (Deborah Loewenberg Ball ; Imani Masters Goffney ; Hyman Bass 2005).

Frances M. Butler, Susan P. Miller, Kit-hung Lee, and Thomas Pierce report at their review that "The National Council of Teachers of Mathematics (NCTM) developed standards for mathematics curriculum in 1989 and again in 2000 (National Council, 1989, 2000). These standards demonstrate a shift from a behaviorist approach of teaching rote learning of facts and procedures to a constructivist approach, emphasizing the development of conceptual understanding and reasoning. Teachers are encouraged to give students hands-on experiences connected to the real world rather than emphasizing algorithms and abstract routines. Perhaps now more than ever, limiting mathematics instruction to rote computation practice will deprive students with disabilities from competence in important mathematics concepts and, thus, prevent them from succeeding in inclusionary settings and using mathematics effectively in real-world activities." So according to Roh, Kyeong Ha it is important to bear in mind that the successful experiences of students in handling their own personal knowledge also helps them solve math problems (Shoenfeld, 1985; Boaler, 1998). The PBL is a student-centered learning style. When teachers use the

above theory, help students to focus on solving problems within a real context, encouraging them to consider the case in which the problem exists when trying to solve it (Sheryl MacMath, John Wallace, and Xiaohong Chi , 2009). The Karweit's (1993) definition focuses on the fact that the learning framework is designed so that students handle things and solve problems in a way that reflects the nature of these problems in the real world. The research supports the effectiveness of learning environments that are rich in meaning (Carraher, Carraher & Schleimer, 1985; Lave, Smith & Butler, 1988). Resnick (1987) notes that the school placed emphasis on symbol manipulation and subtractive process rather than learning framework used in the world outside school. She says that the problem is that symbols separated by references to the real world. Being out of context, have no meaning for students.

Not only the games in general, but also the digital games of learning purpose, include goals (eg winning the game, find the treasure) that allow students to devise strategies to feel proud of their achievements and accomplishments and be rewarded with the continuation of the game (Gee, 2003). And this whole process is conducted by the players through active exploration and experimentation. Trial and error is not a punishment for them but works as a feedback in the world of digital gaming. In addition, digital learning games offered for the purpose of individualized learning approach and can be used in accordance with your personal rate of each player, and for this reason the majority does not set a time deadline of the end game. Simultaneously, they can provide a multitude of options for feedback through sounds, images, verbal messages, etc. being able to respond to the different learning styles (Becker, 2005a).

### **Method**

This research is a case study. It lasted two years, during which there was a systematic attempt to investigate the qualitative elements that contribute to the development of strategies in solving problems which are related to the operation of division.

### **Research Questions**

The aim of this research is to examine the extent to which students with mild mental retardation may initially recognize and then successfully handle division problems after being involved in processes of solving such problems in

a different context. The different context that the software “Hidden Treasure” sets.

Moreover we aim to investigate whether the students are able to understand the conception of division through this process.

### **Participant**

The survey involved a student of the second class of high school with mild mental retardation (IQ 58). The student could easily handle the operations of addition and subtraction. With regard to multiplication he could understand it as a repeated addition. This strategy enabled him to successfully handle multiplication problems. He was unable to grasp the concept of division since he could not understand the part –whole relationships in the multiplicative structure.

### **Research Tools**

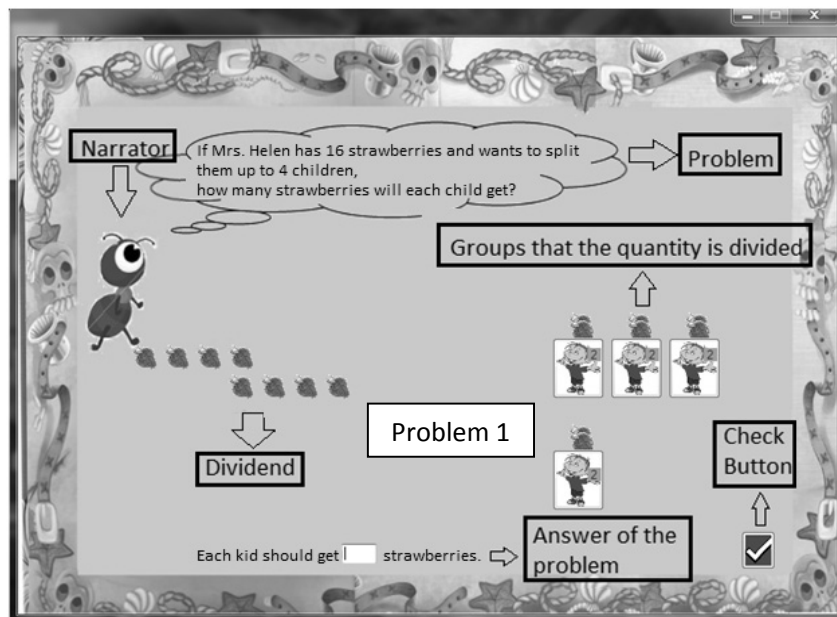
#### **Interview Guide**

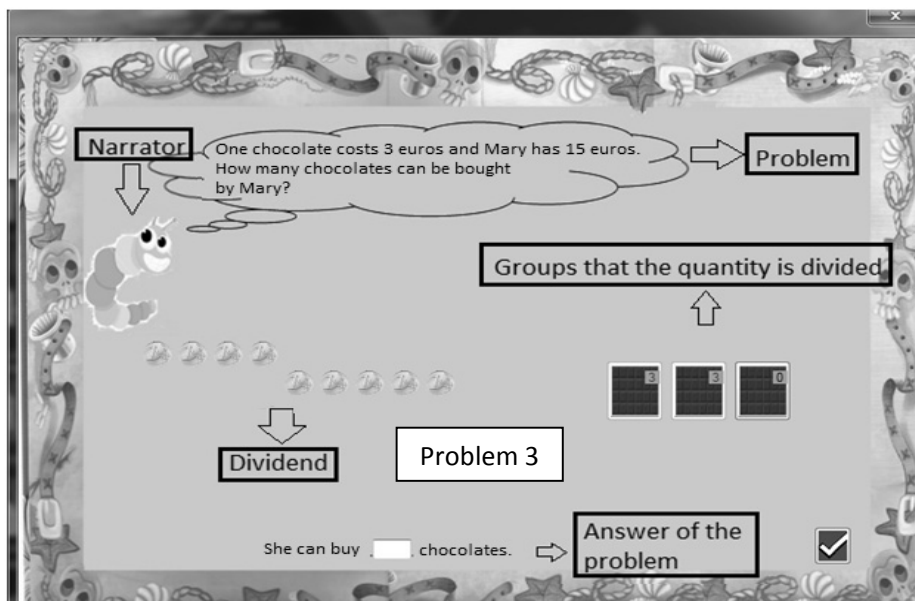
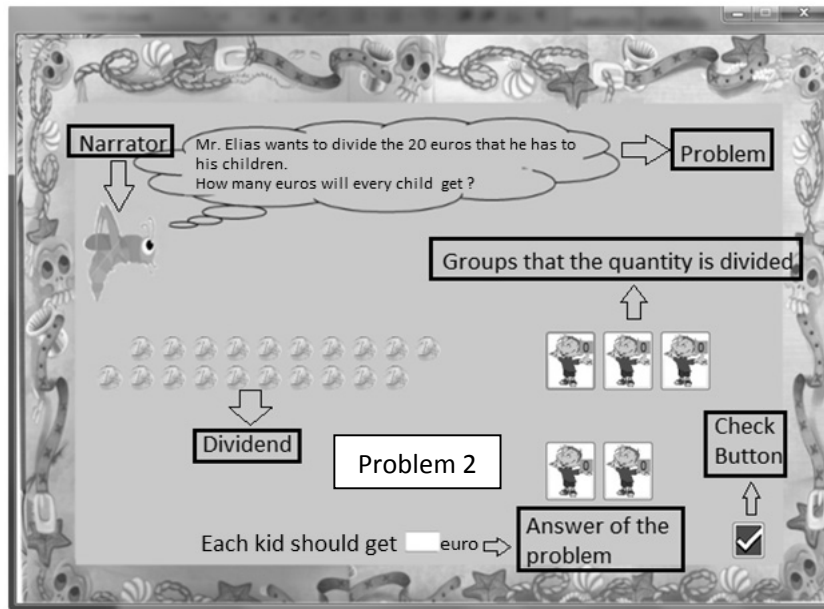
It was used a questionnaire which contained 10 questions. The questions were related to explanations of the solutions proposed by the student to the various problems he asked to solve. Through this questionnaire we were able to discern whether the student works mechanistically, or indeed has developed problem-solving strategies. The questions are summarized in the appendix.

#### **Software “Hidden Treasure”**

The software used was designed for this student and refers only to specific groups of students. It contained two types of problems each of which had a different solution process. However in both categories the notions of divisor, the dividend, the quotient and the rest visualized and the student could handle them as if he had the same shared objects in his hands. Also, the narration is very important. The narrator changes at each activity and each time helps the student to get a step closer to the treasure. The problem involves various divisions and not just divisions which deal with money. As a result student and teacher are given the opportunity to modify the problem in any new use by changing images, the narrator and the object to be shared. As students participate in the construction of the game create their own customized variables, and therefore a particular personalized instructional intervention.

Below there are three of the ten problems of software "Hidden Treasure". While the first two have to do with quotative - measurement divisions the third represents a partitive - distribution division.





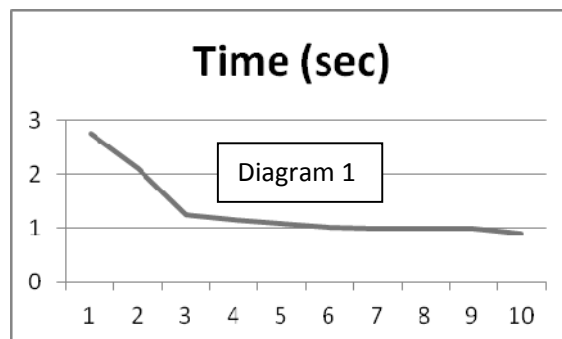
### Analysis

For the analysis of instructional interventions we used the filming of these interventions. Every dialog and movement was analyzed in order to have qualitative data of the relationships.

### Results

#### *Respond Time*

The only explanation requested by the student had to do with the way that he distributes the objects. While we agree that the student must be "fair", every time an object is given to each child until they run out of items. It is striking how quickly the student responds to various problems. The response time is reducing significantly from problem to problem. The diagram below provides us with information that has to do with time.



#### *Helping Context*

Also we should mention that the student recognized that these are division problems since he was sharing items to equal groups. The student recognizes the problems that had been given out of the game's context and thus recognizes the game's helping role as he could not answer the problems before. The

visualization and the new context help the student to handle the problem and "see" the solution.

#### *Narration*

The student pays particular attention to the narration. He is interested in it, and so he listens carefully during the game. He asked where I found the images and if he can draw and put his own images instead of them. He really enjoyed not only playing the game but also drawing new pictures and making a new game by himself.

#### *Getting to Arithmetic*

The student has already handled several problems in the game and has already acknowledged that these are division problems. This fact gives us a great push to try to link two different representations. The one representation is for example  $36 : 9$  and the other is the visual representation of the 36 items that are divided into 9 groups to find out how many items each group has. Through different representations of the same operation we manage to achieve deep and diverse knowledge of the subject.

#### *Connection to multiplication*

Students with mental retardation have difficulty in understanding reverse processes, so it is very difficult for them to link multiplication with division. Through the software it is easier to push them to understand the connection between the operations. When students share a quantity of 8 items in 4 children understand not only that each child will take 2 objects, but also that if he gets all items back again he will have the initial quantity of objects. So one he senses not only that  $8:4 = 2$ , but also that  $4 \times 2 = 8$ .

#### *What makes partitive problems so difficult?*

As Catherine Twomey Fosnot and Maarten Dolk mention to understand that distributing, or dealing out, to a given number of groups produces equal or fair sharing requires that children comprehend the one-to-one correspondence involved. Further, they must consider the number of groups, the number in the groups, and the whole-all simultaneously! The part-whole relationships involved make this a big idea. Children frequently see adults deal playing cards

out one-to-one, but that does not mean they understand that this strategy results in an equal number of cards for each player. In fact, they often need to check by counting to see if the resulting number of cards in each group is the same. Only by exploring many partitive contexts and reflecting on their actions and the results, do children come to construct a “dealing out” strategy. This is exactly what is offered by the software.

*Being out of the Software’s Context*

The most encouraging of all findings was that the student has acquired a permanent strategy for solving division problems through visualization of objects that are shared. When after 2 weeks he was given the following problem he managed to solve it by portraying it as shown below:

Problem:

Mr. Nick has 12 apples and wants to divide them to his 3 children. How many apples would each child get?

The student constructed not only apples but also children. He gave an apple to each child till he run out of apples. When he shared all apples he said that every child will get 4 apples.

Similarly he solved the partitive problems that were given to him without facing any difficulty.

The most important of all was that even though we were outside the context of the Software, the student seeking meaning in operations through the context in which he initially gave meaning to these problems.



Figure 1



### Discussion

Through this research several issues are highlighted such as the fact that children with mild mental retardation are able to gain a deeper conceptual understanding of the concept of division and also to recognize and solve problems with divisions. We have strong evidence that while moving in this direction we can push students to develop strategies to solve problems and make them able to see that a division problem is not a problem of successive abstractions, as equally a multiplication problem would be a problem of successive additions. This is quite important since we can provide students with different strategies rather than counting or trial and error strategy.

However much interest would present a survey which will not be associated with sharing divisions. As in real life, most division problems have remainders that need to be dealt with in context. Children who are used to doing problems like these see mathematics as mathematizing. They find ways to think about their lives mathematically. They do not perform procedures that are nonsensical to them, and they treat remainders in relation to the contexts (Catherine Twomey Fosnot and Maarten Dolk). And also it would be interesting to interpret students' responses to various problems while having to divide a quantity equally and there are surplus items that we do not know how students would possibly handle. In conclusion it is important to emphasize that understanding of mathematics by students with disabilities is essentially the realization of democracy in practice. Each student has the right of access to knowledge that it's use will be immediate in their everyday life.

### Appendix

#### *Questionnaire*

<b>Question</b>	<b>Answer</b>	<b>Comments</b>
1. What process do we use to solve the problem?		
2. Why do we use this process?		
3. Is there any similarity between the first and the second problem? Which one?		
4. Do you like the game?		
5. If you could change		

something what it would be?		
6. What method would you use to solve the problem?		
7. If you'd give the following problem: Helen has 24 candies and wants to split them up to her 6 friends. What would you do to find out how many candies will each get?		
8. Is there any connection between division and multiplication?		
9. How can you explain the connection?		
10. How can we get to Arithmetic?		

**References**

- Baroody, A. J. (1996).** Self-invented addition strategies by children classified as mentally handicapped. *American Journal on Mental Retardation*, 101, 72-89.
- Barroody, A. J. & Ginsburg, H. P., (1986).** The relationship between initial meaningful and mechanical knowledge of arithmetic” In J. Herbert (Ed.) *Conceptual and Procedural Knowledge: the case of mathematics* (pp 75-112), Hillsdale, NJ: Erlbaum.
- Bird, G. & Buckley, S., (1994).** Meeting the educational needs of Children with Down’s Syndrome. Portsmouth: University of Portsmouth
- Dykens, E. M. , Hodapp, R. M. & Evans, D. W. (1994).** Profiles and Development of Adaptive Behavior in Children with Down Syndrome. *American Journal of Mental Retardation*, 98, 580-587
- Gelman, R., & Cohen M. (1988),** Qualitative differences in the way Down Syndrome and normal children solve a novel counting problem. In L. Nadel(ed.) *The psychobiology of Down’s Syndrome* Cambridge, MA:MIT Press

## Conceptual and procedural strategies in rational number tasks and their relation to ninth graders' approaches to the study of mathematics

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### Abstract

In the present study we examined ninth graders' procedural and conceptual strategies in rational number tasks and their relationship to students' approach (deep/superficial) to the study of mathematics. We found individual differences-sometimes extreme-in the way that students combine procedural and conceptual knowledge for rational numbers. Moreover, it appears that these differences may be associated with differences in students' approaches to studying. Specifically, our findings indicate that students who consistently use only procedural strategies adopt superficial approaches to studying, whereas students who consistently use conceptual strategies adopt deep approaches to studying.

### Theoretical background

The term "number sense" was coined to describe deep conceptual understanding of numbers that goes beyond mere procedural competence and the ability to use this understanding in flexible ways (e.g., McIntosh, Reys, & Reys, 1992; Verschaffel & De Corte, 1996).

The distinction between rational number sense procedural and conceptual components, is grounded on the distinction between procedural and

conceptual knowledge. Procedural knowledge is defined as the ability to execute action sequences to solve problems. On the other hand, conceptual knowledge is defined as the implicit or explicit understanding of the principles that govern a domain and of the interrelations between knowledge units in the domain (Rittle-Johnson, Siegler, & Alibali, 2001). There is a lot of discussion regarding which type of knowledge develops first and there is evidence in favour of contradictory views (Rittle-Johnson et al., 2001).

Hallett, Nunes and Bryant (2010) suggested that a possible explanation for the contradictory findings is that not much attention is paid to the individual differences in the way students combine the two kinds of knowledge.

Schneider and Stern (2010) highlighted another methodological issue: Typically it is assumed that conceptual and procedural components of knowledge in a domain (e.g., rational number sense) can be measured independently of each other via the use of appropriate tasks. It is possible, however, that solutions of conceptual assessment tasks might, to some degree, also reflect procedural knowledge and vice versa. Moreover, for tasks administered in paper-and-pencil tests, it is often impossible to decide whether the student used an algorithm or not.

In the present study, we assessed ninth graders' number rational sense. We used tasks assessing conceptual and procedural aspects of rational number sense as reported in the relevant literature (e.g. McIntosh et al. 1992). However, taking into account the methodological difficulties mentioned above, we designed a qualitative study in order to investigate the kind of strategies that the individual student uses, regardless of the procedural/conceptual nature of the task. Again, we relied on relevant literature to identify procedural and conceptual strategies (Yang, Reys, & Reys, 2007). Procedural strategies are related to rules and exact computation algorithms whereas conceptual strategies rely on conceptual components of rational number sense.

Similarly to Hallet et al. (2010), we hypothesized that there are individual differences in the way students combine the two kinds of knowledge. We predicted that there are extremely different profiles, namely students that consistently follow procedural approaches and fail in conceptual tasks

and students with poor procedural knowledge that successfully apply conceptual strategies.

In addition, we attempted to investigate possible reasons underlying the predicted individual differences. One such reason could be students' mathematics-related beliefs (Op 't Eynde, De Corte, & Verschaffel, 2002). We reasoned that students who view mathematics as a set of rules and procedures to be followed are more likely to belong to a procedural profile than students who view mathematics as a body of notions and procedures that are meaningfully interrelated. We adopted Stathopoulou and Vosniadou's (2007) model that predicts the indirect influence of beliefs on student's understandings via their approach (deep/superficial) to studying. A deep approach to studying involves goals of personal making of meaning, deep strategy use (e.g., integration of ideas), and awareness of understanding or the lack of it. A superficial approach involves performance goals, superficial strategy use (e.g., rote learning), and low awareness of understanding. We hypothesized that procedural profile students follow superficial approaches to studying, whereas conceptual profile students follow deep approaches.

### **Methodology**

#### **Participants**

The participants were seven students at grade nine from schools in the area of Athens. By grade seven, they had been taught all the material related to rational numbers, including fractions and decimals operations, equivalent fractions, comparing and ordering rational numbers, turning a fraction to decimal and vice versa. They had also encountered many problems pertaining to various aspects of rational numbers and had been exposed to various representations, including the number line.

#### **Materials**

We used nineteen rational number sense tasks grouped in three categories. The tasks of the first category (e.g., ordering) could be solved by standard procedures taught at school. The second category included tasks targeting conceptual knowledge, such as translating between representations, and

estimating the magnitude of rational numbers. The tasks of the third category required deep conceptual understanding or the combination of conceptual understanding and procedural fluency. For example, there were tasks targeting the dense ordering of rational numbers and problems that require the understanding of rational numbers as quantitative relations.

In addition, we developed fourteen items so as to investigate the kind of approach (deep/superficial) students adopt in the study of rational numbers as well as of mathematics in general. The items were presented as scenarios describing a situation that the student had to react to (e.g. “*A younger student asks for your help with the comparison of fractions. What would you do to help him?*”).

### **Procedure**

Each student participated in two individual interviews. In the first, the students dealt with the rational number sense tasks. They were asked to think aloud and explain how they reached their answers. About a week later, they were interviewed about their study approaches. Additionally, they were asked to comment on the responses of the first questionnaire (certainty about the solution, awareness of their performance in tasks). All interviews were recorded and transcribed.

### **Results**

Based on students’ responses in all tasks three student profiles were defined. As shown in Table 1, the students in the first profile dealt successfully with all tasks, moving flexibly between procedural and conceptual strategies. The students in the second profile responded correctly to all tasks that could be solved via the application of procedural strategies, but failed when this was not the case. Only one student (hereafter,  $S_1$ ) was placed in the third profile.  $S_1$  used consistently conceptual strategies, sometimes rather advanced, such as such as the use of sophisticated representations. For instance, when asked «*how many numbers are there between  $2/5$  and  $3/5$* », she responded «*If we locate them on the number-line a gap is created. In this gap, there are infinitely many numbers*». However,  $S_1$  failed in all tasks that required procedural fluency (e.g., computations).

Table 1 Students' performance and kind of strategies used in rational number sense tasks

	Performance		
	Profile A (n=3)	Profile B (n=3)	Profile C (n=1)
Rational number sense tasks	Procedural- Conceptual	Procedural	Conceptual
Tasks assessing procedural aspects	Success	Success	Failure
Tasks assessing conceptual aspects	Success	Failure	Success
	Kind of strategies		
Tasks that could be solved with paper-and-pencil	Procedural/ Conceptual (Selection of the efficient strategy)	Procedural	Conceptual

For the second phase of the study, we present results for two students, namely  $S_1$  and one student from group c (hereafter,  $S_2$ ), who had similar school grades.  $S_2$  failed in all tasks that presupposed conceptual knowledge. For example, she estimated that  $3/8$  is closer to  $1/2$  than  $7/13$  explaining that «*the denominator 8 is closer to 2*».

The criteria for identifying  $S_1$  and  $S_2$ 's studying approaches were defined in terms of their goals, strategies, and awareness of understanding (Stathopoulou & Vosniadou, 2007). The analysis of their responses to all items showed that  $S_1$  adopted a deep, whereas  $S_2$  a superficial approach to studying.

Specifically,  $S_1$  valued personal meaning making as a goal (e.g., “*It is important to make sense of what you study in mathematics*”, “*If you understand the meaning of fraction then you can compare fractions*”). On the other hand,  $S_2$  appeared school-performance oriented (e.g., “*What I would*



advise a younger student is to focus on what is likely to be asked in the exams”).

S<sub>1</sub> valued active involvement and strategies aiming at integration of ideas and in (e.g. “*Mathematics is not about rote learning, you have to put your mind to the work*”, “*It is important to solve unfamiliar problems on your own*”). On the contrary, S<sub>2</sub> repeatedly referred to the importance of memorization and described her study habits with expressions such as the following: “*I look at what we have done at school and the exercises, so that I remember how to solve them*”).

Finally, S<sub>1</sub> detected all the rational number sense tasks that she had answered incorrectly and was aware of the fact that she lacked procedural fluency. On the contrary, S<sub>2</sub> was confident that she had answered all the rational number sense tasks correctly and that she had a firm understanding of rational numbers in general.

### **Discussion-Conclusions**

The results presented support our hypothesis that there are individual differences, even extreme, in the way that students combine conceptual and procedural knowledge of rational numbers. Our findings also indicate that differences in the level of rational number sense development may be associated with differences in approaches to studying. This possible relationship, as well as its connection to students’ mathematics-related beliefs, needs to be further investigated.

### **References**

- Hallet, D., Nunes, T., Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology*, 102, 345-406.
- McIntosh, A., Reys, B.J., & Reys, R.E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, 12, 2-8.
- Op ’t Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students’ mathematics-related beliefs: A quest for conceptual clarity and a comprehensive categorization. In G. C. Leder, E. Pehkonen, & G.

- Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp.13-17). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Rittle-Johnson, B., Siegler, R.S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346-362.
- Schneider, M., Stern, E. (2010). The developmental relations between conceptual and procedural knowledge: A multimethod approach, *Developmental Psychology, 46*, 178-192.
- Stathopoulou, C., & Vosniadou, S. (2007). Conceptual change in physics and physics-related epistemological beliefs: A relationship under scrutiny. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Reframing the conceptual change approach in learning and instruction* (pp. 145-164). Oxford, UK: Elsevier.
- Yang, D., C., Reys, R., & Reys, B. (2007). Number sense strategies used by pre-service teachers in Taiwan. *International Journal of Science and Mathematics Education, 7*, 383-403.

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## **Going beyond the classroom's walls: the electronic forum as a lever for an adequate use of the rules of mathematical reasoning**

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### **Abstract**

One of the goals of learning mathematics is the development of reasoning. Closely related to reasoning, the notion of proof is also fundamental in the learning of mathematics. However, in spite of the importance placed on the capacity to reason mathematically, several students are confronted with difficulties during the development of proofs. The study of these difficulties makes it possible to determine important aspects linked to the production or the evaluation of proofs, aspects which play an important part in democracy in the mathematics classroom (for example, the social and the formal aspect). This study examines the impact of the use of an electronic forum on the development of algebraic validation skills with 13 and 14 year old students from New-Brunswick and Quebec (Canada). The results lead us to believe that the use of the electronic forum facilitates an adequate use of the rules of mathematical reasoning.

### **Problem**

Numerous authors agree that the development of reasoning is essential to the mathematical activity (Hanna, 2000; Martin & McCrone, 2001; Ministère de l'Éducation du Loisir et du Sport, 2005; National Council of Teachers of Mathematics, 2005). Greatly linked to reasoning, the notion of

proof is also fundamental in learning mathematics, because it allows the establishment of the validity of mathematical arguments and confers a sense to various concepts through logical explanation of the work done (Martin & McCrone, 2001; Miyazaki, 2000). Indeed, it is through the development of skills related to the learning of proof that students learn to clearly explain why a statement is true (or false) or why a proposed solution leads to a correct answer (McCrone, Martin, Dindyal, & Wallace, 2002; Miyazaki, 2000; National Council of Teachers of Mathematics, 2005). Therefore, it is fundamental for students to develop an "attitude of proof" (Brousseau, 1998), to learn to use mathematics as tools allowing them to accept or to reject propositions and models which are proposed to them. However, in spite of the importance given to the development of various types of reasoning, several students are confronted with difficulties during the development or the evaluation of proofs (Balacheff, 1987, 1999; Duval, 1991; Galbraith, 1981; Healy & Hoyles, 2000; Miyazaki, 2000; Sowder & Harel, 1998; Weber, 2001).

The study of the difficulties encountered by students as well as research relating to the development of various types of reasoning make it possible to determine three requirements to the production of proofs: the social requirement, the temporal requirement as well as the formal requirement. These requirements, more precisely the social and the formal requirements, play an important part in democracy in the mathematics classroom. Indeed, the social requirement takes into account the importance for students to participate in a mathematical community and to negotiate mathematical significance, which is an integral part of the process of developing proofs. The formal requirement asks for students to base their answers on mathematical facts instead of, for example, an authority figure or a general impression (be it of the mathematical content itself or of the person who submitted it).

Within the framework of this research, we are interested in the evaluation of proofs in an electronic forum, an online communication tool which provides an environment for interactions between students and teachers. The electronic forum can thus be viewed as a concrete situation of citizenship. The choice to use this type of technology is based on researches' results showing that the use of such a tool allows the overtaking of certain limits met in the school system (among others, the constraint of time) while taking into account characteristics considered determining for the development of proofs (for example, the social aspect).

### **Theoretical framework**

Several theoretical elements are taken into account in our research. Firstly, the theory of didactical situations (Brousseau, 1986, 1988, 1995, 1998, n.d.) permits us to identify as well as describe the various types of situations (action, formulation, validation and institutionalization) that a student can meet. This theory allows us, among others, to clearly identify the characteristics of the situations of validation, situations which are at the heart of our research. Secondly, the rules of mathematical reasoning (Arsac et al., 1992) guide our analysis of the arguments used by students to (in)validate proofs or to classify them from the most convincing to the least convincing. These theoretical elements lead us to specify our research questions in the following way:

1. What rules of mathematical reasoning do students mobilized, in the classroom (paper-pencil) or in an electronic forum, when they develop proofs linked to an algebraic problem?
2. What rules of mathematical reasoning do students mobilized, in the classroom (paper-pencil) or in an electronic forum, to validate or invalidate a proof or a solution developed to answer an algebraic problem?

### **Methodology**

#### Population

This research project took place in 2009 in four grade 8 classes. We worked with two classes from Quebec and two classes from New-Brunswick (Canada). A class from New-Brunswick and a class from Quebec constitute the experimental group (2 teachers and 57 students) and the two other classes form the control group (2 teachers and 62 students). The presence of both groups permits us to compare the work of students and to observe the influence that the electronic forum can have on that work. The discussions of the control group strictly take place in the classroom while part of the interactions of the experimental group take place in the classroom whereas the other part takes place in the electronic forum. It is important to note that the total discussing time is the same for both groups. During the online discussions, students can comment or question both messages from their classroom colleagues and from students from the other province. Such interactions between students of two provinces allow the overtaking of the classroom's physical barriers.

### Experiment

The experiment takes place in several stages. At first, students from the control group and the experimental group individually answer questions in a pretest (in the classroom). Afterward, four activities pertaining to algebra (each consisted of multiple questions) are completed over a period of four months. These activities are integrated into regular teaching. The work that students have to achieve during these activities is inspired by the work of Arzac and al. (1992). Students have to develop proofs, validate their solutions and those of others and, finally, compare various solutions to the same problem in order to determine the ones that are the most convincing. In short, they have to convince themselves and then convince others. The experiment ends with a post-test that, as the pretest, is completed individually in the classroom. Within the framework of this communication, we limit our attention to the problems where students have to evaluate proofs or order proofs from the most convincing to the least convincing.

### Data analysis

The data collected in the pretest, in the various activities and in the post-test give us a portrait of the work done by students in both groups during the experiment. Traces kept in the electronic forum allow us to follow the work of the experimental group throughout the mathematical activities. To insure the internal coherence of the data, analysis of the information linked to the development of proofs is made from a unique code (a coding where every segment is associated with one code) where the unit of analysis is a sentence, a segment of sentence or a block made of several sentences in which an idea appears. Links are then made between these ideas and the rules of mathematical reasoning.

### **Results and analysis of the results**

On four occasions, students have to order proofs from the most convincing to the least convincing and justify their classification. In all these cases, the solutions presented to the students are fictitious and each type of proof from Balacheff's typology (1987) is represented. The percentage of students of the control group and the experimental group which mobilize a rule of mathematical reasoning to justify their classification is sensibly the same in the pretest and the post-test. Certain differences however are noted with the use that students make of these rules. In spite of the fact that in the two groups, between the pretest and the post-test, it is possible to observe a decline in the percentage of comments which present an adequate use of a rule

of mathematical reasoning and an increase in the percentage of comments which violate a rule, it remains that, in general, students of the experimental group adequately use the rules of mathematical reasoning in a greater percentage than students of the control group. Perhaps the traces of the exchanges in the electronic forum make it possible for the students to become aware of various arguments used in favor or in discredit of a proof. Thus, as soon as they determine the validity of a proof, they are exposed to several justifications which do not necessarily have the same weight from a mathematical point of view. They also have access to messages which evaluate the relevance of these justifications. Since students of the experimental group show a greater adequate use of the rules of mathematical reasoning than students of the control group, it is plausible to think that the electronic forum, because of an exposure to a greater number of arguments and an evaluation of these arguments, allows the development of a greater comprehension of the use which must be made of the rules of mathematical reasoning.

On three occasions, students are asked to validate or invalidate proofs that are fictitious or that have been submitted by their peers. The results show that the percentage of comments in which students mobilize rules of mathematical reasoning to justify their choices are higher with the control group than with the experimental group. However, a more important percentage of comments given by students of the experimental group than by students of the control group use these rules adequately, particularly the one that states that in mathematics, examples which check a statement are not enough to prove that it is true. During the online exchanges, students of the experimental group are exposed to various justifications which aim to validate or invalidate statements or evidence and these justifications are then evaluated by the students. In the light of what precedes, the conjecture according to which the electronic forum, because of the diversity of the messages which are presented in it, has an influence on the use which students make of the rules of mathematical reasoning seems increasingly plausible.

### **Conclusion**

The majority of students from both groups use argumentation, instead of mathematical reasoning, to justify their classification or to (in)validate a proof. However, throughout the experiment, when students do use the rules of mathematical reasoning, it is possible to observe that students of the experimental group show a higher rate of effectiveness than students of the control group. Such results lead us to believe that the electronic forum may

have an influence on the way students view and use the rules of mathematical reasoning.

Moreover, the discussions that take place in the electronic forum allow the observation of phenomena which escape us on paper. Indeed, online, it is possible to notice a certain evolution with regard to the same question. For example, in the electronic forum, some students specify that such or such example seen online allowed them to understand, while the first answer they had submitted was erroneous. A more detailed analysis of the online interactions also allows us to notice an improvement in the quality of the published messages, because the discussions bring students to clarify their solutions. So, although certain data are absent in the electronic forum (compared with the data available with discussions that take place in the classroom), these kinds of online discussions give us access to relevant information which allows us to better understand students' reasoning and which also allows them to better understand.

In conclusion, the electronic forum seems to enrich the situation of formulation by favoring interactions. In certain cases, it seems to represent a lever for the social aspect of proof. Furthermore, on paper, it is strictly the results of the discussions in class that are collected. In the electronic forum, written interactions (which do not consist inevitably in the final result) are recorded. The forum can thus represent an interesting tool for conceptualization. Therefore, it seems relevant to pursue researches in order to continue to explore the influence of the use of an online communication tool in mathematics and more precisely the influence that such a tool can have on algebraic validation skills.

### References

- Arsac, G., Chapiron, G., Colonna, A., Germain, G., Guichard, Y., & Mante, M. (1992). *Initiation au raisonnement déductif au collège*. Lyon: Presses universitaires de Lyon.
- Balacheff, N. (1987). Processus de preuve et situations de validation. *Educational Studies in Mathematics*, (18), 147-176.
- Balacheff, N. (1999). Is Argumentation an Obstacle? Invitation to a Debate. France: Preuve: International Newsletter on the Teaching and Learning of Mathematical Proof.
- Brousseau, G. (1986). *La relation didactique : le milieu*. Paper presented at the de la IV<sup>ième</sup> École d'Été de didactique des mathématiques, IREM Paris 7.



- Brousseau, G. (1988). Le contrat didactique : le milieu. *Recherches en Didactique des Mathématiques*, 9(3), 309-336.
- Brousseau, G. (1995). *L'enseignant dans la théorie des situations didactiques*. Paper presented at the de la VIII<sup>ième</sup> École d'Été de Didactique des Mathématiques.
- Brousseau, G. (1998). *Théorie des situations didactiques*. Grenoble: La Pensée Sauvage, éditions.
- Brousseau, G. (n.d.). La théorie des situations didactiques (à paraître dans *Interaction mathématiques*, Genève). Retrieved 30 janvier, 2006, from [http://dipmat.math.unipa.it/~grim/brousseau\\_montreal\\_03.pdf](http://dipmat.math.unipa.it/~grim/brousseau_montreal_03.pdf)
- Duval, R. (1991). Structure du raisonnement déductif et apprentissage de la démonstration. *Educational Studies in Mathematics*, 22(3), 233-261.
- Galbraith, P. L. (1981). Aspects of Proving: A Clinical Investigation of Process. *Educational Studies in Mathematics*, 12(1), 1-28.
- Hanna, G. (2000). Proof, Explanation and Exploration: An Overview. *Educational Studies in Mathematics*, 44(1-2), 5-23.
- Healy, L., & Hoyles, C. (2000). A Study of Proof Conceptions in Algebra. *Journal for Research in Mathematics Education*, 31(4), 396-428.
- Martin, T. S., & McCrone, S. S. (2001, 18-21 octobre). *Investigating the Teaching and Learning of Proof: First Year Results*. Paper presented at the de la 23<sup>rd</sup> Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Snowbird, Utah.
- McCrone, S. S., Martin, T. S., Dindyal, J., & Wallace, M. L. (2002, 26-29 octobre). *An Investigation of Classroom Factors That Influence Proof Construction Ability*. Paper presented at the de la 24<sup>th</sup> Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Athens, GA.
- Ministère de l'Éducation du Loisir et du Sport. (2005). Programme de formation de l'école québécoise. Enseignement secondaire, premier cycle. Retrieved 20 mars 2006, from [http://www.mels.gouv.qc.ca/DGFJ/dp/programme\\_de\\_formation/secondaire/prformsec1ercycle.htm](http://www.mels.gouv.qc.ca/DGFJ/dp/programme_de_formation/secondaire/prformsec1ercycle.htm)
- Miyazaki, M. (2000). Levels of Proof in Lower Secondary School Mathematics. *Educational Studies in Mathematics*, 41(1), 47-68.
- National Council of Teachers of Mathematics. (2005). More and Better Mathematics for All Students. Retrieved 11 novembre 2005, from <http://www.nctm.org/>

- Sowder, L., & Harel, G. (1998). Types of Students' Justifications. *Mathematics Teacher*, 91(8), 670-675.
- Weber, K. (2001). Student Difficulty in Constructing Proofs: The Need for Strategic Knowledge. *Educational Studies in Mathematics*, 48(1), 101-119.

## **Democratizing pre-service teacher education: The understanding of the institutional context of teaching**

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### **ABSTRACT**

In this paper we report some of our results from a current design experiment. Our overall goal is to support and analyze the learning of a group of seven student- teachers. Based on prior research we planned to support the learning of this group along five directions. This paper focuses on one of these directions and attends to the group's evolving understanding of the institutional context of teaching as it relates to the construction of instructional practices building on students' mathematical reasoning. Student-teachers' attempts to resolve the conflicts between their intentions to support students' reasoning and the instructional priorities of the current institutional context, reveal the significance of supporting their evolving understanding of the institutional context. At the same time, the democratic aspects of their work emerge, as they seek to change the institutional context.

### **INTRODUCTION**

In our mathematics education courses, we aim to support pre-service teachers in developing instructional practices that build on students' mathematical reasoning. We contend that this goal ensures the promotion of democracy in a classroom<sup>1</sup>. A classroom wherein students' reasoning is at the center of instructional decision making, mathematical knowledge is not im-

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<sup>1</sup> Without pretending that we are experts in political philosophy, this contention becomes more plausible, considering Castoriadis' (1995) views on democracy. By resorting to the development of ancient Greek democracy, he documents how this society was the first example on autonomy. As he elaborates his perspective on democracy, he ends up by considering a society as genuinely democratic when it recognizes itself as the creator of its own institutions.

posed but is created by the concerted efforts of the classroom community members. Students do not accept mathematical ideas because the teacher's authority or the textbook has pronounced them, but they build mathematics based on mathematical argumentation. They are supported to constitute a community of validators in which they participate as increasingly autonomous members (Cobb & Yackel, 1996).

Our previous experience in teaching mathematics-education courses alludes that student-teachers who are called to compare this form of instructional practice to the traditional form of teaching mathematics, find it easy to discern its democratic character. More specifically student-teachers, by comparing, for example, video-recorded-students'-interviews attending traditional and nontraditional classrooms, manage to realize the differences in students' mathematical reasoning. In traditional classrooms, students are following rules without reason, while in nontraditional, they show their conceptual understanding. We believe that student-teachers' desire to understand what is involved in a nontraditional form of instruction, is partially motivated by their need to avoid the non-democratic consequences of typical mathematics instruction. However, we find out that the traditional character of the institutional context of teaching in Greece (Boufi & Skaftourou, 2009), engenders student-teachers' worries, as soon as they become aware of their obligations in teaching mathematics, based on their students' interpretations and solutions. They do believe that giving priority to students' thinking will cause the reaction of principals, future teacher colleagues, supervisors, parents, etc. Thus, they seem to expect that a change of the typical instruction will be inhibited. From their experiences as students themselves, they know that the typical mathematics instruction is based upon the fidelity of textbook reproduction.

Student-teachers' worries are reflected in a number of research findings documenting the power of the institutional context. More specifically, in some of these studies the institutional context is not viewed as a pre-designed organization but as emerging from teachers' activities while they participate in the taken-for-granted aspects of school life (Engeström, 1998). By adopting an observer's perspective, teaching is framed as a distributed activity and the institutional context is analyzed through the-identification of the communities involved with the teaching of mathematics as well as the investigation of their interconnections (Cobb, McClain, Lamberg, & Dean, 2003). In this sense, teachers' activities in the classroom are found not merely influenced by but partially constituted by the institutional contexts (Cobb & McClain, 2006; Remillard, 2005; Stein & Spillane, 2005). It

should be noted that analyses of this type can guide the design of institutional changes that promote the reform of mathematics teaching. At the same time, by adopting an actors' perspective, researchers document that the extent to which in-service teachers understand the relationship between the institutional context and their instructional practices is an important aspect of their learning (Dean, 2005). The analyses of these evolving understandings allow researchers/educators to co-participate with teachers in the institutional change.

Apart from our experience as educators, the above research findings indicate the need to consider the institutional context in which student-teachers are going to work. The instructional practices we want them to develop are in conflict with the instructional priorities of the school. Student-teachers, if their field experiences are not appropriately designed, will remain distrustful of the feasibility to change traditional practices in teaching mathematics and as a consequence, their learning will not be sustained (Boufi & Koutromanos, 2012, in press). By assuming, that practicing teachers are the only group student-teachers interconnect, we could intervene and renegotiate the conditions of their field experiences. On the other hand, pre-service teacher development will be ineffective if we do not focus on their evolving understandings of institutional constraints and their influence on the instructional practices they develop. These understandings point to the democratic aspects of pre-service teacher education. Student-teachers' struggle to transcend the institutional constraints is not based on an antagonistic relationship to the practicing teachers. Instead, we support them to understand teachers' actions as reasonable within the institutional context of their work<sup>2</sup>. Both of these concerns lead to the directions of support that we included in designing our pre-service teacher development program.

By employing the design research methodology in supporting and investigating student-teachers' learning, we get involved in a current design experiment (Cobb, Zhao, & Dean, 2009). We are closely collaborating with a group of seven student-teachers, as they are preparing themselves to teach mathematics in a school of Athens. This experiment is in three phases in which student-teachers: (1) teach individual students, (2) observe teachers in the classrooms in which they are going to teach, and (3) teach in the classrooms. In each of these phases, student-teachers' learning is supported along

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<sup>2</sup> Student-teachers' inquiry of the rationality in teachers' actions, reminds us of one of the four interrelated characteristics of democracy, mentioned by Oikonomou (2011). This characteristic is the fair control of all authorities.

five interrelated directions: (1) the understanding of the institutional context of teaching, (2) the building of a community, (3) the development of a mathematical background, (4) the understanding of students' reasoning, and (5) the adaptation of instructional sequences to the students' needs. Our purpose in this paper is to report our findings concerning student-teachers' developing understanding of the institutional context of teaching.<sup>3</sup> In the rest of this paper, we shall first discuss the theoretical framework and methodology we will use for analyzing our data. Then we will present the results of our analysis.

### **THEORETICAL FRAMEWORK AND METHODOLOGY**

In supporting and accounting for the learning of the student-teachers group, we view this learning in collectivist terms. In particular, focusing upon the normative or taken-as-shared understanding of the group, we try to document their evolving awareness of the institutional context and the way it affects the instructional practices they are developing. In the same context, Cobb, Zhao, and Dean (2009) speak about institutional reasoning norms. These norms are not "solo" accomplishments. They emerge as regularities in student-teachers' interactions, while student-teachers participate in activities designed to support the exchange of their ideas about the institutional context.

In presenting their method for developing conjectures about social norms in a classroom, Cobb, Stephan, McClain, and Gravemeijer (2001) emphasize the importance of searching for instances where a student appears to violate a norm in order to see whether his or her activity is constituted as legitimate. In these cases the conjecture has to be revised. Otherwise, if the violation of a norm is constituted as illegitimate activity, the conjecture is supported. In a similar way, we tried to delineate the institutional reasoning norms established by the student-teachers group.

### **SUMMARY OF RESULTS<sup>4</sup>**

As we analyzed the data collected from our teaching experiment, the changes detected in the student-teachers' views of the institutional context

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<sup>3</sup> The data that our results are based upon, consists of the video-recorded work of the seven student-teachers at the school, field notes of the mathematics education sessions held at the University of Athens, the entries of the student-teachers' communication in a specially designed Wiki, notes on our meetings with the student-teachers' group, and our diaries concerning our plans and activities.

<sup>4</sup> A more detailed and complete report of our findings will be given in our presentation.

and its relation on their developing instructional practices, can be considered as an important aspect of their learning.

At the outset of our collaboration, student-teachers did not seem to recognize the constraining power of the institutional context. For example, when negotiating our access to the school, they characterized practicing teachers' reluctance, to accept us in their classrooms, as a sign of their deficits.

In the context of their teaching experiments with individual students from the classrooms they were going to teach, they started to become aware of the complex and demanding nature of supporting their students' learning. At the same time, in their interconnections with the practicing teachers, they realized that the institutional context did not allow them to appreciate their students' work<sup>5</sup>. By acting as brokers, we encouraged the interaction between student-teachers and practicing teachers. However, student-teachers came to doubt the feasibility of teaching mathematics in the practicing teachers' classrooms, by focusing on students' reasoning. The institutional context manifested through the status of the textbooks, as practicing teachers have made it clear, did not seem to be controllable.

By the end of the first phase, student-teachers became familiarized with their students' multiplicative reasoning<sup>6</sup>. In addition, our support to deepen their understanding of the significant mathematical ideas involved in early multiplicative reasoning helped them feel more confident in supporting their students' mathematical development. From their entries in the Wiki as well as from our meetings, we find out that their initial difficulties in supporting students' development were now considered more as a challenge rather than insurmountable. Thus, at the beginning of the second phase of our teaching experiment, they had already developed some criteria for judging the adequacy of pedagogical arguments. As student-teachers visited the practicing teachers' classrooms, they were engaged in several activities we had designed. These activities offered them the opportunity to better understand the problematic aspects of the traditional teaching of mathematics as a consequence of the institutional setting. Moreover, as they identified instances

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<sup>5</sup> Cobb, McClain, Lamberg, and Dean (2003) characterize the interconnections between the communities of practice identified within a school and district by focusing on: 1) boundary encounters, 2) brokers, and 3) boundary objects. In our case, students' work from the student-teachers' teaching experiments was designed to function as a boundary object.

<sup>6</sup> Each of the student-teachers met seven times with the student he or she was teaching. Five of the students were second graders and two of them third graders. All of the sessions focused on the topic of multiplication and division.

in which students' reasoning was not capitalized upon, they accounted for practicing teachers' ineffectiveness as a matter of the institutional constraints or the lack of effective assistance. More significantly, they could propose ways of supporting students' learning. In addition, when preparing themselves for teaching, they seemed to be in a position to anticipate difficulties due to the students' routine ways of acting in their classrooms and figured out ways to overcome them. Thus, the institutional context was no longer beyond their control.

As student-teachers were involved in the third phase of our teaching experiment, they came to perceive institutional changes not as just feasible, but accomplishable too. During the second phase, student-teachers collected data concerning students' multiplicative ideas. These data came from a written test and an oral assessment that took place in students' classrooms. Based on these data, student-teachers were in a better position to conjecture a learning path for students' further learning and the means to support them along this path. As they were teaching for four consecutive days, they acted as designers in adapting the textbooks and the tasks they had already used with their individual students to the needs of the classroom. In this process they had help from us. After each lesson, we had meetings with the student-teachers. In these meetings, we focused our discussions on crucial classroom events and used them as a means to support them in their attempts to adapt their instructional moves to their classroom situations. On the other hand, in the final discussions student-teachers had with the practicing teachers, it became apparent that the complexity of their attempts to enact alternative norms and practices in their traditional classrooms was not appreciated by them. This lack of appreciation did not discourage student-teachers. With our advices, they kept sharing their plans with the practicing teachers and explaining to them the rationale of their teaching. As a consequence, practicing teachers who were observing them did not intervene or impede their teaching in their classrooms, thus giving student-teachers the opportunity to assume that the institutional context of teaching can be changed through informing, discussion, and negotiation.

### CONCLUSION

From student-teachers' discussions in the Wiki and in our meetings, we noticed that there was an evolution in their understanding of the institutional context of teaching mathematics and its relationship to their developing teaching practices. Thus, as teacher educators, we have to understand the institutional context of teaching mathematics and support student-teachers'



learning of its constraining power in their attempts to develop teaching practices based on students' reasoning. Through a democratic process of resolving the conflict between the instructional priorities of a traditional institutional context and the goals of nontraditional teaching, student-teachers will have a chance to overcome its non-democratic status quo.

### REFERENCES

- Boufi, A., & Koutromanos, G. (in press). Supporting future teachers to teach mathematics based on students' reasoning: The understanding of the institutional context of teaching. In *Proceedings of the Panhellenic Conference for the Quality in Education: Trends and Prospects* [in Greek].
- Boufi, A., & Skaftourou, F. (2009). Anticipating teachers' learning within the institutional setting of their work. In Tzekaki, M., Kaldrimidou, M., & Sakonidis, H. (Eds.), *Proceedings of the 33<sup>rd</sup> Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 201-208. Thessaloniki, Greece: PME.
- Castoriadis, C. (1995). *Human domains*. Ypsilon Editions [in Greek].
- Cobb, P., & McClain, K. (2006). The collective mediation of a high stakes accountability program: Communities and networks of practice. *Mind, Culture, and Activity*, 13, 80-100.
- Cobb, P., McClain, K., Lamberg, T., & Dean, C. (2003). Situating teachers' instructional practices in the institutional setting of the school and school district. *Educational Researcher*, 32(6), 13-24.
- Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, 10, 113-164.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31, 175-190.
- Cobb, P., Zhao, Q., & Dean, C. (2009). Conducting design experiments to support teachers' learning: A reflection from the field. *Journal of the Learning Sciences*, 18, 165-199.
- Dean, C. (2005). *An analysis of the emergence and concurrent learning of a professional teaching community*. Unpublished doctoral dissertation, Vanderbilt University, Nashville, TN.
- Engeström, Y. (1998). Reorganizing the motivational sphere of classroom culture: An activity theoretical analysis of planning in a teacher team.

- In F. Seeger, J. Voigt and U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp 76-105). Cambridge University Press.
- Oikonomou, G. (2011). The conditions of direct democracy. In G. Oikonomou (Ed.), *The birth of democracy and today's crisis* (pp. 59-98). Euroasia Editions [in Greek].
- Remillard, J.T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, 75(2), 211-246.
- Stein, M.K., & Spillane, J.P. (2005). Research on teaching and research on educational administration: Building a bridge. In B. Firestone & C. Riehl (Eds.), *Developing an agenda for research on educational leadership* (pp. 28-45). Thousand Oaks, C.A: Sage.

## COULD STUDY AND PLAY OF CHESS IMPROVE SOCIAL INTERACTIONS? REPORT OF AN ITALIAN CASE STUDY.

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### Abstract.

*Aim of this work is to improve the social interactions within a math classrooms (6<sup>th</sup> grade), introducing a chess activity during the curricula hours.*

*The theoretical framework upon which this research is based consists of: Vygotsky's Theory of child development (Vygotsky, 1986), the knowledge objectification theory (Radford, 2006) and theory of configural concepts (a personal review of Fischbein's theory of figural concepts).*

*We will discuss the results of an experimentation that has the purpose to create an appropriate environment where the students develop the abilities to solve and pose problems.*

### 1. Introduction

This work born from the conviction that all didactical proposals, in math education, must consider the relationships within the classroom, and how it's possible enhance social interactions. What we will want observe with this work, is how a collaborative activity (and also individual) in a mathematical context could be improved by a collaborative activity in a chess context.

As proposed by D' Amore (D' Amore, 2004) will be considered 4 situations of thinking:

- Autonomous situation with motivation and volition (SA<sub>m</sub>);
- Collaborative situation with motivation and volition (SC<sub>m</sub>);
- Autonomous situation without motivation and volition (SA<sub>n</sub>);
- Collaborative situation without motivation and volition (SC<sub>n</sub>).

The choice of chess activity was due to the fact that this discipline create an highly motivating

context in which the students deal with problem solving and problem posing activities. Furthermore, chess elements like geometrical ones, are not purely conceptual or figural elements, but mental elements that sharing similar conflict and difficulties in recognize and handling (Fischbein, 1993).

The motivation to do chess activities don't lies only on game nature of this discipline, but also because it could be a "bridge" between the family and classroom.

In fact, the diffused idea that chess is a great training for the mind, can push parents to encourage their children to participate actively in those activities. This could also be a way for parents to "believe more in the school" and especially to have arguments to share with their children.

Thus, chess activity could be improve the language between parents and students in an age in which this could be very difficult.

## **2. Chess and zone of proximal development**

Entering in the epistemological nature of the game of chess, we want to extend the Vygotsky's paradigm to the learning phases of chess, distinguishing between playing and studying chess.

As mentioned above, the problem-solving activities in a chess context, can be considered a condition in which the students work with motivation and volition.

*How this, could help the learning/teaching activities in a geometrical context?* To answer to this question, we want to enter deeply in the cognitive activity of a student during playing and studying chess. In this paper we want to summarize some key points on these processes.

### *Play chess*

When an individual play a chess game, he/she will brings knowledge and competences, all studied rules (or create by himself/herself) and all emotional choices that led him/her to promote or exclude certain variations.

For us this is the *actual level of individual cognitive development*.

Fundamental feature of the chess games is that there are two players, and this obvious considerations will allow us to define the zone of individual potential development.

In fact, this can be identified by the meeting of the two zones of actual development. With the term "meeting" of course we aren't referring to the mere union of the two levels of development, but we will re-

fer to what can be produced by the interaction of knowledge, competences and emotions used by the players to win the game.

For example, when a player  $P_1$  (the "strongest") and a player  $P_2$  (the "lowest")<sup>1</sup> play a game, they create a zone of potential development that is valid for both players.

This *zone of potential development* will depend by the actual development zones of the two players, by motivations and expectations of the players that are playing that game (tournament game, friendly game, game between teacher and pupils), by the psychological situation of each of them (presumption, concentration etc.) and also by their age.

The zone of potential development, and also the proximal one, will be generated by the two contenders (peers) that to win the game, they produce knowledge and/or skills that they haven't before, or they had but they didn't know when and how use them.

In this paper we will not dwell on the various dynamics that can exist between the two players and their zones of development during the game, but we deal with those dynamics which include the language as a mediator between the actors, and so the study activities.

### *Study chess*

In this section we consider the collaborative phases in chess studying in classroom activities or between two chess players who have finished a game.

During the chess lessons, similar to those of mathematics, the teachers are attempting to provide knowledge and skills, computational techniques and criteria for evaluating positions. But there is a big difference between the two disciplines, in chess, the concept of "best move" isn't always unique.

In fact, during the game, is usual try to search the best move or best variant, but many times happens that the players restricts his research to find a good move or however a not bad one.

Because this determination is more subjective than objective, how we define the best move during the time of study involving the interaction of many individuals?

We define best move (or best variant), one that is socially and democratically accepted by the collectivity, which is understood by all individuals and that could modify the zone of effective development of them.

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<sup>1</sup> It is supposed that  $P_1$  have a greater zone of actual development than  $P_2$

This definition covers the discussed moves during the lesson, but also the not discussed one, which will give safety to every student through the silence of the remaining collective.

So, fundamental moment for a chess player, is the comparison of ideas and analysis not only with the rest of the class or with its game opponent, but also with himself. This is not a “paradox”, in fact when a chess player ends a game and analyze it, it’s possible that he/she doesn’t recognize himself how the moves maker. He / She doesn’t understand the reason of certain moves or how he/she does some mistakes. This phenomena depends by the emotional sphere of the game, that doesn’t afflict the player during his/her analysis.

In fact, for us, one of the main skills of a chess player is to ask to himself (and to other people) question of a metacognitive type, to accept criticism and thus to extend the reflection activity not only to the game, but also to its analysis and sharing them with the context.

Thus, using the previous considerations we can say that every chess games could be thought as an “anticipated exercise”.

### 3. Case Study

In this section we will discuss an experimentation that have the purpose to create a learning environment in which the students develop the abilities to solve and understand increasingly challenging problems in chess and geometry.

These activities were born from the idea that certain individual skills (like the democratic access to some ideas by the learners, the cognitive abilities or the metacognitive ones) are shareable by similar (cognitively speaking) disciplines.

But, why have we chosen chess? There are 2 great reasons:

- a) During chess activities the students do several intellectual and social activities:
  - They compete between them;
  - They pose and solve problems;
  - They create and evaluate strategies;
  - They have funny;
  - Etc...
- b) Chess and Geometry are (cognitively speaking) similar disciplines, because they deal with something different by mere concepts or figural

representations, they deal with something that we have defined *Configural Concepts*<sup>2</sup> (Ferro, 2012a; 2012b).

So, in this experimentation, we have proposed a chess activities (6 lessons of 4 hours for each of them) to a math classroom of 6<sup>th</sup> grade (24 students).

The activities included a progressive synergy between chess and math activities, in which we propose both types of tasks. It provides three steps:

1. Proposing of geometrical tasks in collaborative situations (group of 3 or 4 students);
2. A chess activity that gradually become geometrical activities;
3. Proposing of geometrical tasks in collaborative situations (group of 3 or 4 students).

In line with the theoretical framework of objectification of knowledge<sup>3</sup> (Radford, 2003) we have analyzed the activities using a semiotic approach and then we have video-recorded them.

In first step of experimentation we have observed that students had some difficulties in collaborative activities to solve geometrical problems. In fact, often, there was one students that take the task paper, read it, think and give him/her answers for the problem.

The chess activities provides 4-5 hours for teach the fundamental rules of chess. After, we have proposed autonomous situations in which the students try to solve easy chess problems with the aim to trust in themselves. So, we have proposed collaborative situations to solve chess problems starting from tasks of the previous difficult level (easy) and we have observed that in more cases they don't solve problems that they can solve autonomously.

In fact, when more individual (more than one) look at a position their judgment depend by their perspective about the chessboard (Ferro, 2012a),

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<sup>2</sup> A configural concept consists of a networking of knowledge about an object, whose meanings depends on the configuration of its parts, including the relationships that an individual uses to perceive and explain them.

The organization of the parts of a configural concept depends on the objective of the active reflection of the problem; this organization is mediated by artifacts, body, language, and signs.

<sup>3</sup> According to which thinking is above all a form of active reflection on the world, mediated by artifacts, from the body, language, signs, etc..

and so it's possible that they haven't different or conflicting ideas to solve the problems.

To enhance their collaboration we say them that every group find out one answer, and this will be compared with the other groups ones. After some lessons (3 lessons and so about 5-6 hours), we have seen that they collaborate actively during the task, but also when they talk about the games that they play in "free time" (in the last hour of every lesson they play free).

Thus, we have gradually expanded the groups (3 – 5 – 10 students per group) and introducing some geometrical task. What we have seen is that they use the same (democratically and respectful) way to discuss about the problems in the geometrical task.

The last two (geometrical) lessons were with one group, the totally of the classroom, in which they try to solve the geometrical tasks with a strong collaboration and a respect for the answers of the other students. Furthermore, to find the answer to give to the teacher, they make a dense network of questions of a metacognitive aspects about their mistakes and results, improving their language and critical thinking.

A last result was obtained from the discussion of the answers, indeed, when the teacher discuss the wrong answers, the students that want to give another answer (maybe the right answer), don't have denied the classroom answer, but at most they were perplex because they don't understand the reason of their changing of evaluation.

#### **4. Conclusions**

In this work we have shown an experimentation in which we want to see if chess activities may affect the ability of the student to structure questions of a metacognitive type in other contexts, and in particular in the geometrical one.

We have chosen chess because it generate a context an highly motivating context, in which the students could improve social interactions through collaborative and autonomous situations.

Thus, it was created a motivating learning environment in which we have focused our attention on the importance of strategy in chess and geometry.

The students have collaborated to find the correct variation (in chess) and the correct construction (in geometry). So, we have enhanced their critical process aging on what Duval (Duval, 1998) calls operative apprehension, that is a cognitive process that involves the operations on the figure (mental or physical) that give "insight" on solving a problem.



In these activities, fundamental is the role of the democratic access to that ideas that the students need to solve the problems.

They learnt that the social interaction between them is a strong feature of the classroom, and in this way they become firstly learners and then citizens.

### References

- D'Amore B., Fandiño Pinilla M.I., Marazzani I. (2004). "*Esercizi anticipati*" e "*zona di sviluppo prossimale*": *comportamento strategico e linguaggio comunicativo in attività di problem solving*. *La matematica e la sua didattica*. 2, 71-95.
- Duval, R (1998), *Geometry from a Cognitive Point of View*, En C. Mammana and V.Villani (Eds.) *Perspective on the Teaching of the Geometry for the 21<sup>st</sup> Century* (pp.37-51). Dordrecht, Netherland: Kluwer Academic Publishers.
- Ferro M., (2012a). *Chess thinking and configural concepts*. *Acta Didactica Universitatis Comenianae – Mathematics* (In press)(ADUC-M).
- Ferro M., (2012b). *Concetti configurali: un nuovo modello per l'insegnamento/apprendimento della geometria*. QRDM, QUADERNI DI RICERCA IN DIDATTICA MATEMATICA. (In press)
- Fischbein, E. (1993). *The theory of figural concepts*. *Educational Studies in Mathematics*. 24(2). 139-162. Kluwer Academic Publisher.
- Radford L. (2003). *Gestures, Speech, and the Sprouting of Signs: A Semiotic-Cultural Approach to Students' Types of Generalization*. *Mathematical Thinking and Learning*. 5(1), 37-70.
- Radford, L. (2006). *Elements of a cultural theory of objectification*. *Revista Latino americana de Investigación en Matemática Educativa*, 9,103 – 129 (Special issue on semiotics, culture and mathematical thinking. Available at: <http://laurentian.ca/educ/lradford/>).
- Vygotsky, L.S.(1986). *Thought and language*. Cambridge, MA: MIT Press.

**ICT enhanced Data and Graphs comprehension activities in the kindergarten. Preparing the citizens of modern democracies.**

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**Introduction**

Data analysis, graph construction and sense making constitute important competences for every citizen in modern society. Public opinion polls and election results in democratic societies produce large data sets that are often presented in the form of graphical synopses, revealing the direct relationship between graphs and democratic functions. Furthermore, printed and electronic media (e.g. newspapers, magazines, television programs, internet sites, advertisements, research reports) convey data graphs daily, affecting citizens' opinions and decision making. Additionally, in business management, decision making based on graphically presented, up-to-date data is more and more crucial. Finally, in science, data processing and graphical representation of variables are basic methodological components. Data analysis and graphs are key competences for advanced Statistics, making them important for social sciences as well.

Graph comprehension, construction and use, in general, are included in fundamental adult literacy and especially in document literacy (OECD, 1995) which is defined as "*the knowledge and skills required to locate and use information contained in various formats, including ... tables, and graphics*" (Murray et al., 1997, p. 17).

This paper investigates the possibilities of developing data modeling and graphing competences of young children in the kindergarten. In contrast to the claims of Inhelder and Piaget (1964) that preschoolers skills in data clas-

sification and seriation are very pure, more recent researchers show that even very young children can successfully engage in related learning activities provided these are authentic and interesting (Macnamara, 1982). Data processing and graphs have significant role in many Mathematics curricula worldwide even in PreK-2 stage. As an indicative example, in NCTM, 2000, pp. 108, the expectations for the Pre-K2 stage state:

*In prekindergarten through grade 2 all students should:*

- *pose questions and gather data about themselves and their surroundings;*
- *represent data using concrete objects, pictures, and graphs.*

As far as the learning activities are concerned in NCTM, 2000, it is proposed that students should collect, organize and analyze data from their real life experiences in order to answer questions related to their interests. Data analysis and graph processing provides opportunities for kindergarten children to apply several mathematical concepts and skills such as counting, classifying, comparing. Furthermore, since data can be drawn by using themes from various disciplines (e.g. social science), graphs can be applied in interdisciplinary learning activities.

Graphs and information processing appears also in ICT educational standards. For example ISTE National Educational Technology Standards for Students, 2007, states:

*Students apply digital tools to gather, evaluate, and use information.*

*Students:...*

*b. locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media. ...*

*d. process data and report results.*

This reveals that data analysis and graph processing is a field that ICT and Mathematics education naturally meet and questions rise for both research communities.

### **Educational research about graphs comprehension**

Data processing and graphs have attracted the interest of many researchers. For example Friel et. al., (2001), presents a study about critical factors influencing graph comprehension and instructional implications. The paper also offers an extended review of the related researches. As far as the learning activities are concerned, the researchers propose that students should actively collect, organize and analyze authentic data and construct simple graphs in order to support their thinking. They also propose a three stage

model of graphs comprehension. The first level “*reading the data*” refers to the ability to decode basic information depicted directly on the graphs. The second level “*reading between data*” refers to the ability to make comparisons and/or operations on the data presented in the graphs with the aim of finding relationships (e.g. the most, the least, greater than, more often etc.). The last level “*reading beyond data*” involves trends finding, predictions making, and in general using the information derived from the graphs to answer relevant questions. More recently Monteiro, & Ainley, (2003), emphasize in critical sense making of graphs permitting citizens to quest on the reality behind and beyond the graphs.

According to the researchers (Friel et.al., 2001; Curcio 1987) in kindergarten the graphs could be invented by the children or be conventional picture, bar, line and circle graphs. Kindergarten children should build graphs at the beginning by using real objects and then gradually by proceeding to more abstract forms like painting tiles on grids or software. The teacher should both, guide children by posing questions for the interpretation of graphs and encourage children as well to formulate their own data collection problem cases and questions.

### **ICT and graphs**

Data analysis and modeling in education using ICT has been studied by researchers since 1990 (Hancock & Kaput, 1990; Hancock, et. al., 1992). The researchers primarily aimed to the design, specification and development of software suitable for children and learning. Tabletop and TabletopJr software are two well-known outcomes of those efforts. Tabletop permits the design of simple data tables and the production of corresponding graphs; since the use of Tabletop software requires writing and reading, it is not developmentally appropriate for young children. TabletopJr is aimed to kindergarten children but does not support the manipulation of user defined data. Modern ICT technology (e.g. general purpose software as well as especially designed educational software e.g Tom Snyder Productions/Graph Club) permit the easy and quick production of graphs that are based on data defined and collected by children. Furthermore ICT tools enable the easy translation of one graph type to others or the production of several forms of graphs for the same data set. In most cases graphs are dynamically connected to the data, meaning that any changes on the data tables are reflected on the graph and vice versa, any changes on the parameters of the graph (e.g. the height of a bar), will accordingly change the data set values. A first impact of ICT in graph processing is the time saving that gives more oppor-

tunities for concentration on the interpretation of graphs answering questions about the data instead of spending time for graph construction. ICT can enhance both purposes of graphs: analysis and communication (Kosslyn, 1985).

From the research review it appears that more studies on the use of ICT towards the direction of enhancing data modeling and graphs comprehension in kindergarten need to be performed. There are many questions open especially on the interaction of modern ICT tools with the real object activities and the children responses to such learning experiences. There is also the need for specific case studies in order to help teachers design, adapt and successfully implement learning activities in their specific class conditions.

### **Aim**

The aim of this paper is to consider the research directions that are open with regard to the integration of ICT enhanced learning activities for data modeling and graph comprehension in authentic and real kindergarten conditions. More specifically, the paper presents research design ideas for discussion in the conference along with the preliminary results that will be available by the date of the presentation.

The proposed research will examine the following questions: What are the available types of software for kindergarten graphing? Are they consistent to the research results available? How could software tools for graphing be integrated in learning activities for real kindergarten conditions? Are there any difficulties for the children and/or the teachers during implementation of such activities? How do children interact during these learning activities? Does the software affect children interaction quality? Is there any impact of using Interactive White Boards during graphing in kindergarten because of the participation of the whole class? Does ICT anyway help children engage more effectively in graph processing and enhance their learning? Are there any new features of the graphing software that could help enhance children learning and interaction?

### **Methodology**

The research questions will be approached using case studies in real kindergarten. Initially the participants will be assessed about their graphing knowledge and their graph comprehension level. A series of learning activities will be designed for the use of real objects and ICT combination in graphing. Then the children, with the guidance of teacher, will implement the activities. The activities will be based on authentic scenarios for decision

making and children will be observed in all phases. Graph construction and comprehension will be assessed using children products and answers to questions, as proposed by other researches. Finally the software use by the children will be analyzed in order to find out any new features that would make it more efficient for the kindergarten children's developmental level.

### Summary

The paper discusses possible research directions on enhance learning of data and graphs in kindergarten using ICT. The results will be of interest to researchers studying ICT enhanced learning approaches, educational data modeling, statistics education and graphs sense. In addition the results could also prove useful to teachers' educators, teachers wishing to work with data and graphs, and software designers that want to investigate new features for this kind of software.

### References

- Curcio, F.R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education*, 18, 382-393.
- Friel, S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education*, 32(2), 124-158.
- Hancock, C., & Kaput, J. J. (1990). Computerized tools and the process of data modeling. In G. Booker, P. Cobb, & T. N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference* (Vol. 3, pp. 165172). Mexico City: International Group for the Psychology of Mathematics Education.
- Hancock, C., Kaput, J. J., & Goldsmith, L. T. (1992). Authentic inquiry with data: Critical barriers to classroom implementation. *Educational Psychologist*, 27, 337-364.
- Inhelder, B. and Piaget J. (1964). *The Early Growth of Logic in the Child: Classification and Seriation*, London: Routledge and Kegan Paul
- International Society for Technology in Education (ISTE). (2007). *National educational technology standards for students* (2nd ed.). Washington D.C.: International Society for Technology in Education.
- Kosslyn, S. M. (1985). Graphics and human information processing: A review of five books. *Journal of the American Statistical Association*, 80, 499-512.

- Macnamara, J. (1982), *Names for things: A study of Human Learning*, Cambridge, Massachusetts: MIT Press
- Monteiro, C. & Ainley, J. (2003). *Developing Critical Sense in Graphing*. Proceedings of III Conference of the European Society for Research in Mathematics Education. available at [http://fibonacci.dm.unipi.it/cluster-pages/didattica/CERME3/proceedings/Groups/TG5/TG5\\_monteiro\\_cerme3.pdf](http://fibonacci.dm.unipi.it/cluster-pages/didattica/CERME3/proceedings/Groups/TG5/TG5_monteiro_cerme3.pdf)
- Murray, T. S., Kirsch, I. S., & Jenkins, L. B. (1997). *Adult literacy in OECD countries: A technical report for the first international adult literacy survey*. Washington, DC: National Center for Education Statistics, U.S. Government Printing Office.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*, Reston, Va.:NCTM
- Organization for Economic Co-operation and Development (OECD). (1995). *Literacy, economy, and society: Results of the first international adult literacy survey*. Ottawa: Statistics Canada.

## **Can digital games democratize access to mathematics learning? Tracing the relationship between learning potential and popularity.**

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### **Introduction**

Digital games are considered to be an attractive and popular activity to both young and older children (Prensky, 2003). As a consequence the educational community has showed interest over the impact of ICT in general and of the digital games especially to children's learning and development. The issue proved quite controversial. Healy (1998) argued that the extensive use of PC's from preschool children poses numerous dangers to the development of their creativity and mental abilities in general. On the other hand, zealous proponents of digital games, as Gillen and Hall of Manchester's Metropolitan University, argued that even violent games can benefit children's creativity (Slater, 2001) showing some kind of pedagogical thoughtlessness. Papert (1996), having adopted a more pragmatic view, claims that the real question today is which the appropriate uses of ICT are for children, in order their extensive exposure to ICT's to be beneficial for them. Barab et al., (2007) advocates that if we exploit the appeal that video games have upon children using them as medium of education, we could develop competences required for the modern citizens e.g. decision making, problem solving, mathematical communication etc.

Moreover, the widespread use of internet and its use as distributor of digital games enable more and more children to access them. This fact sup-



ports the assumption that educational digital games could democratize the learning of mathematics. It is a common belief that learning of mathematics is hard and the providing of high quality mathematical education is a privilege for few. On the contrary, today it seems that using digital games designed for learning and which are accessible through the internet could deliver high quality mathematical education to vast population of children, overcoming didactical and financial obstacles.

In this paper we examine the possibility of detecting mathematical learning potential of digital games, whether they have been designed for learning or not. Moreover, we are investigating the relationship between the learning potential and the popularity of games. Results show that the popularity and availability can be inversely proportional to the mathematical learning potential in some games. Consequently, the role of educators proves to be essential for the utilization of the appropriate games that could help students develop mathematical learning.

What follows is a brief review of digital games for learning mathematics. Thereafter, we describe “applied ludology”, the method that was used for the detection of the learning potential of digital games. The paper ends with the case study of the detection of learning potential of two specific digital games in order to assess its efficiency and to investigate the relation among popularity and learning potential.

### **Digital games for mathematical learning**

The positive effects of the use of ICT’s in mathematics learning have been supported by several researchers (Battista & Clements, 1984; Clements, 1987; Yelland, 1999). Early research on the use of digital games in schools recorded that the majority of them were drill and practice games (Clements et al, 1993). More recent studies argue that digital games promote understanding of mathematical concepts, while their structure shapes the quality of interactions among children that use them jointly. Digital games can develop mathematical skills like graph’s understanding (Gros, 2007) and problem solving (Squire, 2005).

There are also well known projects about digital games in mathematical learning, e.g. as Electronic Games for Education in Math and Sciences, EGEMS (<http://www.cs.ubc.ca/nest/egems/index.html>) of Canadian University of British Columbia or Mathematical and Equitable Game Software (MEGS, 1996-1999) (Rubin,1999) (<http://mathequity.terc.edu/>). Those projects have studied games design so as to integrate mathematical learning and have also produced some successful games that are still on use (e.g. The

logical journey of Zoombinis) (Hancock & Osterweil, 1996). Nevertheless, researchers argue that there is still much work to be done (Yelland, 2002), especially in designing specialized learning games in combination with the development of modern curricula.

### **Research rational**

Taking into account the above remarks it appears purposeful to find methods for the detection of the learning potential of games. These methods will be able to inform us, beyond the explicit intentions of the manufacturers, about what can someone expect from a particular game in terms of learning.

### **Detection of the learning potentials of the game - The applied ludology method**

In this paper we adapt “applied ludology” (Jarvinen, 2007), a method developed especially for the content analysis of digital games. We present its application in specific games in order to detect their mathematical learning potential. Our aim is to evaluate its applicability and to investigate the relationship among learning potential and popularity of the games. The method admits that a) any kind of game can be identified through a limited number of structural features called game elements and b) the experience of playing a game can be analyzed with a set of “psycho-ludological” concepts.

Applied Ludology includes a toolset of seven methods known as Rapid Analysis Method (RAM), of which we have used the followings:

*A) Game elements identification:* In order to understand how a game works we must find out what are the parts of the system, the game elements.

*B) Game mechanics and goals identification:* Game mechanics are referred to the actions that a player has to perform through the game. They correspond to verbs: choosing, shooting etc. The analysis of game mechanics and their classification to primary mechanics, submechanics and modifier mechanics can help us detect and classify game’s goals.

*C) Player ability sets identification:* Aims to the definition of the required player ability sets through their experiences while playing a game. In particular, we investigate the uncertainty factors which are defined as: the abilities of a player that contribute in the uncertainty of his successful use of game mechanics.

### **The case of RAM on “Super Mario Bros” and “Lure of the Labyrinth”**

For the purposes of this analysis we have chosen two digital games a) *New Super Mario Bros*, as well known popular game, which although hasn't been designed for educational purposes, it is considered to be simple enough and can help to the easy understanding of the analysis method b) *Lure of the Labyrinth*, an educational digital game for mathematics, in order to find out whether the RAM method can detect the content argued by the game's designers.

We have adapted RAM's third method where we specified player's abilities set as uncertainty factors which derive from Carroll's (1993) model of psychomotor human abilities. For the presentation of the mathematical required player's abilities we used NCTM's Standards (2000).

### **Results – Discussion**

Summarizing the application of RAM for the learning potential detection of the two games we notice:

a) Learning content of “New Super Mario Bros” is considered to be suitable for children of age 3-6 years old. The competences of the players which have been detected are considered as conquered for the majority of the oldest pupils. This is a digital game which promotes partly several very simple mathematical skills, e.g. number concept, counting, sense of time, simple orientation abilities.

b) Regarding the game “Lure of the Labyrinth” and particularly the puzzle “Manager's cafeteria” we found that requires the development of significant mathematical learning e.g. fractions multiples, prime factorization, understanding and using ratios and proportions to represent quantitative relationships. In addition it helps children understand concepts significance in everyday activities using engaging and realistic story plot. Lure of the Labyrinth promotes high order skills such as problem solving, decision making but also the collaboration among teams.

Through this particular research of games it can be seen that mathematical learning potential can be conversely proportional to the popularity, thus the educational system is responsible for choosing to get children in contact with the appropriate game in order to ensure equal opportunities for the learning of mathematics. Internet can provide access to games for vast children population but the selection of games with high quality learning potential has to be supported by appropriate educational structures. The application of learning potential detection methods, like applied ludology, can help educators, parents, and guardians to ensure that the engagement of the chil-

dren with digital games will not be only mere entertainment but also learning.

In the future, we would like to inquiry how content analysis can contribute to learning/instructional design and to the establishment of design principles and methods for the development of educational games. Using content analysis it is possible to find correlation of games' features to their learning results.

### REFERENCES

- Battista, M. & Clements, D.H. (1984). A Case for a Logo-based Elementary School Geometry Curriculum. *Arithmetic Teacher*, 36, 11-17.
- Barab, S.A., Bransford, J.D., Greeno, J.G., Gee, G.P. (2007). Embodied cognition : A more meaningful ontological unit . *Proceedings of the 2007 AERA Annual Meeting*.
- Clements, D.H. (1987). Longitudinal Study of the Effects of Logo Programming on Cognitive Abilities and Achievement. *Journal of Educational Computing Research*, 3, 73-94.
- Clements, D.H., Nastasi, B. & Swaminathan, S. (1993). Young Children and Computers: crossroads and directions from research. *Young Children*, 48(2), 56-64.
- Gros, B. (2007). Digital games in education: The design of Games-Based learning environments. *Journal of Research on Technology in Education*, 40(1), 23-38.
- Hancock, Chris & Osterweil, Scot. (1996). Zoombinis and the Art of Mathematical Play. Originally published in *Hands On!* 19(1). Cambridge, MA: TERC.
- Healy, J., (1998). *Failure to Connect: How Computers Affect Our Children's Minds for Better and Worse*. New York: Simon & Schuster.
- Jarvinen, A. (2007). Introducing applied ludology: Hands-on methods for game studies. *Proceedings of Digra 2007 Conference*, 134-144. Retrieved from <http://www.digra.org/dl/db/07313.07490.pdf>
- NCTM. (2010). *Principles and Standards for School Mathematics*. Retrieved from <http://www.nctm.org/standards/content.aspx?id=4294967312>
- Papert, S., (1996). *The connected Family: Bridging the Digital Generation Gap*. Atlanta:Lonstreet Press.
- Prensky, M., (2003). Digital Game-Based Learning. *ACM Computers in Entertainment*, 1(1), 1-4

- Provelengios, P., Fesakis, G. (2011, October). *Educational applications of Serious Games: The case of the game "Food Force" in primary education students*. Paper presented at "5<sup>th</sup> European Conference on Games Based Learning". Greece.
- Rubin, Andee. (1999). *What Is MEGS? Through the Glass Wall Website*. Cambridge, MA: TERC.
- Slater, J., (2001). *Violent games spark creativity*. Retrieved from: [http://www.tes.co.uk/search/story/?story\\_id=353505](http://www.tes.co.uk/search/story/?story_id=353505)
- Squire, K. (2005). Changing the game: What happens when videogames enter the classroom? *Innovate: Journal of Online Education*, 1(6).
- Yelland, N. (2002). Playing with Ideas and Games in Early Mathematics. *Contemporary Issues in Early Childhood*, 3(2), 197-215.
- Yelland, N. (1999). Reconceptualising Schooling with Technology. *Information Technology in Childhood Education*, 1, 39-59.

## **Space-discourse relationships in the classroom: towards a multi-semiotic analysis**

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### **Abstract**

Drawn on the notional system of the Systemic Functional Linguistics (Halliday 1978) and on “Spatial Pedagogy” (Lim and al. 2012), in this paper we attempt to investigate the relationship meaning which enacts the use of space by the teacher and the students in two mathematical classrooms. According to our assumption, space is significant as the material site where various semiotic resources of the teacher (language, movement etc) are embodied and instantiated and as such it is related to different conceptions and attitudes of his pedagogical attitude.

### **Introduction**

Harré and van Langenhove (1999) describe positioning as the ways in which people use action and speech to arrange social structures. As such, the notion seems to refer both to physical positioning (Goodwin, 2007) as well as metaphorically to represent the interpersonal relationships. An important aspect of positioning theory is that it recognizes that interpersonal relationships, especially relationships between teachers and students, necessarily involve issues of control, authority and power expressed explicitly or implicitly through the structuring of action and language use in the teaching-learning event.

In the arena of social semiotics, recent studies challenge the traditional view that teaching and learning are primary linguistics accomplishments (Schleppegrell, 2007) and extend the scope of investigation to also include other semiotic resources (e.g. gestures, position and/or movement in space etc) and modalities (e.g. visual, oral, etc) used in the classroom (Kress et al.,

2001, O'Halloran, 2000, 2005). As Norris (2004) suggests «all movements, all noises and all material objects carry interactional meanings as soon as they are perceived by a person». In this sense, classroom communication, as any other type of communication, seems to be more what is said or heard and to also include what is perceived.

The experience of teaching and learning is constructed through the teacher's use of multiple semiotic resources which along with his language use are embodied in his pedagogy. Different organization and managing of the available semiotic resources reflect specific forms of power and participation in the teaching and learning event or to express it in terms of Appelbaum (2008), they create different "spaces of relation" which allow students to construct different understandings for the social relationships and participation in society.

In modeling the meaning potential of varied semiotic resources the relative studies adopt the triadic stratification of MAK Halliday (1978) whereby the meaning of language- and eventually of any other semiotic resource in use- evolves through three distinct and interdependent metafunctions: *the ideational meaning*, which is the expression of our ideas about the world, *the textual meaning* for the organization of meaning into a coherent unit (text) and the *interpersonal meaning* which is the enactment of social relations.

### **Regarding the educational space**

Solomon (1992) sustains that the selected space arrangements reveal conceptions and stances about [pedagogical] principles and as such they correspond to [educational] attitudes and practices. Kress and al. (2005) describe how a teacher's slow and deliberative movements into his classroom function as "invigilator", in their own term as "a patrol". Matthiessen (2010) states that the teacher's positioning in the different material sites in the classroom realizes "semiotic distance" and as such it establishes social interpersonal relations with the students. In terms of social distance, Hall (1966) in his foundational work on proxemics suggests four general categories of space, according to the typical distance and the degree of physical contact between the participants in the communication: a) *the Public* b) *the Social-Consultative* c) *the Casual-Personal* and d) *the Intimate space*. Regarding Hall's taxonomy, Lim and al. (2012) claim that in the context of the classroom, most communication takes place in the Social-Consultative space, which construes the formal relationship between teacher and students. Therefore in order to further investigate the semiotic potential

of Social-Consultative space they develop a further sub-division within this category, namely they propose to distinguish: a) *the Authoritative space*, b) *the Personal space*, c) *the Supervisory space* and d) *the Interactional Space*. Their coding is based both on the characteristics of the material site and the degree of distance between teacher and students as well as on the type of activities that are typically taking place there (e.g the Authoritative space is usually the furthest from the students in terms of proximity and corresponds to the space in the front center of the classroom. It is also the space where the teacher usually positions himself to provide instructions). It is underlined then that while each material site in the classroom has its own affordance and constrains as semiotic resource, the physical spaces could be reconfigured by the nature of activities and interactions into a new semiotic space with a different set of meanings. Their functional meanings reside into the co-deployment with the other semiotic resources and their functional integration in the teaching-learning processes.

### **Methodological issues**

The data for this study include two, hour-long video-recorded lessons from two different classes. The two lessons have several similarities between them, which make them comparable in terms of the data analysis. Both lessons are: a) from the 6<sup>th</sup> grade, b) on the same topic c) they comprise the same number of students (19 students per class), d) they are conducted in classrooms of similar capacity, e) they are addressed to students with the same mixed linguistic and mathematical ability. With respect to the teachers' profile, there is a difference in gender but not in their teaching experience. Both have five years of experience in teaching.

The research questions underlying the study are the following:

- a) Which semantic configurations are established through the teachers' classroom management (desks arrangement/teachers position into classroom space). What types of interpersonal relationships do they establish?
- b) What is the correspondence between the space's semantic configurations and the other semiotic resources used in the deployment of teaching-learning processes?

In exploring the use of classroom space as semiotic resource we attempt:  
a) a statically based analysis which considers the stationary position of the teacher in the specifics locations, its semantic relationships to the students' position and the exact time he or she stayed in each location during the lesson  
b) a more dynamically inspired analysis which through the different



stages of the lesson (microgenres: Christie, 2005) is looking for the potential reconfigurations of spaces' semantics due to their interaction and integration with other semiotic resources used by the teacher during his teaching.

### **The results**

#### Class 1

*C1 Desk arrangement:* the desks were united in pairs with four students sitting around them and as such it is constructing the meaning of a potential collaboration between the students.

*C1 Teacher's stationary positions:* in terms of semiotic distance between the teacher and the students, the teacher of C2 class positions himself mainly into the Authoritative and only for a while into the Personal Space in the classroom. During the entire lesson he is standing: a) in front of the whiteboard, (authoritative space), b) behind his desk (personal space) c) to the left of his desk (authoritative space) d) to the right of his desk (authoritative space / Supervisory function).

The teacher of C1 makes use of the Authoritative space for giving a formal type of teaching: he initiates and solves a problem by himself in the whiteboard while he is trying to show to the students the new mathematical relation which they have to learn (1<sup>st</sup> microgenre). While he is modeling the solving procedures of such type of problem (2<sup>nd</sup> microgenre) he is moving behind his desk, namely in the Personal Space which functions as such for only two seconds, that is the time he needs to take and find the appropriate page in the textbook. After that sort time, this material site turns to be an Authoritative Space: The teacher stands up behind his desk keeping the textbook (motion as semiotic resource) and reading the guide of a typical procedural solution of the types of problems he had just taught (mediation resources). By his intonation he is emphasizing the attention the students should give to the "authority" of the book (paralinguistic resource). In addition, he has the conversational control by initiating every communicative turn during this 2<sup>nd</sup> microgenre (Chaviaris, Stathopoulou, Gana, 2011). When he initiates the 3<sup>rd</sup> microgenre, by giving the students a problem to solve in groups – which were formed by the typical arrangement of their desks, and without any possible pedagogical reconsideration-, the teacher moves to the right of the desk and he is standing there supervising from the distance of his position the students working. His posture may to some point give a sense of casualty but the phrasing of his discourse support his authoritative teaching profile ("*I want you to solve a problem for me*", "

*I want to see who feels strong enough and certain of himself to be in the whiteboard”, “don’t forget to adopt the typical procedure I’ve just told of”.*

### Class 2

*C2 Desk arrangement* the desks are arranged in the shape of a rectangle with the students seating around to all the sides so as to have visual contact with the whiteboard; those who sit in the two larger sides never have eye contact with each other. Around the rectangle there is enough space for circulation.

*C2 Teacher’s stationary positions:* in terms of semiotic distance between the teacher and the students, the teacher of C2 spend most time into the Interactional Space and makes use of the Authoritative Space only when she introduces the new mathematical knowledge.

The teacher of C2 selects to begin the lesson by relating the new information to the relative students’ mathematical background. During this lesson’s microgenre she is standing alongside the students desks, that is into the typical Interactional Space which however functions as a Supervisory Space: she often looks at the whiteboard making comments to a pupil’s writing there (gaze modality/linguistic resource). At the same time she is regulating the recall of mathematical concepts for the whole class by a continuing prompt and questioning (linguistic resources). In that case the space proximity between teacher and students reduces the meaning of power and authority asserted by the actions of the supervision and construe a “structured informality” (Lim and al 2012) which encourage students’ participation. In the next microgenres there is an alteration in the use of spaces, with the authoritative space being used only for a limited time during the establishment of new knowledge. In short, the interpersonal relationship asserted by the use of classroom’s space along with her linguistic choices (eg the high use of modality and adjuncts constructs solidarity with her students), her challenging of the textbook “authority” (she points to a misstatement), her minimal distracting movements encourage students participation in the classroom.

### **Discussion**

The first outcomes of this study confirm the hypothesis that although the teachers apply the same Curriculum Standards, the use of the classroom space that every teacher consciously or unconsciously choses during his teaching along with his discursive choices, and the use of other semiotic

resources (gesture, gaze etc) construct different learning experiences and therefore different representations about notions such as authority, power and participation in society. Teachers' sensitization to the interpersonal meanings that are dictated by the use of space (and other semiotic resources) could contribute to shaping a democratic mathematics education; an education providing equal access to every student independently of his aptitude or origin. The use of space for articulating an authority discourse, prevents the inclusivity and rights of the students, thus undermining the democratic access of all the students in mathematics education.

### References

- Appelbaum, P. (2008). *Embracing Mathematics*. London: Routledge Falmer.
- Christie, F. (2005). *Classroom discourse analysis*, London and New York: Continuum.
- Chaviaris, P., Stathopoulou, Ch. and Gana, E. (2011). A socio-political analysis of mathematics teaching in the classroom. *Proceedings of CIEAEM 63 Conference* "Facilitating access and participation: Mathematical practices inside and outside the classroom" (in press) <http://www.mathforlive.net/CIEAEM63.php>
- Goodwin, C. (2007). Participation, stance and affect in the organization of activities. *Discourse and Society*, 18(1): 53–73.
- Hall, E. (1966). *The hidden dimension*. New York: Doubleday
- Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. London: Edward Arnold.
- Harré, R., and van Langenhove, L., (Eds.) (1999). *Positioning theory: Moral contexts of intentional action*. Oxford: Blackwell
- Matthiessen, C.I.M. (2010). Multisemiosis and context-based register typology: Registeral variation in the complementarity of semiotic systems. In Ventola, E. and Moya, J. (Eds.), *The world told and the world shown: Multisemiotic issues*. Palgrave Macmillan:Hampshire.
- Norris, S. (2004). *Analysing multimodal interaction: A methodical framework*. London and New York: Routledge.
- O'Halloran, K. L. (2000). Classroom discourse in mathematics: A multisemiotic analysis. *Linguistics and Education*, 10(3):359-388.
- O'Halloran, K. L. (2005). *Mathematical discourse: Language, symbolism and visual images*. London and New York: Continuum.

- Kress, G., Jewitt, C., Ogborn, J., and C. Tsatsarelis. (2001). *Multimodal teaching and learning: the rhetorics of the Science Classroom*. London: Continuum
- Kress, G., Jewitt, C., Bourne, J., Franks, A., Hardcastle, J., Jones, K., and E. Reid. (2005). *English in urban classrooms: A multimodal perspective on teaching and learning*, London: Routledge Falmer.
- Lim. F. V., O'Halloran, K. L., and A. Podlasov. (2012 in print). Spatial Pedagogy: Mapping Meanings in the Use of Classroom Space. In *Cambridge Journal of Education*. (retrieved from [http://multimodal-analysis-lab.org/?page\\_id=217](http://multimodal-analysis-lab.org/?page_id=217))
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading and Writing Quarterly*, 23(2):139-159.

## **Divided minds: Adults' anxiety towards mathematics**

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### **Abstract**

This presentation reports on the findings of a research concerning anxiety towards doing mathematics experienced by adults who have returned at school, assuming that negative emotions and particularly mathematics anxiety drive persons to avoid learning and using mathematics and thus to self-exclusion from participating in many aspects of social, cultural and civic life. As the evidence of this research indicates a considerable number of adults reports no anxiety when carrying out number manipulations or operations in everyday life mathematical activities, although a remarkable variation is found according to the situation and the tasks involved. On the other hand a high level of anxiety has been reported when the adults are thinking or are carrying out activities related to school mathematics. As a main conclusion may be claimed that mathematics anxiety is “taught” by school mathematics practice and “learned” by students and finally becomes a constituent aspect of their “mathematical self-determination”.

### **1. Background of the study**

This presentation is based on the findings of a research into negative emotions and anxiety towards mathematics reported by adults who were studying at a Greek Second Chance School, at the time of this research. These students aged between 18 and 30 have not completed the 10 year compulsory education cycle and therefore they lack both the basic knowledge and the specific competencies to fully benefit from available

opportunities in training, labour, social and cultural life, thus experiencing a situation or are faced by an immediate threat of social and/or cultural exclusion as well as of civic marginalization (Eurydice, 2010).

The research aimed at tracing adults' anxiety towards mathematics as a school subject and as an essential component of everyday life activities, assuming that persons experiencing mathematics anxiety are avoiding learning and using mathematics and thus are gradually excluded, subjectively or objectively, from participating in many aspects of social, cultural and civic life.

Anxiety is a psychological and physiological state of a person characterized by somatic, emotional, cognitive, and behavioral components. Mathematics anxiety, in particular, has been studied as a topic in mathematics education research for more than 30 years and it has been defined and described in the relevant literature in many and differing terms. For instance, Richardson and Suinn (1972, p. 551) described mathematics anxiety as "*involving feelings of terror and anxiety that interfere with manipulation of numbers and the solving of mathematics problems in a wide variety of situations*", Tobias & Weissbrod (1980, p. 65) defined mathematics anxiety as "*the panic, helplessness, paralysis, and mental disorganization that arises among some people when they are required to solve a mathematical problem*" and Spicer (2004, p. 1) stated that mathematics anxiety is "*an emotion that blocks a person's reasoning ability when confronted with a mathematical situation*". At the same time, many scholars have correlated or even identified mathematics anxiety with mathophobia, "*an irrational and impeditive dread of mathematics*" (Lazarus, 1974, p.16).

Defined in the one or the other way, mathematics anxiety is reflecting a vicious cycle of mathematics avoidance that leads to educational and societal mathophobes" (Williams, 1988, p.96).

Most of the researchers in the field are in agreement that we may distinguish two dimensions of mathematics anxiety on the basis of its context. Mathematics anxiety related to mathematical / numerical activities of everyday life situations and mathematics anxiety related to school mathematics, courses, lessons, textbooks, tasks, tests and examinations. Rounds and Hendel (1980, p. 142) describe this distinction as two factors of mathematics anxiety. The first, named "mathematics course or mathematics test anxiety", reflects apprehension about mathematics courses as well as about anticipating, taking and receiving the results of school mathematics tests and the second factor, named "numerical anxiety", refers to anxiety

generated by everyday concrete situations requiring some form of number manipulation, use of elementary arithmetic skills in practical situations as well as to practical skills necessary for making money decisions.

## **2. Research methodology**

In the research reported in this presentation they have been participated 191 adult students from 3 Second Chance Schools at Athens and Thessaloniki. A questionnaire extracting items from Richardson-Suinn Mathematics Anxiety Rating Scale (MARS) as revised for adolescents (Suinn & Edwards, 1982) has been developed and employed for our data collection.

The Mathematics Anxiety Rating Scale for Adolescents (MARS-A) includes 98 items which are brief descriptions of behavioural situations, as for instance “thinking that a mathematics lesson is forthcoming”, “solving a mathematics problem in the classroom”, “adding two three-digit numbers while someone looks over your shoulder” or “checking your bill in a restaurant”, in response to which people are expected to indicate different levels of anxiety from “not at all” to “very much” anxious. According to its inventors, since different kinds of anxiety lead to different effects on intellectual performance the MARS-A scale aims to be situational-specific and trace the particular anxiety producing factors. In Evans’ view the MARS scale is the more appropriate indicator to be used for the study of the variation in numerate performance and anxiety across contexts (Evans, 2000).

As most of the researchers in the field are in agreement that we may distinguish mathematics anxiety related to mathematical / numerical activities of everyday life situations and mathematics anxiety related to school mathematics activities we have included in our questionnaire questions referring to these two contexts. Each question has been phrased and posed in two variations. Questions referring to numerical activities carried out in a private situation, e.g. calculate a sale price and questions referring to numerical activities carried out in a public situation, where the person is under observation by others, e.g. checking a bill in a restaurant.

After all, our questionnaire included 28 questions aimed at tracing adults’ anxiety towards mathematics plus demographic questions, as are sex, age, family status, employment etc.

It has to be emphasised, however, that tracing mathematics anxiety and probing its sources on the basis of adult students’ self-reports as well as concluding on the basis of quantitative data drawn from these self-reports

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have to be considered from a critical standpoint. On the other hand, the lack of research into adults' mathematics anxiety in Greek literature credits this research with a rather pioneering value.

### 3. Selected findings

In the following they are presented and briefly commented two sets of findings concerning mathematical / numerical activities and tasks carried out respectively in specific everyday life and school mathematics contexts.

- (a) Considerable percentages of adults ranging from 20% to 70% acknowledge that they felt anxious when involved in particular everyday life numerical activities in various contexts. For instance, half of the participants in this research report that they felt anxious when they are obliged to read and comprehend numerical data, as those contained in a home electricity bill and significant percentages of adults feel anxiety when having to carry out number operations, e.g. finding the balance of a payment (39%) or calculate a sale price (40%). Significant, as well, are the percentages of adults reporting anxiety when they have to estimate or compare numerical data or results, e.g. two different priced holiday trips (36%) or home expenses (56%). On the other hand, adults in high percentages report that they feel no anxiety when carrying out calculations and numerical estimations in playing games of chance (70%), shopping calculations (80%), checking bills (70%) or work out someone's age (74%). It seems that for adults the numerical manipulations and the number operations acquire various meanings depending on the context in relation to their significance for their personal or family decisions.
- (b) The participants in this research report a considerable anxiety when thinking or carrying out activities related to school mathematics, which according to the evidence is much stronger than - and in statistical terms significantly different from - the anxiety felt when involved in everyday life numerical activities. It is characteristic the percentage of adults which reports that they feel anxiety even reading the word "mathematics" (36%). The solution of problems in mathematics classrooms seems to be an anxiety producing activity in a rather high percentage (60%). It is characteristic that even listening to another person explaining the solution of a mathematics problem is reported as an anxious event by the participants of this research (43%), and even more an anxious activity is considered the explaining of how a mathematics problem has been solved (56%), especially in cases where



there the adults feel uncertain about its solution (62%). Furthermore, the mathematics classroom per se seems to be a source of anxiety for a significant number of adults. The waiting for the mathematics teacher to arrive (25%), the attending a mathematics lesson (33%) or the thinking of a forthcoming one (33%) are reported as anxious situations.

Finally, according to the answers offered by the adults who participated in this research their mathematics anxiety is not found to be significantly differentiated when the numerical activities are taking place in a “private” or in a “public” context. Likewise, no significant differences are found according to the demographic characteristics of the participants.

#### **4. Main Conclusions**

As the evidence of this research indicates a considerable number of adults reports that they feel no anxiety carrying out number manipulations or operations in everyday life numerical activities, despite a remarkable variation according to the situation and the tasks involved.

On the other hand a state of personal anxiety has been reported when the adults are thinking or are carrying out activities related to school mathematics. Anxiety which is much stronger in problem solving situations, mainly related to school evaluation tests and exams.

This is endorsing the claim that mathematics anxiety is situationally specific and not transsituational (Rounds & Hendel, 1980), therefore it is not transferred from the one situation to the other. Thus, the non-anxious feelings of coping with everyday life mathematics may be quite difficult to be transferred to mathematics classroom situations simply by introducing imitations of everyday life situations to mathematics lessons. So from this psychological point of view, the contribution of real life situations introduced into school mathematics is questionable.

The reported by the participants of this research as sources of their anxiety were mainly associated to their difficulties in comprehending mathematics texts, their negative experiences from school mathematics, their uncertainty of answering mathematical or numerical questions and their fear of disapproval by others, their feelings of a deficiency concerning their mathematics knowledge and skills and their beliefs about the uselessness of school mathematics in out of school contexts.

These factors claimed by adult students for their mathematics anxiety confirms a rather negative effect of school: the mathematics anxiety is “taught” by school and “learned” by students and finally becomes a constituent aspect of their “mathematical self-determination”. This effect

may not be exclusively attributed to a sole component of school mathematics education but it seems a product of an established pedagogy of mathematics as a social practice.

An interpretation of this process grounded on the cultural practice theory may be based on the following idea. Assuming that knowledge is situated within particular contexts then particular people have difficulties with school mathematics as a result of a discontinuity between schooling and other cultural contexts in their lives (Brown, Collins & Duguid, 1989, Lave, 1988). In accordance, these people develop negative thoughts, emotions and behaviours towards mathematics considering it exclusively as a school subject matter. As a consequence, they exclude themselves initially from school mathematics and finally from school *and* mathematics. The incorporation of inclusive practices in mathematics classrooms which recognise a variety of mathematical ideas, understandings and vocabularies widespread in everyday life activities of people is a demand from the standpoint of a democratic mathematics education. Such equitable practices allow all students to recognize and participate in career opportunities and preparations offered by school and at the same time offer them chances for intellectual exploration and development within their own understanding of mathematics and everyday life.

### References

- Brown, J.S, Collins, A. and Duguid, P. (1989), Situated Cognition and the Culture of Learning, *Educational Researcher*, 18 (1), 32-42.
- Evans, J. (2000), Adults 'Mathematical Thinking and Emotions: A study of numerate practices, London:Routledge/Falmer,Taylor & Francis Group.
- Eurydice (2010), Structures of Education and Training Systems in Europe, Greece, 2009/10 edition, European Commission.
- Lave, J. (1988), Cognition in practice: Mind, mathematics, and culture in everyday life, Cambridge, UK: Cambridge University Press.
- Lazarus, M. (1974), Mathophobia: Some personal speculations. *The National Elementary Principal*, 53(2), 16-22.
- Pries C. & Biggs Bobbie T. (2001), Can Instructors Help Learners Overcome Mathematics Anxiety? *American Technical Education Association Journal*, 28(4), 6-10.
- Richardson F. & Suinn R. (1972), The mathematics anxiety rating scale: psychometric data, *Journal of Counseling Psychology*, 9, 551-554

- Rounds, J. and Hendel, D. (1980), Measurement and Dimensionality Of Mathematics Anxiety. *Journal of Counselling Psychology*, 27, 138-149.
- Spicer, J. (2004), Resources to combat math anxiety, Eisenhower National Clearinghouse, *Focus* 12(12).
- Suinn, R. M. & Edwards, R. (1982), The measurement of mathematics anxiety: The Mathematics Anxiety Rating Scale for Adolscents-MARS-A. *Journals of Clinical Psychology*, 38(3), 576-580.
- Tobias, S. & Weissbrod, C. (1980), Anxiety and mathematics: An update. *Harvard Educational Review*, 50, 63-70.
- Williams, W. V. (1988), Answers to Questions about math anxiety, *School Science and Mathematics*, 88 (2), 95-104.

## **LES JEUX MATHÉMATIQUES POUR DÉVELOPPER LA DÉMOCRATIE DANS UNE CLASSE DE MILIEU DÉFAVORISÉ**

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Ce texte relate les conclusions d'un projet de co-développement autour des jeux mathématiques réalisé en milieu défavorisé pluriethnique. Cette recherche collaborative avec des enseignants et des didacticiens visait à créer des trousse de jeux mathématiques pour faire le pont entre l'école et la famille. Les retombées furent plus nombreuses que prévu avec entre autres le changement des représentations des enseignants sur l'enseignement des mathématiques et la revalorisation du jeu comme pratique d'enseignement. La mise à l'essai a permis de constater la mobilisation de stratégies et d'habiletés de haut niveau et l'appivoisement de l'erreur comme outil d'apprentissage, mais également que des discussions émergeaient avec les élèves et que le concept d'honnêteté était abordé. Des ajustements à la gestion de classe devaient être effectués afin d'intégrer adéquatement le jeu dans l'enseignement. Les jeux mathématiques apparaissent comme une pratique d'enseignement transdisciplinaire où il est possible d'aborder la démocratie et le respect de règles de vie en société.

### **Problématique**

Au cours d'un projet de codéveloppement autour des jeux mathématiques, le groupe de participants (enseignants, ressources professionnelles et chercheurs universitaires) concluent, entre autres que l'utilisation des jeux mathématiques provenant de diverses cultures est source de motivation pour les élèves et permet aux enseignants de développer une nouvelle gestion de classe pour intégrer le jeu dans leur enseignement (Poirier, 2011).

Nous nous sommes alors demandé si l'utilisation des jeux mathématiques dans des classes de milieu défavorisé comme pratique d'enseignement permettait également de développer des valeurs humanitaires.

### **Cadre conceptuel**

A priori, il peut sembler inhabituel d'effectuer un rapprochement entre l'enseignement des mathématiques et la démocratie. Toutefois, dès l'époque de la Grèce antique, le philosophe Platon accordait une grande importance à l'apprentissage des mathématiques pour former la raison. Sur l'entrée de l'Académie, son école, on pouvait y lire "Nul n'entre ici s'il n'est géomètre".

Au Québec, le programme de formation du primaire (Ministère de l'Éducation, 2001) contient des domaines de généraux de formation, c'est-à-dire " un ensemble de grandes questions que les jeunes doivent affronter " (Ministère de l'Éducation, 2001, p.42). La démocratie est abordée comme problématique transdisciplinaire à travers le domaine général de formation Vivre ensemble et citoyenneté. L'enseignant doit développer chez les élèves: la valorisation des règles de vie en société et des institutions démocratiques, l'engagement dans l'action dans un esprit de coopération et de solidarité et la culture de la paix (Ministère de l'Éducation, 2001, p.50)

Pour Dupperet (2007), l'apprentissage des mathématiques est une forme d'apprentissage de la démocratie. Premièrement, au cours de la réalisation d'une activité mathématique, nous avons recours à une démarche intellectuelle semblable à celle des philosophes grecs en développant et construisant notre pensée grâce à une succession de débats où l'on soumet nos idées et réfute les diverses solutions. Ensuite, les activités mathématiques permettent le développement de comportements experts transférables à d'autres champs par la recherche de la meilleure stratégie, du modèle le plus pertinent en raison d'une constante confrontation au non-savoir. Enfin, l'activité mathématique contribue à l'éducation civique en raison des aptitudes développées pour analyser, traiter et transformer des informations. Le jeu ne pourrait-il pas être lui aussi un contexte d'apprentissage où l'on développe les interactions sociales ?

Malloy (2002) dégage les quatre grandes qualités pour qu'un enseignement des mathématiques soit démocratique. Il faut que cette

éducation soit accessible à tous les élèves, repose sur le postulat que tous les élèves peuvent apprendre si on leur fournit les bonnes conditions, puisse permettre aux élèves d'apprendre de façon significative et substantielle et aide les élèves à construire des outils leur permettant de devenir des citoyens productifs et actifs. Le jeu pourrait-il permettre d'atteindre ces quatre objectifs?

De plus, Mounier (2005) fait plusieurs rapprochements entre le débat démocratique et le débat mathématique: " Le groupe doit se mettre d'accord sur une solution, les propositions de chacun sont discutées - débouche sur la rédaction d'affiches qui sont ensuite le support d'un débat sur la validité des solutions proposées" (Mounier, 2005, p.50). Ainsi, selon Legrand (Mounier, 2005) "L'intérêt d'un tel enseignement pour fonder une démocratie c'est donc de proposer au futur citoyen des modes de mise en accord qui fait appel à ses responsabilités d'être pensant; recherche d'accords qui ne reposent ni sur l'autorité absolue d'un supérieur infallible, ni sur l'abandon à l'irrationnel, à la chance ou la malchance, à la volonté divine.

Epstein (2001, p.19) affirme "[qu'un] enfant ne joue pas pour apprendre, mais apprend parce qu'il joue ". Le jeu permet entre autres de développer certains repères sociaux qui permettent d'apprendre à faire ensemble. Garon (2001) soutient que le jeu entraîne la socialisation ainsi que des progrès cognitifs et sociaux. Entre sept et onze ans, les enfants progressivement ont recours à des jeux de règles pour lesquels on doit comprendre et respecter certaines conventions. L'enfant doit apprendre à ne pas modifier au cours du jeu certaines règles à son avantage. En accédant à la pensée réversible, l'enfant comprend les consignes et anticipe les conséquences de ses gestes en situation de jeu. Des conflits peuvent survenir en cours de jeu et l'enfant doit exprimer ses idées de façon cohérente et coordonner son point de vue avec celui de ses adversaires. Il s'agit de nombreuses habiletés sociales qui continueront à être utiles pour la vie à l'âge adulte.

### **Objectif général de l'étude**

Au cours de ce projet nous avons examiné si l'utilisation de jeux mathématiques comme pratique d'enseignement dans une classe de milieu défavorisé permet développer des activités "culturellement juste" afin de créer le pont entre la culture de la famille et la culture de l'école. Les activités mathématiques serviraient "d'interfaces culturelles" entre le milieu

familial et, entre autres, les valeurs humanitaires que l'on retrouve dans le programme de formation de l'école québécoise.

### **Méthodologie**

Tout d'abord, trois jeux ont été utilisés dans ce projet. *Mankala*, un des meilleurs jeux de stratégie (Zaslavsky, 1997), fait appel au dénombrement, à la correspondance terme à terme et à l'anticipation. Ce jeu est encore joué sur le continent africain. *Referme les boîtes* fait appel à l'addition et à l'anticipation. Ce jeu était joué par les matins français lors de la traversée de l'Atlantique pour venir en Nouvelle-France. *Barrage* est un jeu de déplacement dans l'espace et de stratégie qui aide les enfants à acquérir des habiletés nécessaires à la résolution de problème et à la prise de décision (Zaslavsky, 1997). Ce jeu est l'ancêtre du Tic-Tac-Toe et remonte à l'époque des pharaons égyptiens.

En collaboration avec le *Programme de soutien à l'école montréalaise*, deux chercheurs universitaires, deux ressources professionnelles et vingt enseignants ou conseillers pédagogiques ont pris part à cette recherche collaborative de janvier 2009 à juin 2010. Des rencontres préliminaires ont permis de discuter de la problématique de l'enseignement/apprentissage des mathématiques spécifique au milieu défavorisé. Certains concepts mathématiques intervenant dans les jeux ont été revisités en plus de discuter des procédures et stratégies mises de l'avant par les élèves de 5 à 12 ans lorsqu'ils jouaient.

L'expérimentation constituait en une rencontre de l'équipe pour élaborer les jeux d'après un canevas de base qui leur était présenté par les chercheuses. Ensuite les enseignants mettaient à l'essai les jeux dans leurs classes dans les semaines suivantes. Lors de la rencontre suivante, l'équipe effectuait un retour sur la mise à l'essai et on planifiait des variantes de ces jeux ou de nouveaux jeux. Les verbatim des rencontres filmées de l'équipe du projet de codéveloppement autour des jeux mathématiques ont été retranscrits et leur contenu analysé afin de connaître leur opinion sur le sujet et certains éléments qu'ils relevaient de l'expérimentation en classe. Pour compléter l'expérimentation, des entrevues individuelles seront menées avec quelques participants.

### **Discussion et conclusions**

Les analyses préliminaires permettent d'affirmer que le jeu offre un contexte d'apprentissage riche, ludique et varié comme le disait Garon. En effet, le jeu n'était plus vu comme seulement une récompense à la fin de la journée ou de la semaine, mais faisait l'objet d'un enseignement nécessitant une nouvelle gestion de classe.

L'équipe a aussi déclaré que le jeu peut devenir une situation d'apprentissage complexe qui sollicite des stratégies et des habiletés de haut niveau tel qu'avancé par Dupperet (2007). Le jeu met en place des interactions sociales qui permettent, entre autres, de développer le langage mathématique contribuant à l'éducation civique comme le mentionne Dupperet (2007). Quant aux similitudes entre le débat démocratique et le débat mathématique de Mounier (2005), nous croyons que ce type de débat a été vécu en contexte de jeux mathématiques lors de la présentation des règles ou lors du changement de règles sur approbation de l'ensemble des joueurs.

L'équipe de recherche (chercheur, ressources professionnelles, conseillers pédagogiques et enseignants) arrive également à la conclusion que le jeu permet aux élèves de développer leur sentiment de compétence et leur autonomie. Nous en venons donc à la conclusion que l'intégration du jeu mathématique a eu un impact sur les élèves. De ce fait, dans plusieurs classes certains élèves faibles en mathématiques ont développé des stratégies informelles qui leur ont permis de gagner aux jeux de stratégies et d'être les meilleures de leur classe. Le contexte du jeu mathématique a également permis le développement de valeurs humanitaires grâce aux nombreuses discussions entourant le fonctionnement des jeux et les stratégies les plus pertinentes.

### **Principales références**

- Dupperet, Jean-Claude. (2007). *Mathématiques et valeurs*. CRDP.
- Epstein, Jean. (2001). Rôle du jeu en collectivité. In Catherine Van Nieuwenhoven (Ed.), *Pourquoi tu joues ? : Rôle du jeu dans le développement de l'enfant* (pp. 727). Louvain-la-Neuve: Presses universitaires de Louvain.
- Garon, Denise. (2001). Analyse développementale de supports ludiques. In Catherine Van Nieuwenhoven (Ed.), *Pourquoi tu joues ? : Rôle du*



- jeu dans le développement de l'enfant* (pp. 727). Louvain-la-Neuve1: Presses universitaires de Louvain.
- Malloy, C. (2002). Democratic access to mathematics through democratic education : an introduction. In L. English (ed.) *Handbook of International Research in Mathematics Education* (pp 17 – 26). Lawrence Erlbaum Associates, Publishers, New Jersey.
- Ministère de l'Éducation. (2001). *Programme de formation de l'école québécoise (version approuvée) - Éducation préscolaire et enseignement primaire*. Québec: Gouvernement du Québec.
- Mounier, Georges. (2005). Débat mathématique, Débat démocratique: Une expérience d'enseignement à de futurs professeurs d'école. *Repères IREM*(60 - juillet 2005), 47-56.
- Poirier, Louise. (2011). Projet de codéveloppement autour des jeux mathématiques. *Vie pédagogique*(158), 39-40.
- Zaslavsky, Claudia. (1997). World Cultures in the Mathematic Class. In Arthur B. Powell & Marilyn Frankenstein (Eds.), *Ethnomathematics : challenging eurocentrism in mathematics education* (pp. 307-320). Albany: State University of New York Press.

## **Using drama techniques for facilitating democratic access to mathematical ideas for all the learners**

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### **subtheme 2**

#### **Introduction**

The critical view of mathematical education promotes the need for a political vision of mathematics education and argues that its main objective should be people support for the creation of citizenship through experience and active participation in school mathematics education. According to Skovsmose (2005) mathematics education has the potential to contribute to the development of critical citizenship and support democratic ideals, as well as the development of citizens who can participate actively and responsibly in discussions and processes in which the individual is required to take part in personal and public decisions (Skovsmose 1996:1267). The democratic aspect of mathematics requires the development of students critical ability, which is of vital importance for the development of democratic citizenship (Ernest, 2008).

The notion of “democratic access” to mathematical ideas by the learners has been the focus of many researchers. Democratic access is the creation of the appropriate learning conditions where all students have the chance to become critical thinkers and decision makers and develop the ability to solve and understand increasingly challenging problems in classroom setting, but also in their life conditions (Skovsmose & Valero, 2002). One of the main questions that emerged, concerns the kind of learning environments and classroom practices that are needed in order to promote a democratic access to mathematical ideas for all students.

Gerofski (2009) claims that a classroom where students’ active participation and ideas are clearly valued might prepare kids to live in a more participatory, activist political system where their voices and ideas are

taken seriously. She suggests to reconfigure our conception of mathematics classroom, learning to include participatory performance both as a model and the means to stimulating democratic participation in our societies.

We claim that Drama in Education<sup>1</sup> techniques promote participatory performance and can give a context that offers innovative and creative ways for the students to develop critical thinking about ‘mathematics in society and history’. We also claim that the question ‘*which is the geometry which best describes our physical world*’ is an intriguing one that can motivate students and promote critical thinking.

Active involvement in the teaching of non-Euclidean geometries enables students to perceive Mathematics as a creation under constant negotiation, modifying thus their epistemological beliefs about mathematics and provokes the dominant belief that Euclidean geometry is the only model which interprets and represents our real world, shaking thereby and other certainties.

### **The research**

This paper focuses on exploring the dynamics of Drama in Education Techniques in teaching Geometry, as a process that promotes “democratic access” to mathematical ideas by all learners. Through a didactical experiment—titled “*Is our world Euclidean*”—we tried: a) to motivate and engage all the students of a mathematics class actively through Drama b) to encourage students to develop a critical stance towards mathematical knowledge, as absolute, objective and infallible knowledge. Using Drama techniques the axiomatization of Euclidean and Non-Euclidean Geometries as far as the history of Euclides’ fifth postulate were discussed.

#### The setting

The research was carried out in a group of 26 (11<sup>th</sup>-grade students) from different directions of studies in the 2<sup>nd</sup> High School of Ilion (Athens, 2010-11).

The method: Exploitation of ethnographic research techniques (observation-interviews) helped us gathering research data, while all students’ presentations were videotaped and analyzed.

#### The teaching experiment

The teaching experiment was carried out in 25 teaching periods, during

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<sup>1</sup> Drama in Education can be a highly structured pedagogical procedure utilizing specific rituals and techniques of dramatic art aiming to focus participants’ attention towards *the process* of participants’ experience and not on the final product (Alkistis, 2000; Wagner, 1999).

seven weeks, in Geometry, History, Language, Literature and Ancient Greek Language classes. The three main pillars were: the mathematics themselves, the persons that have invented them and the historical and cultural context in which these Mathematics have been created and formatted. More analytically:

1. Euclid's Elements and the axiomatization of Euclidean Geometry, 6h.
2. Euclid and the historical, cultural and political frame of his era, 4h.
3. 'History in shadow' The controversy of Euclid's Fifth postulate till 18th century, 5h.
4. János Bolyai, Lobatscevski, Riemann: The founders-creators of non Euclidean geometries, 3h.
5. Hyperbolic Geometry and Poincare model, 6h.
6. Spherical Geometry and the axiomatization of Elliptic Geometry, 1h.

In every unit of the experiment, initially, a lecture was given by us preparing the students for the presentations. Subsequently, the students worked in teams using bibliographical/ICT recourses and then performed their presentations using different drama conventions, as: 'role playing', 'reportage', 'alter-ego', 'interview', dramatization 'Radio-broadcasts'. Furthermore, 'Shadow Theatre' was used for presenting the fruitless efforts of Arabs mathematicians to prove the fifth postulate. Finally, a documentary film entitled 'Our lives with Euclid' was created around these drama-activities.

### Results

Drama techniques helped for new practices to be formed in a class of active students who cooperated towards the development of mathematical knowledge. Students integrated correctly in their performances all mathematical notions that they had to present and from their answers in interviews two months later, it seems that they realized the axiomatization of Euclidean and Hyperbolic Geometry and changed their perceptions about the nature of Mathematics.

- It appeared that all students were motivated to develop **cooperative skills**.

*-(V) It was completely different than just writing stuff, from comprehending something yourself and trying to pass it to others. It was very nice.*

*-(Tz.) In this particular project we all worked, irrespectful the direction of our majors*

The motives for the participation of all students in the teaching experiment, according to them due to the following reasons:

- **To the experiential nature of Drama.**

-(A). *We felt as if we were discussing it to others for them to learn, as we were the teachers, Euclid, Gauss and Lobachevski. We were more involved in profound knowledge.*

- **To Drama creative character**, which gave them the opportunity to express themselves in their own way and for everyone to bring out his special abilities and talents. Students felt that the sketches were their own creation and due to this they wanted to have the best result in their presentations.

-(A) .. *without Drama **this would not be a part of us.** Through Drama we prepared it, we presented it, **we felt it as our thing.***

-(Vir.)...*it was something new, it was so very different for everyone,... and very creative **and everyone could show his abilities**, the different ones that everybody has.*

- **To the atmosphere of game, joy and fun**, that Drama created.

-(P.) *All this dramatic, theatrical thing is like a game...*

- **To the impact that drama had on students' identities as mathematics learner**, stimulating their confidence as individuals who could express their opinion, participate in the writing of the texts and in presenting of the different skits.

-(H.) *Yes, surely (I felt more capable) because **before I hardly participated, while here I said my opinion, I wrote and I took part in the presentation.** I couldn't think of this before*

- **To the cooperation of students and the sense of security that this cooperation offered to them.**

-(Tat.) *Definitely, a team works better than a person alone and has far better results, so everyone felt more sure of himself, had no anxiety and knew that he he had some help, because a team and work together and this gives them more courage.*

During the preparation of the presentations, students through working in teams had opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics, ie. to cultivate critical thinking skills. As NCTM (2000, p. 60) propose, they communicated to learn mathematics, and they learned to communicate mathematically.

-(St.) *Some of the concepts were too difficult for me to explain them to the others, but trying to help the others you teach yourself..., **trying to explain it to the other, you actually see how it's done.***

Drama also cultivated students abilities and skills, necessary for a

responsible citizen in a democratic society; Students for preparing their Drama presentations, had to learn to listen carefully to the others, make decisions concerning the Drama convention they should use, distribute the roles in different sketches and decide what elements of the issue were necessary to be contained in their texts. Students thus learned to help each other, work together to take collective decisions, appreciate the abilities of the other, have responsibility as members of a team and offer the team according to their capabilities and particular inclinations.

*-(Vir.) And everybody could show his skills, the different abilities that one has. Through the project not only the qualities of children more involved with mathematics could be seen, but there were some moments when the knowledge of students of humanities major were needed and so every one helped each other and the cooperation had a very good result.*

*- (Chr.) ... it was hard for someone to be indifferent. Not because he wouldn't like to participate, but somehow as an obligation to other classmates.*

*-(Vir.) This feeling that we are all together and we must do our best for us to be then satisfied and for audience and those watching to be delighted.*

The teaching experiment theme raised students interest, who were surprised and impressed by the existence of other geometries changing their beliefs about geometry.

*-(P.) It was something that I couldn't imagine, because the different geometries are not encountered in the physical world, so you don't think anything that goes beyond the five senses...so at first it just seemed strange to us but then it surprised us.*

*- (S.) The fact that I saw and other geometries, basically because I like strange things, it picked my interest, i.e. to see a geometry I have never seen before, i.e. the segment that was a curve.*

A large percentage of the students challenged Geometry and more generally Mathematics as a science of absolute truth.

*-(St.) Certainly the plasticity of mathematics emerged and how mathematics are created and are changed depending on the needs of mathematician, of scientist and of human been generally. It is clear that mathematics is a complex creation which is not restricted to follow only one way around the understanding of reality.*

*- (Ang.) Finally there are and other views and we cannot say which is absolutely right and which not.*

### Concluding Remarks

Our research offered considerable evidence of the effectiveness of the use of Drama techniques as an alternative approach in the creation of the appropriate learning conditions where all the students participate in teaching/learning process and become critical thinkers and decision makers in the classroom setting.

We believe that with the aforementioned activities with Drama in Education techniques all students experienced mathematics, not only mentally but emotionally and physically. It seems that such projects can help to achieve the major goals of education, the global participation of the students in courses promoting critical thinking and decision making not only restricted in the classroom setting.

-(S.) *I believe it is worthwhile to have such things in schools for the children to learn more and so we will not say that in a class there are ten people who are good students and everyone else we have them on the sidelines, but we can say that every child is special somewhere, that has a distinct inclination...*

### References

- Alkistis (2000). *Drama in Education*, Ellinika Grammata, Athens.
- Ernest, Paul (2008). What does the new philosophy of mathematics mean for mathematics education, *Proceedings of 25<sup>th</sup> Panhellenic Congress in Mathematics Education of Hellenic Mathematical Societ*, Volos 21- 22- 23, November 2008.
- Gerofski, Susan (2009). Performance Mathematics and Democracy, *Educational Insights*, 13(1).
- National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM. <http://standards.nctm.org/>
- Skovsmose, Ole (2004). Critical mathematics education for the future, *ICME 10*, Copenhagen.
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas, in L. English (Ed.), *International Research in Mathematics Education* (pp. 383-408).
- Skovsmose, Ole (1996). Critical mathematics education, in A. J. Bishop; K. Clements; C. Keitel; J. Kilpatrick; C. Laborde *International handbook of mathematics education*, Dordrecht, Kluwer, p. 1257-1288.
- Wagner B., (1999). *Dorothy Heathcote. Drama as a Learning Medium*, Heinemann, Portsmouth.

## **Democratic education for blind students**

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### **Abstract**

Nowadays it is required that we provide equal opportunities of education to all the students no matter what their disability or special need is. During the last years, in our country, there has been an increasing interest for Special Education and new Curricula for all categories of students with special needs, blind ones included. This particular paper examines how democratic is the mathematical education for blind students in Greece through the study of the Curricula.

### **Introduction**

Within the frame of the common educational policy that is being followed, it is considered (Warnock committee, ref.:Norwich, 1980) that the aim of education should be the same for all the children, including those with special needs, and targeting to the development of their potential. However, White (1982, ref. :Norwich, 1980) while examining whether the aims of education can be the same for all, he concluded that for the children with special needs the demands of education shouldn't be high.

The learning abilities of the professional and social inclusion of the students with special needs will be shaped according to the education that they will receive (Tzouriadou, 1995). For this reason it is imperative to realize



the need for substantial and systematic approach of their education with the existence of appropriate Curricula that respond not only to their special needs, but also to every child separately, so as to achieve the aim of individualized education

Democracy in education is important for the social equality and justice (Malloy 2002). The equality of opportunities calls for the differentiation and adjustment of the educational system by taking measures in order to allow for the special needs of students to be taken into consideration. The establishment of special structures for children with sensory disabilities started when it became apparent that these children are not incompetent, but they could learn with the assistance of special measures that would be taken for them, through education more adjusted to the particular difficulties that they present and based on the rest of their senses. In Greece this was put into practice with the designing of special Curricula for every category of students with special needs (Mapping – Curricula of Special Education, 2004).

In this particular paper it is examined how democratic is the mathematical education for the blind children in Greece through the study of the Curricula.

### **Designing of the Differentiated Curriculum**

Democratic education should include everybody in the Curricula (Malloy, 2002), since all students, everywhere in the world, have a right in education and learning (Skovsmose&Valero, 2002).

Since the 1970's, in the USA and in many countries in Europe, Curricula suitable for every category of children of special education started to be designed (Lambropoulou, Panteliadou, Markakis, 2005), following the transition from the medical to educational sector concerning the approach of issues related to people with special needs with the concept of integration, social inclusion and acceptance (Tzouriadou 1995). The principle of integration, states that the students with visual problems and with no mental disabilities or other additional special needs can attend the General School, following the same Curricula with modifications, simplifications and adjustments of methods and means of teaching based largely on the modern technology and in hearing stimulation (Kouroupetroglou, 2004, Polyxronopoulou, 2003, Kroustalakis, 1997), supported by the teacher of the class or by a member of the Special Teaching Staff.

Nevertheless, today in Cyprus, Germany, Denmark and Sweden according to Hapoupi, Grosbøll, Wagner και Rönnbäck, (through personal communication with them), each of them in charge of special education in their

countries, respectively, there are no Curricula for blind students in Maths. The blind students follow the Curricula that apply to the general population. The teachers are supported by school advisors that attend the classes and provide guidance for the adjustment of the teaching to the needs of these students and the tools that can be used every time. Unified Curricula exist also in Hungary and Norway (Csocsán, 2005). In England, the blind students attend the same subjects and take exams in the exact same topics with the rest of the children (Cobb, 2002, ref. Csocsán, 2005, p.p:196).

In Greece differentiated or special Curricula for every category of special needs were designed, indicating a different curriculum focused on the special educational needs of the students (Zoniou–Sideri, Karagianni, Nteropoulou–Nterou, Papastaurinidou&Spandagou, 2005), having as their main philosophy the equal rights for inclusion and education in a school for all (Mapping – Curricula of Special Education, 2004). Nevertheless, no special Curricula were designed for the people with visual problems but there have been alterations in the Curricula of general education which were modulated for the needs of these people in order to lift the obstacles obstructing the accumulation of knowledge (pi-schools, 2011). The aims of the Curricula of the General School remained the same, but there has been an analysis of aims and addition of sub-aims. Moreover, there has been a differentiation, redefinition and adjustment of activities according to the needs of blind students, thus creating the Differentiated Curriculum for blind students. (2004).

This Differentiated Curriculum contains instructions of support (mobility, orientation), development of autonomy skills and is accompanied by indicative lesson plans and suggestions for teaching material (Lambropoulou, Panteliadou&Markakis, 2005). More specifically, as far as Mathematics are concerned, we are presented with some special modifications, simplifications, additions or modulations not only in teaching methods and means (Westphalen, 1982), but also in the material.(Liodakis, 2000), in order to mitigate or counteract the difficulties that occur to people with visual problems during the learning procedure. There is a reference to Braille system, however the typology of mathematical symbols is absent, because there is an endeavor to introduce the Nemeth code that is used internationally.

The material and the means employed play an important role since they are considered necessary for the support of learning of blind children. It is proposed to use special paper, relief shapes, geometrical instruments with relief indications, counterfeit coins with relief signs, speaking scales, speaking thermometers, speaking pocket calculator, miniatures, models in scale, maquettes and other material that can be utilized by the teacher and can be

used by blind students.

The Ministry has decided to form a group work in order to make teaching material for blind students. Teachers of students with serious visual problems can apply and ask for the necessary material that will support the procedure of teaching/learning mathematics.

In reality it is suggested that children with a partial loss of vision should attend classes in the same classroom, while for the blind children it is recommended that they should attend a special school. The blind children when finishing the Special Primary School continue to the general High School, since there are no special schools in Secondary Education.

### **The development of Differentiated Curricula**

Although the Differentiated Curriculum for blind students in the Kindergarten and Primary school was designed to complement the Curricula of the General School and to be used at the same time, this didn't happen in all the levels of its development. For instance, there is no apparent analysis of aims and sub-aims for the mathematics, as it is mentioned in the same Curricula.

It is evident that the emphasis has been put on the material that is to be given to blind students. Crucial points are those of functionality of the material (e.g. hourglass "open" in the sandbox), the method to approach it (e.g. with a funnel they transfer sand from a container to another after they have timed and chosen the amount of time they wish their hourglass to last), the way to design the mathematical activities (many activities are required to be based on the hearing ability, such as the comparison of the duration of activities which began at the same time, the comparison of the velocity of movement of things in relation with time when the distance is equal etc) and also of the general instructions (e.g. the use of geometrical models in the description of spaces gives the students the possibility to recreate their world geometrically and stereometrically. Or the miniatures and the maquettes help them to understand sizes and distances etc). Nevertheless all these are only mentioned theoretically. In reality no material is provided to support the teaching procedure but it needs to be ordered from the special team.

### **Conclusion**

For the children with special needs the equality of opportunities cannot be achieved through Curricula that were shaped for the general population since they should take into account that these children differ greatly on the one hand as far as their needs are concerned and on the other hand as for their potential

and skills. The aims may be common for all, however, the means with which the new knowledge will be taught and conquered should differ (Hart, 1992, Meijer, χ.χ., Jahnukainen, 2004). The concepts and the subjects should be taught according to the needs and abilities of the students.

The democratic education should include equal participation, equal rights and equal encouragement for success. This cannot be achieved through a theoretical modification of the Curricula. It demands that this modification be put to practice during their development and management, through the provision of a specially designed supportive material and activities.

Because of the difficulties that are encountered by a blind child there should be a reinforcement of the learning opportunities that are provided in the general Curricula and special emphasis should be placed in activities, programs, materials and aims that are beneficial for these children (Hart, 1996).

#### **Bibliography:**

- Csocsán, E. (2005). The program of teaching mathematics and its application to students with serious visual problems in the general education, in: Education and blindness. Modern tendencies and prospects. Assiduity: A. Zoniou-Sideri and. H. Spandagou, Translation: A. Stamoglou, Athens:Greek Letters, p.p.: 191-204.(in greek)
- Hart Tutor, S. (1992). Differentiation. Part of the problem or part of the solution?, *Curriculum Journal*, 3(2), p.p.: 131 – 142.
- Hart, S. (1996). *Beyond special needs. Enhancing children's learning through innovative thinking*. London:Paul Chapman.
- Jahnukainen, M. (2004). *Special education and the need of individual transition planning in Finland*. Retrieved from: [openetti.aokk.fi/o4eu/Special%20ed%20and%20transition.doc](http://openetti.aokk.fi/o4eu/Special%20ed%20and%20transition.doc).
- Malloy, E. C. (2002). Democratic Access to Mathematics Through Democratic Education: An Introduction. English D.L. (ed.) *Handbook of International Research in Mathematics Education*. Lawrence Erlbaum Associates, Publishers. London:Mahwah, New Jersey.
- Meijer, C.J.W. (χ.χ.). *Special Needs Education in Europe: Inclusive Policies and Practices*. Retrieved from: [http://www.bmas.de/portal/21958/property=pdf/rede\\_cor\\_meijer.pdf](http://www.bmas.de/portal/21958/property=pdf/rede_cor_meijer.pdf).
- Norwich, B. (1980). *Reappraising special needs education. Special needs in ordinary schools*. London:Cassell.
- Skovsmose, O. and Valero, P. (2002). Democratic Access to Powerful Mathematical Ideas. English D.L. (ed.) *Handbook of International Re-*

- search in Mathematics Education*. Lawrence Erlbaum Associates, Publishers. London: Mahwah, New Jersey.
- Westphalen, K. (1982). *Reformation of analytic programs: Introduction to the reformed curriculum based on the experiences of Bavarian model*. Thessalonica: Kyriakidi. (in greek).
- Zoniou-Sideri, A., Karagianni, P., Nteropoulou-Nterou, E., Papastaurinidou G., Spandagou, H., (2005). The importance and the role of analytic program in the integrative education of blind children in Greece, in: *Education and Blindness. Modern tendencies and prospects*. Assiduity: A. Zoniou-Sideri and Spandagou, Athens:Greek Letters, p.p.: 205-217. (in greek).
- Kouroupetroglou, G. (2004). A complete model of Inclusion of Blind Students in the School via Advanced Systems of Information technology. (Free statement in the 10th Pan-Hellenic Congress of Physics, Loutraki). (in greek).
- Kroustalakis, G. (1997). Children with particular needs in the family and the school. Psychopedagogical intervention for one of advisory parents and teachers. Athens:OFFSET. (in greek).
- Lambropoulou, V., Panteliadou, S., and, Markakis E. (2005). Mapping-Curricula of Special Education: They were late one day... *Scientific Yearbook ARETHAS Volume III* (243-262). University Patras, Faculty Humanitarian and Social Sciences, Patras. (in greek).
- Liidakis, D. (2000). Educational programs for blind people. Athens:Pathway. (in greek).
- Polychronopoulou, P. (2003). Children and adolescents with special needs and possibilities. Modern tendencies of education and special support. Volume A'. Athens:Pathway. (in greek).
- Tzouriadou, M., (1995). Children with special educational needs. Psychopedagogical approach. Thessalonica:Prometheus. (in greek).
- YPEPTH, Department of Special Education, (April 2004). Differentiated Curricula for blind students. Athens:OEDB. (in greek).
- Ministry of National Education and Religions, Pedagogic Institute, department of Special Education. (30/3/2004) Program: "Mapping-Curricula of Special Education (2003SE04530072)". Person in charge of Action:Souzana Panteliadou. Volos. (in greek).

**Web page:**

[http://www.pi-schools.gr/special\\_education\\_new/html/gr/8emata/analytika/analytika.htm](http://www.pi-schools.gr/special_education_new/html/gr/8emata/analytika/analytika.htm)(9/2/2011,15:08).

## **Pupils talk about equity in mathematics class regarding the issue of questioning.**

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### **Abstract**

*This article explores aspects of the process of questioning which could limit the potential of student voice in the mathematics classroom. More particularly, we focus on the way pupils experience practices of questioning in the mathematics classroom from an equity perspective.*

### **Theoretical Background**

One of the links suggested between mathematics education and democracy (Christiansen, 2007), is the development of a democratic and egalitarian culture in the classroom. Within this framework, one special issue stressed is the examination of the power relationships constructed and maintained through the patterns of classroom discourse and questioning (Myhill & Dunkin, 2005) which can affect the susceptible democratic atmosphere of the mathematics classroom.

Regarding these power relationships between pupils, teacher and school mathematics activities, Hardy (2004) argues that the teacher creates a situation of supervision in which pupils' actions are exposed to the control of the teacher, who publicly approves or disapproves of pupils' answers to his/her questions. Pupils do not only "answer" to the teacher's demands, they are identified with an answer and learn to identify themselves with an acceptable or not behavior and way of thinking (Valero, 2008).

Teacher questioning behaviors are crucial with regard to which pupils learn. "If questions are vehicles for thought, then the questioning process determines who will go along for the ride" (Walsh & Sattes, 2005, p.9). Researchers argue that teachers tend to call on high achievers more frequently than low achievers, which provides these academically able pupils with an additional edge (Walsh & Sattes, 2005). In addition, the mathematical ideas of high-achievement students, during a mathematical

discussion, seem to be more respectable than the proposals of low-achievement students (Kafoussi et al., 2010).

If we piece together the fact that teachers have greater expectations for high-achievement pupils with the finding that ‘pupils who regularly ask and answer questions do better on subsequent achievement tests than pupils who do not’ (Walsh & Sattes, 2005), we should be concerned about the equal opportunities that all pupils have in the mathematics class.

The use of questions has been criticised on grounds of its effect on the pupils’ affective experience of mathematics (Boylan, 2002). Low achievers have to deal with high levels of ambiguity and risk when they respond, so they learn not to volunteer and not to answer when called upon and possibly, to ask fewer mathematical questions (Walsh & Sattes, 2005).

It also has been found that almost all pupils are concerned with the public and private shame of the possibility of being wrong (Boylan et al, 2001) which influences their participation in the mathematics class. Moreover, pupils feel nervous about asking questions as this can lower their mathematical status in small groups (Ivey, 1997). For a more engaged or meaningful participation to occur, pupils need to feel a level of security and freedom from anxiety. Mutuality between the question asker and question responder must be established in order to overcome the threatening, power-laden nature of question asking and to make questions less threatening for pupils (Hardy, 2004).

Furthermore, in order to realise any kind of democratic life in the classroom, pupils should be assigned with greater mathematical authority. They must feel and understand the importance of their contributions to the learning community of the classroom. Authority and control over interactions must be more equitably shared (Boylan, 2002).

### **Research context**

This work is part of a broader research program about professional development, focused on the pedagogical mathematical knowledge of seven teachers, based on their needs.

During the second year of the Program we focused on the issue of questioning. It is this challenge that this paper reports, by intervening in Greek primary mathematics classrooms, to shed light on an important component of mathematical discourse and more specifically on the way that pupils see and feel about questioning regarding issues of equity. The rationale underlying this research is based on the assertion that in order to create and maintain the democratic atmosphere in the mathematics classroom, teachers need to take into account pupils’ beliefs, perceptions and feelings about the learning

environment of their classroom (Waxman & Huang, 1996).

At the first phase of the research, researchers observed for one month the classes of three teachers of the community, and audio recorded their mathematical instructions. At the second phase, pupils' interviews were conducted. This paper focuses on pupils interviews and presents data from the class of one teacher of the community, with 2 years of educational experience, teaching a 6<sup>th</sup> grade class with 17 children.

The basic questions of interview were:

*Who asks more questions in the classroom, the teacher or the students?*

*Which pupils are the most difficult questions directed to?*

*Does the teacher learn from pupils' answers to his/her questions?*

*Would you like to be in a mathematics classroom, in which the teacher doesn't ask questions and assigns the work of questioning to pupils?*

### **Discussion**

Pupils opinions about questioning reveal power issues that limit the democratising potential of pupils' voice in the mathematics class. One aspect that was noticed is the dominance of teacher's authority. Pupils consider their teacher as the source of information. They recognize the teacher's role as the role of the expert in mathematics. They don't believe that teachers learn with and from their pupils. Pupil's voice reveals this perception.

*"...we learn, but the teacher doesn't learn anything from our answers to his/her questions because he already knows everything".*

Pupils maintain that their teacher sometimes treats differently high and low achievement pupils as he has different expectations from them.

*"...our teacher tries to help the low achievement pupils and he talks to them nicely, waiting for them, however he has more expectations from us (the high – achievement pupils). He has the expectation from us to answer his questions..."*

Furthermore, pupils develop implicit recognition of teachers' beliefs underlying the selection of pupils to which difficult questions are directed.

*"I think that our teacher asks all pupils difficult questions but... sometimes he says: Theodora will answer this question because she is for difficult stuff..."*

When the learners' perspectives are considered on what influences their active participation in the mathematics classroom, it is found that most pupils (14 out of 17) are concerned with the shame of the possibility of being wrong and a feeling of anxiety in view of the fear of disappointing



their teacher. These feelings results in the need for a self- confirmation of the response. They want to be sure about their answer before announcing it to the teacher and the class.

*"I want to be absolutely sure about my answer... about what I am going to say"*.

*"I am assailed by doubts. I am not sure about my answer and I am afraid that it is wrong. I feel hesitation because someone may laugh at me. Even when I know the answer... I don't have the certainty that I know it. I raise my hand but with hesitation"*.

Similar reasons stand, regarding pupils' practice of not asking many questions in the mathematics class.

*"sometimes I am afraid of sounding stupid when asking a question. I think that this is the reason why most pupils don't ask many questions in our class. But I don't know why we do not feel liberated in this classroom. In the previous years we had more strict teachers. Now, this teacher is closer to us"*.

When asked to play a part in the questioning process pupils feel they are at a loss, for they have never even considered what a classroom in which the teacher stops asking questions and pupils take on the process of questioning would be like. Below, a pupil shares her thoughts about this imaginary class.

*"I would not like a classroom like this, because a teacher is a teacher and a pupil is a pupil. The role of the teacher in the mathematics classroom is to pass on the subject matter to pupils, to help them when they are stuck at problems and to ask them questions. The role of pupils is to attend, study, answer the questions and be consistent with their school duties"*.

The reframing of mathematics classrooms is necessary so that both the relationship between participants and the relationship of the participants with mathematics is changed (Povey, 2003). This means teachers and pupils should be equal partners in the teaching-learning process. In this way, pupils will have the opportunity to communicate their views and to feel they have a say in their own learning and in the direction of classroom discourse (Ellis & Malloy, 2007). Making the vision of a democratic mathematics classroom a reality for all students, is both an essential goal and a significant challenge.

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## References

Boylan, M, Lawton, P. & Povey, H. (2001). "I'd be more likely to speak in class if...": Some students ideas about increasing participation in

- whole class interactions. In M. Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, (vol. 2, pp. 201-208). Utrecht: Freudenthal Institute.
- Boylan, M. (2002). "Teacher questioning in communities of political practice. *Third International Mathematics Education and Society Conference* (Vol. 1, pp. 182-191). Helsingor, Denmark.
- Christiansen I. (2007). Some tensions in mathematics education for democracy (pp.49 – 62). *TMME*, Montana University.
- Ellis, M., & Malloy, C. (2007). Preparing teachers for democratic mathematics education. In D. Pugalee, A. Rogerson, & A. Schinck (Eds.), *Proceedings of the Ninth International Conference: Mathematics Education in a Global Community* (pp. 160- 164). Charlotte, NC.
- Hardy, T. (2004). "There's no hiding place". Foucault's notion of normalization at work in a mathematics lesson. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 103-119). Greenwich: Information Age.
- Ivey, K. (1997). Warning: Asking questions may lower your mathematical status within groups. *Proceedings of the Nineteenth Annual Meeting: Psychology of Mathematics Education* ( vol. 2, 619- 625). North America, Ohio State University.
- Kafoussi, S., Chaviaris, P., & Dekker, R. (2010). Factors that influence the development of students regulating activities as they collaborate in mathematics. *International Journal for Mathematics in Education*, 2.
- Myhill D. & Dunkin F. (2005). Questioning Learning, *Language and Education*, 19 (5) 415- 427.
- Povey, H. (2003). Teaching and learning mathematics: Can the concept of citizenship be reclaimed for social justice? In L. Burton (Ed.), *Which Way Social Justice in Mathematics Education?* (Vol. 3, pp. 51-64). Westport, Connecticut: Praeger.
- Valero, P. (2008). Discourses of power in mathematics education research: Concepts and possibilities for action. *PNA* 2(2), 43-60.
- Walsh, J. A., & Sattes, B. D. (2005). Quality Questioning: Research – based practice to engage every learner. *Thousand Oaks*, CA: Corwin.
- Waxman, H.C., & Huang, S. (1996). Motivation and learning environment differences in inner-city middle school students. *Journal of Educational Research*, 90(2), 93-102.

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## GREEK SCHOOL TEXTBOOKS AND CHILDREN WITH ASPER- GER SYNDROME: THE CASE OF MULTIPLICA- TION

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### **Abstract**

In this study, which was based on a case study of a child with Asperger syndrome in a pilot research, we investigated whether the multiplication tasks existing in Greek textbooks of 3rd and 4th grade help children with AS to gain multiplicative reasoning. The results showed lack of appropriate tasks which appeared in the pilot research that help children with AS to develop multiplication strategies.

### **Theoretical background**

Disorder of Asperger described as cerebral dysfunction (Aylward et al., 1999). It is a pervasive developmental disorder and belongs to the continuum of autistic spectrum disorders (ASD). Individuals with Asperger Syndrome (AS) present social difficulties such as social interaction, social communication and social imagination (Wing & Gould, 1979). They also present sensory sensitivity and motor clumsiness (Wing, 1981; Frith, 1991; Attwood, 2007). Most research, which was based on psychological and psychometric tests for academic achievement has indicated that children with AS demonstrate average mathematical ability and difficulties in problem solving tasks (Chiang & Lin, 2007). They also present difficulties in numerical operations (Griswold et al., 2002) and they can not apply the mathematical knowledge in a real - life situation (Jordan, 2003). Individuals with AS, while presenting basic skills in reading and oral expression, however, seem to have difficulty in reading comprehension (Griswold et al.,

2002; Attwood, 2007). They are more detailed and visualisers (Cumine et al., 1998; Attwood, 2007).

Children up to age 9 to 10 years use more intuitive model of repeated addition to problems of asymmetric multiplication situations (Bell et al., 1984; Greer, 1992; Mulligan, 1992). The development of multiplicative reasoning requires the student to create "composite" units, which can be utilized as material in other cognitive operations (Steffe, 1988). Initially, the student usually counts by one (*numerical composites*), then he/she identifies his/her composite experience as "one thing" (*abstract composite units*) and then not only understands the composite units as one thing, but may continue the numbering of the composite units from an intermediate element of numbers' order (*iterable units*) (Steffe, 1988; Bryant & Nunes, 2009). The student who can construct iterable units is also able to use them as "material" in further cognitive operations such as the relationship between part - whole between two different iterable units (e.g. if he/she knows the  $7 \times 6$ , then to find  $7 \times 7$  by counting 6 seven times and seven times 1) (Boufi, 1996).

The purpose of our research, based on a case study of a child with Asperger's syndrome, was to investigate whether the multiplication tasks existing in Greek textbooks of 3rd and 4th grade (8-10 years old) help children with AS to gain multiplicative reasoning. Our research question was closely related to the promotion of learning opportunities for children with AS-based tasks in school textbooks.

### Method

Our pilot study, conducted in August 2011 in a case study (9 years old boy who had finished 4th grade, with AS and medium performance in mathematics - according to WISC- Wechsler Intelligence Scale for Children) investigated what types of multiplication tasks may be more effective in children with AS aged 10 years to develop spontaneously multiplication strategies for understanding the relationship of part-whole between two iterable units. For this purpose, we used the following categories for multiplication tasks (Noulis & Kafoussi, 2011).

1. *Manipulative material* (MM) (forming groups (FG) with representation of the factors with two materials and the formation of rectangular (FR) with representation of factors with one material)
2. *Pictorial Representations* (PR) (FG, FR by realistic (RC), realistic artificial (RAC) and mathematical context (MC) (Tatsis & Skoumpourdi, 2009) - and number lines (NL) - structured (S), semi-

structured (SS) and empty (E)-Skoumpourdi, 2008)

3. *Word Problems* (WP) (corresponding to PR tasks - FG, FR & NL)
4. *Numerical Calculations* (NC) (horizontal (O) and vertical (V))

Each task was given in three phases ( $a \times b$ ,  $(a + 1) \times b$  and  $(a + 1) \times (b + 1)$ ) to investigate if the subject understands the iterable units and uses the advantage of understanding the relationship of part-whole.

In this investigation, we tried to categorize the multiplication tasks given in textbooks of 3<sup>rd</sup> and 4<sup>th</sup> grade, according to the classification of the above pilot study, both in the “Student’s book” (SB) and in the “Student’s Notebook” (SN). The research focused on those grades because in the 3<sup>rd</sup> grade learning of multiplication facts, two-digit number by one-digit and two-digit by two-digit product is carried out, and the 4<sup>th</sup> grade focuses on multiplication strategies.

### Results

Analysis of textbooks showed that in the third grade there were 14 chapters (28 didactic hours in the “Student’s book” (SB) and in the “Student’s Notebook” (SN), involving a total of 103 multiplication tasks (51 in SB and 52 in SN). The following table shows the classification of tasks.

Table 1. Number of tasks per category of multiplication tasks in 3<sup>rd</sup> grade textbooks

MM (1 task)		PR (16 tasks)									WP (35 tasks)			NC (51 tasks)	
FG	FR	FG			FR			NL			FG	FR	NL	O	V
		RC	RAC	MC	RC	RAC	MC	S	SS	E					
0	1	0	1	1	0	1	13	0	0	0	31	1	3	45	6

This table shows that the MM and PR tasks are in a very small percentage: 0.1% and 15% respectively in all the tasks. In the tasks of PR (15.5%), tasks of a realistic context are absent, while artificial and realistic context covers only 12.5%. From the PR tasks, 87.5% are in a MC and they are given in the form of squares, bullets and mosaics.

From WP (34%), 60% are problems where some data within images or shapes are given, but not as part of PR tasks. FG tasks in WP are 31, while only 2 in PR. FR tasks in WP is 1, while in PR are 14 and NL tasks in WP are 3, while in PR is 0. This indicates a mismatch in the three subcategories (FG, FR and NL) of WP and PR. The numerical calculations cover 49.5% of

total tasks.

In the fourth grade, there were 5 chapters (8 didactic hours) in SB and in SN and a total of 35 tasks (16 in SB and 19 in SN). Table 2 shows the classification of the tasks.

Table 2. Number of tasks per category of multiplication tasks in 4<sup>th</sup> grade textbooks

MM (0 task)		PR (2 tasks)									WP (14 tasks)			NC (19 tasks)	
FG	FR	FG			FR			NL			FG	FR	NL	O	V
		RC	RAC	MC	RC	RAC	MC	S	SS	E					
0	0	0	1	0	0	1	0	0	0	0	9	1	4	14	5

Table 2 shows again that tasks of MM and PR are almost nonexistent (0% and 6% respectively). Regarding the word problems (40%), approximately 80% require estimate or construction problem with given numbers. The tasks of WP and PR show again mismatch in two of the three subcategories (FR and NL). The numerical calculations cover 54% of total tasks.

The subject of AS case study presented spontaneous multiplication strategies in the categories presented in the table below.

Table 3. Spontaneous multiplication strategies of the subject of the pilot research

MM (2 tasks)		PR (9 tasks)									WP (6 tasks)			NC (6 tasks)	
FG	FR	FG			FR			NL			FG	FR	NL	O	V
		RC	RAC	MC	RC	RAC	MC	S	SS	E					
0/1	0/1	1/1	-	0/1	1/1	1/2	1/1	0/1	0/1	0/1	0/2	0/1	0/3	0/3	0/3

Table 3 shows that the subject with the AS in our pilot research, developed spontaneously multiplication strategies only in the PR tasks and with success in a realistic context. Also in the WP tasks, matching PR tasks, he tried to develop such strategies, but he failed, probably because of difficulties in reading comprehension (Noulis & Kafoussi, 2011).

### Discussion

Based on the analysis of textbooks and the results of the pilot research

it could be argued that the existing multiplication tasks do not provide equal learning opportunities for children with AS. There are few PR tasks and moreover complete absence of the realistic context of them, which help children with AS to develop strategies as they are people who think in pictures (Attwood, 2007). There are no tasks with manipulative material that might help some children with AS in the development of multiplicative reasoning. On the other hand, the WP tasks do not correspond to a few PR. While most of the WP tasks correspond to the formation of groups, most PR tasks are of rectangular formation. Moreover, mainly in the 4th grade, WP tasks of school textbooks require students to make a lot of assumptions or construct problems that confuse children with AS (Tanguay, 2006).

Overall, there appears no consistency between tasks of school textbooks, which might lead children with AS in a deeper understanding of multiplication situations and facilitate their participation in the classroom.

### **References**

- Attwood, T. (2007). *The Complete Guide to Asperger's Syndrome*. Jessica Kingsley, UK.
- Aylward, E., Minshew N., Goldstein, G., Honeycutt, N., Augustine, A., Yates, K., Barta, P. & Pearlson, G. (1999). MRI volumes of amygdale and hippocampus in non- mentally retarded autistic adolescents and adults. *Neurology*, 53, 2145 – 2150.
- Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and context. *Educational Studies in Mathematics*, 15, 129-147.
- Boufi, A. (1996). Child's multiplication thinking as teaching basis. *Proceedings of the first Hellenic Conference of Mathematical Education*, 261 – 276 (in greek)
- Bryant, P. & Nunes, T. (2009). Multiplicative reasoning and mathematics achievement. In M.Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of 33rd PME Conference*, Vol. 2, 217-224. Thessaloniki, Greece.
- Chiang, H. & Lin, Y. (2007). Mathematical ability of students with Asperger syndrome and high-function autism: A review of literature. *Autism*, vol. 11, 547-556.
- Cumine, V., Leach, J., Stevenson, G. (1998). *Asperger Syndrome. A Practical Guide for Teachers*
- Frith, U. (1991). Asperger and his syndrome. In U. Frith (ed.), *Autism and*

- Asperger Syndrome*. Cambridge: Cambridge University Press.
- Greer, B. (1992). Multiplication and division as models of situations. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, (pp. 276-295). New York: Macmillan Publishing Co.
- Griswold, D. E., Barnhill, G. P., Myles, B. S., Hagiwara, T. & Simpson, R. L. (2002). Asperger Syndrome and Academic Achievement. *Focus on Autism and Other Developmental Disabilities*, 17, 94–102.
- Jordan, R. (2003). School-Based Intervention for Children with Specific Learning Difficulties. In M. Prior (Ed.), *Learning and Behavior Problems in Asperger Syndrome*. The Guilford Press, NY, 212 – 243.
- Lemonidis, X., Theodorou, E., Nikolantonakis, K. & Spanaka, A. (2007). *3<sup>th</sup> grade Mathematics*. (student's book, notebook a, b & c issue). PI, Hellenic Letters, Athens (in greek)
- Mulligan, J. (1992). Children's solutions to multiplication and division word problems: a longitudinal study. *Mathematics Education Research Journal*, Vol. 4(1) 24 – 41.
- Noulis, I. & Kafoussi, S. (2011). Multiplication Strategies children with Asperger Syndrome develop: a pilot research. *Proceedings of 28th Hellenic Conference of HMS*, Athens, 523 – 537 (in greek)
- Skoumpourdi, C. (2008). Numberline's representation in elementary school's textbooks of Mathematics. *Research in Didactics of Mathematics* 3, 67 – 87. Kedros Education. (in greek)
- Steffe, L. (1988). Children's Construction of Number Sequences and Multiplying Schemes. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades*, (pp. 119 – 140). LEA, NCTM.
- Tanguay, P., (2006). *Nonverbal Learning Disabilities*. Jessica Kingsley, London and Philadelphia
- Tatsis, K. & Skoumpourdi, C. (2009). A study about activities' context of the 1st Grade school textbook of Mathematics. *Proceedings of 3<sup>rd</sup> Conference of GARME*, 383 – 392. University of the Aegean, Rhodes (in greek)
- Vamvakoussi, X., Kariotakis, G., Bobotinou, A.D. & Saitis, A. (2006). *4<sup>th</sup> grade Mathematics* (student's book, notebook, a & c issue). PI, Pataki Publications, Athens (in greek)
- Wing, L. (1981). Asperger's syndrome: A clinical account. *Psychological Medicine*, 11, 115–129.
- Wing, L. & Gould, J. (1979). Severe impairments of social interaction and associated abnormalities in children: epidemiology and classification. *Journal of Autism and Developmental Disorders*, 9, 11-29



## The arrogant exclusion of intuition in the teaching of mathematics: a sideways reflection

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**Abstract** In this paper, we discuss the way that intuition is employed in the teaching of university mathematics. We argue that the exclusion of intuition may prevent many students from entering the mathematical world. We reflect upon historical, empirical and anecdotal evidence to identify didactic principles that may help in the appropriate inclusion of intuition in the university mathematics teaching practices.

### Mathematics and intuition

Intuition appears to be intertwined with mathematical thinking. Hadamard (1945) viewed intuition as an indispensable tool in the mathematicians' research. The role of intuition in mathematical thinking varies, amongst highly successful mathematicians; for example, the 'intuitive' Hermite and the 'analytic' Weierstrass (Poincaré, 1913), Atiyah's preference to build definitions from meaning and Mac Lane's preference to work the opposite way (Mac Lane, 1994). Furthermore, Fischbein (1987) stressed the psychological functions of intuition (positive or negative) in mathematical thinking. The mathematicians' reservations about perceptual intuition can be traced back to Plato. Even Kant's pure intuition became the topic of debate in the 19<sup>th</sup> century, which resulted in the mathematicians' attempt to construct a purely axiomatic logically-founded mathematical structure. Consequently, though intuitive thinking appears to be present in various aspects of *mathematical practices*, the mathematicians seem to prefer discussing the *mathematics product* freed from intuition.

*In this paper*, we discuss the exclusion of intuition in the teaching of university of mathematics and its implications in mathematics learning, the

mathematical community and the corpus of mathematical knowledge itself. We argue that the exclusion of intuition may, on the one hand, prevent many students from even entering the mathematical world and, on the other hand, may prevent many others from fulfilling their potential understanding of the mathematical ideas (cf. Sharygin & Protasov, 2004). Our discussion is clearly linked with the conference theme and, in specific, with the notion of ‘democratic access’ to mathematical ideas by the learners.

### **Intuition, mathematical ideas and axiomatic system**

Though intuition lead to the symbolic formulation of various mathematical ideas (Hadamard, 1945), it also misguided mathematicians, thus rendering it untrustworthy (Hahn, 1956). Consider, for example, Jordan’s (1909) definition of a curve: *A curve is the set of all points of the plane the coordinates  $(x, y)$  of which are continuous functions  $x = f(t)$ ,  $y = g(t)$ , where  $0 \leq t \leq 1$ ,  $t \in \mathbb{R}$ .* Jordan’s definition derived from the symbolic formulation of his intuition, as an attempt to generalise his inductive *concept image* (Tall & Vinner, 1981). Nevertheless, Peano’s example (in 1890) showed that Jordan’s definition produced objects that could not be identified as curves (a square), according to the intuitive understanding of a curve (Engelking & Sieklucki, 1992). Hence, the symbolic formulation enabled the mathematicians to check whether or not this definition is a result of a *generic generalisation* (Harel & Tall, 1991); that is, whether or not this definition is semantically compatible with the intuitive understanding of a curve.

Consequently, our reflection upon historical evidence suggests that the transformation of intuitive ideas into mathematical definitions *within* an axiomatic system draws upon a constant dialectic between intuition and symbolic formulation. Several mathematical objects can be defined as generalisations of inductive intuitive thoughts, but only those that entail the element of ‘generic’ are objectified within the mathematical world i.e. those the logical products of which survive the test within the axiomatic system *and* at the same time are compatible with mathematical intuition. Once this two-faceted consistency is achieved, mathematics may be ‘liberated’ from the ‘chains’ of intuition, thus reaching its ‘true ideal potential’.

Furthermore, this historical development may reveal aspects of a psychological development. For a mathematician, intuition may act as the wheels of an airplane: they are necessary for an airplane to leave the ground, but they are useless in its actual flying. In a similar manner, intuition is necessary to reflect upon the perceptual appearances of mathematical objects, but it is not part of those objects. Moreover, the wheels are necessary for the

plane to be serviced and refuelled, in order to continue safely its flights. Similarly, intuition is necessary for the mathematician, on the one hand, to verify the semantic compatibility of the definition ('service') and, on the other hand, to search for new ideas that may enrich and/or expand the structure of the mathematics construction ('refuel').

### **Intuition and university mathematics textbooks**

In accordance with the exclusion of intuition of the formal mathematical constructions, the school of Bourbaki in the 60s-70s (Davis & Hersh, 1981; Kneebone, 1963) and their purist view of mathematics had a great impact on the textbooks that were used in the universities. However, the New Mathematics movement soon showed its inadequacies. The radical technological advances changed the social and communication framework, with the pictorial representations becoming dominant in these new settings (Healy, 1999). Thus, intuition started to re-appear in widely acceptable mathematics textbooks (for example, Spivak, 1994).

The effect of the socio-political environment in the textbooks is evident when we consider the example of the Polish school of mathematics. Mathematics, in the socialist period of the country, became part of 'popular' culture and a pro-intuitive way of teaching was supported to make mathematics accessible to a broader audience. Thus, a plethora of textbooks (in Polish and in English) were written along these lines; notably, Engelking and Seklucky's (1992) topology. Importantly, these textbooks are not considered to be inferior in their mathematical content and rigour than other purely symbolically written textbooks.

### **Intuition and university mathematics teaching**

The effects of the Bourbaki school in the teaching within university mathematics departments is still evident nowadays, even amongst young professional mathematicians. These teaching practices have a pragmatic effect on the education of future mathematicians. For example, a course that is selected by more than 500 students is gradually 'dropped' by the vast majority of the students, resulting in less than 10 to actually sit the exams. The students argue that the lecturer's choice to present the topics of the course in a pure symbolic manner made it inaccessible for them. The following year, the same course is taught by another lecturer, who prefers to teach by linking the material with the students' intuition; by employing visual representations, meaningful descriptions and operational justifications in his teaching. As a result the vast majority of the enrolled students sit the exams. Thus, it

seems that the number of the students that choose to learn about a topic (successfully or not) appears to depend on the inclusion or not of pro-intuitive teaching practices.

Moreover, Weber (2004) argued that the definition-theorem-proof paradigm usually found in universities may be differentiated amongst three teaching styles: *logico-structural* (focussing on axiomatic definitions and logically derived proofs), *procedural* (focussing on proof techniques and heuristics and, secondarily, on the logical validity of the proof), and *semantic* (focussing on the intuitive meaning and its links with the symbolic formulation). Importantly, Weber noted that different teaching styles seem to be linked with different learning outcomes. Furthermore, different learning and thinking preferences appear to be linked with different proving strategies (Moutsios-Rentzos, 2009).

Consequently, the exclusion of intuition from the teaching of university mathematics may affect accordingly the future mathematicians' way of learning and thinking. Such effects are further strengthened by the fact that that 70% of the tasks included in university mathematics textbooks can be successfully dealt without any semantic references (Bergqvist, 2007).

### **Concluding remarks**

Most of the mathematicians appear to *know* that intuition plays a crucial part in the formation of the mathematical construction. Moreover, they appear to *know* the limitations of intuition and they *know* that mathematics is clearly differentiated from their inductive intuitive origins. Nevertheless, it appears that the mathematicians have *forgotten* their own initial experiences with mathematics. Tall (2002) noted that even "Plato was very young when he was born" (p. 92).

Furthermore, Van Hiele (1986) stressed the fact that the lecturers teach at a different level of abstraction from their students, implies that they effectively speak a different language, thus rendering their communication difficult or even impossible. Moreover, van Hiele noted that this 'technical jargon' may prevent the communication of mathematical knowledge to new members or it may be used as "a smokescreen behind which one tries to protect self-conceit" (p. 126).

We posit that it is the responsibility of the lecturers to share their meanings with their students. The inclusion of intuition in the university pedagogy may help in this effort, as it allows the employment of various means of representation that may help to bridge the communication gap between the students and the lecturer. In this way, more students will be able to fol-

low the changes of level of abstraction that occur within a lecture (Van Hiele, 1986).

Furthermore, it is important to teach students the ways to utilise their intuitions within an axiomatic logically derived system. For example, in our experience, the logically unnecessary addition in the definition of the limit of the phrase ‘no matter how small’ (complementary to the phrase ‘for every  $\epsilon$ ’) may help more students in better accessing the idea of convergence. Subsequently, the students, working within the axiomatic system, may realise the logical redundancy of this addition, which will also help them gaining deeper understanding of the essence of doing mathematics itself. Thus, the students will learn to work with their intuitions and to regard a statement as mathematical knowledge only when it is logically founded within an axiomatic system.

Moreover, we claim that the use of intuition may help the students to realise the counter-intuitive nature of various mathematical objects. For example, visual pro-intuitive representations of the Cantor’s set and Cantor’s ‘step’ function’, appropriately presented by the lecturer, may help in the students’ accessing challenging counter-intuitive mathematical ideas about infinity (countability and uncountability; Spyrou, 2003) and continuity.

In conclusion, in line with the phenomenological principle back to “the things themselves” (Husserl, 1970, p. xxx) –referring to a science that has been hidden behind symbolism and lost its links with its genetic appearance– we argue for the inclusion of intuition in university mathematics teaching practices.

### References

- Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *Journal of Mathematical Behavior*, 26(4), 348-370.
- Davis, P. J., & Hersh, R. (1981). *The Mathematical Experience*. Boston: Houghton Mifflin Company.
- Engelking, R., & Sieklucki, K. (1992). *Topology: a geometric approach*. Berlin: Helderman Verlag.
- Fischbein, E. (1987). *Intuition in science and mathematics: An educational approach*. Dordrecht: Reidel.
- Hadamard, J. (1945). *An essay on the psychology of invention in the mathematical field*. Princeton: Princeton University Press.
- Hahn, H. (1956). The crisis in intuition. In J. R. Newman (Ed.), *The world of mathematics* (Vol. 3, pp. 1956-1976). London: Allen and Unwin.

- Harel, G., & Tall, D. (1991). The General, the Abstract, and the Generic in Advanced Mathematics. *For the Learning of Mathematics*, 11(1), 38-42.
- Healy J. M. (1999). *Endangered Minds*. New York: Simon & Schuster.
- Husserl, E. (1970). *The crisis of european sciences and transcendental phenomenology*. Evanston: Northwestern University Press.
- Jordan, C. (1909). *Cours d'analyse de l'Ecole Polytechnique*. Paris: Gauthier-Villars.
- Kneebone, G. T. (1963). *Mathematical Logic and the Foundations of Mathematics: an introductory survey*. London: Van Nostrand.
- Mac Lane, S. (1994). Response to "Theoretical mathematics: Toward a cultural synthesis of mathematics and theoretical physics", by A. Jaffe and F. Quinn. *Bulletin of the American Mathematical Society (New Series)*, 30, 190-193.
- Moutsios-Rentzos, A. (2009). *Styles and strategies in exam-type questions*. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 145-152)*. Thessaloniki, Greece: PME.
- Poincaré, H. (1913). *Science and hypothesis*. London: Walter Scott Publishing.
- Sharygin, I. F., & Protasov, V. Yu. (2004). *Does the school of the 21st century need geometry?* *Proceedings of the 10th International Congress on Mathematical Education National Presentation: Russia (pp. 167-176)*. Copenhagen, Denmark.
- Spivak, M. (1994). *Calculus (3<sup>rd</sup> edition)*. Houston: Publish or Perish.
- Spyrou, P. (2003). The alternation of representations in case of Cantor set. *Scientia Paedagogica Experimentalis*, 39(1), 83-102.
- Tall, D. (2002) 'Differing modes of proof and belief in mathematics', in Lin, F.-L. (Ed.), *Proceedings of the International Conference on Mathematics: Understanding proving and proving to understand (pp. 91-107)*. Taipei, Taiwan: National Taiwan Normal University.
- Tall, D. O. & Vinner, S. (1981). Concept image and concept definition with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169.
- Van Hiele, P. M. (1986). *Structures and insight: A theory of mathematics education*. Orlando, FL: Academic Press.

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## LEARNING THE DISCOURSE OF ONE'S OWN EXCLUSION

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### INTRODUCTION

The themes of of CIEAEM in 1979, 1985, 1999, 2005, 2011 and 2012 are testimonies of the intense desire in the mathematics education community to contribute to the success of a just society, where every citizen has access to the power of mathematics. In this context students are to be seen as agents of their own knoweldge: they shall be capable of *taking decisions* to use mathematics for participation in democracy. However, agency includes the *opportunity to take a decision not to participate*. Capitalism requires to initiate *some* students into patterns of such decisions not to participate. By this admittedly cynical claim, I try to open up for a critical discussion about some unpleasent truths behind the common-sense everyday discourse about schooling. With Popkewitz (2007) I claim this as a point of departure for the developement of a critical stance:

To make the naturalness of the present as strange and contingent is a political strategy of change; to make visible the internments of the commonsense of schooling is to make them contestable (Popkewitz, 2007, p. xv).

I settle my investigation in a secondary school that can be labeled as *marginalised* or *underpriveleged*. By this, I want to raise the issue of *who* is likely to be set up for taking decisions not to participate. My research question is:

How does an underpriveleged mathematics classroom contribute to the discourse of learning not to participate in democracy?

### METHODOLOGY

According to the analysis of Pais (2012) mathematics education postulates equity and democracy as a final goal of schooling, which is achievable by some kind of evolutionary process that demands *more* research and *more effective* research to overcome exclusion. Its goal (*everyone* benefits from the power of mathematics), but also the tracks leading towards it are taken as *a necessity*. The problem of equity could then be solved if we marshal our efforts to overcome the obstacles, namely the instances of exclusion that are posited as *contingent* over time.

Drawing on the Slovenian philosopher Slavoj Žižek (1991), Pais uses the Hegelian concept of *negative particularity* to invite us to investigate our understanding of *the universal* (schooling) and *the particular* (exclusion). According to Pais (2012), in schooling under Capitalism, exclusion is not only a mistake (albeit contingent) of the system, but an inherent mechanism. The dialectical twist Pais (2012) invites us to take addresses what is commonly considered a necessity as a contingency and what is commonly considered a contingency as a necessity. Thus, "what is *necessary* is precisely the existence of those who fail, [...]. The antagonistic character of social reality - the crude reality that in order for some to succeed others have to fail - is the *necessary* real which needs to be concealed so that the illusion of social cohesion can be kept." (Pais, 2012, p.11). This enables us to regard *failure* "as the symptomal point at which the truth of the system becomes visible." (Pais, 2012, referring to Žižek, 1991, p. 145)

In the following I aim to present a study of educational failure, posited as a necessity of the school system in order to contribute to making visible the internal logic of systematic exclusion by analysing a symptom. I will proceed in two steps:

- 1) Positing resistance to a discourse of one's own exclusion as a symptom of the necessity of educational failure in the particular context.
- 2) Positing the necessity of educational failure in the particular context as a symptom of the institution of schooling.

### THE SOCIAL AND POLITICAL CONTEXT OF THE STUDY

The data reported in this paper stem from videotapes that have been recorded during fourteen consecutive mathematics lessons in September 2009 at a low-streaming school in Berlin, Germany, in a neighbourhood that in the public discourse is often referred to as a ghetto. The fourteen students in the classroom are between the ages of twelve to fourteen. The students in



the classroom all come from cultural minorities and all know the instructional language only as a second language learner. The lowest stream of the German school system is considered as systematically producing educational failure (e.g. Rösner, 2008). The observed teacher acknowledged this in an extensive semi-structured interview. Reporting on her twenty-five year lasting career at this school, students who found a job after graduation have been singular events. Graduation was described as neither the exception nor the rule.

### **ANALYSIS - TWO CASES OF RESISTANCE**

In an exemplary report on the mathematics classroom under analysis, we (Straehler-Pohl and Gellert, 2011) have described the pedagogy enacted as one that, "in order not to overcharge – infantilizes students and – in order to enable classroom management – objectifies students. [...] Learning in such mathematics classrooms adds to the underprivileged conditions that these learners face". I claim that most of the students are aware of the fact that participation in this kind of mathematics education won't bring them back on the road towards participation in democracy. Thus, taking the decision to participate in the classroom activities means to take part in the construction of one's own exclusion: *participation in the activity* means *waiving of democratic participation*. It implies not to make use of the opportunity to take a decision not to participate in a senseless and discriminating activity. I will now provide two examples of students who chose to resist to this *discourse of one's own exclusion* and report on the consequences it had for them.

#### ***The case of Melinda***

Melinda's resistance is characterized by a total refusal of the teachers' authority (most of the times two teachers are present in class). In the beginning of the first math class, Melinda answered the first teacher's request of her mood: "I am feeling bad because today we have class with this teacher [pointing at the second teacher]". During the lesson, Melinda spent quite some time talking to Mariella in a foreign language, which was mostly ignored by the teacher, though two times the teacher spoke out an admonishment in a rather calm voice. When Mariella was demanded to compute a task at the blackboard (887-339), Melinda shouted at her: "what are you doing bitch?". Though understandable quite loud and clear, this interruption remained non-sanctioned. However, a few minutes later, Melinda "col-

lected" (teacher) her third calmly spoken admonishment and got thrown out of the classroom for the rest of the day. The following day, math class took a similar course, resulting in Melinda being thrown out. The third day, Melinda did not appear anymore. She had been expelled from school.

### *The case of Hatice*

On the third day Hatice, who already was known as a truant to the teachers, appeared in class for the first time. In class, Hatice was very quiet and busy, doing the calculations demanded of her by the worksheet (such as 9700-300). The next time Hatice appeared in class, she completed three worksheets in twenty minutes including 186 simple multiplication exercises. The fourth sheet - that was given to Hatice "as a repetition" (teacher) - claims on top of the page that, "it is now getting harder and harder", and concludes at the bottom that, "if you solve all the problems correctly, you are the king of computations". When Hatice came back to her seat and started filling in the solutions on the work-sheet (see Fig. 1), the second teacher asked her to "read the instructions first". However, there were no in-

1. 4509 + 2 =  
3804 + 7 =  
7205 + 9 =

Fig. 1: Task without instructions

4000	4500	1740	2375	+	2	20	200
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Fig. 2: Task with the instruction: "copy the exercises as blocks into your workbook and compute them."

structions for the first 54 exercises. Ignoring Hatice's confusion, the teacher commanded, "read!". It was not before task no. 7 (see Fig. 2), that there was an instruction. Hatice did not show up anymore during the following lessons.

### *Discussion of the two cases*

The case of Melinda seems to be a classical case of intentional resistance (Lanas & Corbett, 2011). She makes very clear that she will not acknowledge the teachers' authority and thus will not participate in any of the activities imposed by the teacher. The teachers on the other side do not use their authority to react oppressively. Admonishments are implicit in some sort of count-down leading towards physical exclusion from the class. The message is clear: Melinda's resistance is her *own choice*, she gets the opportunity to decide to participate in the classroom activity herself. Thus Melinda is constructed as *agentive in her own exclusion*.

The case of Hatice (when being present) is quite contrary. By apparently taking the decision to participate in the activity seriously she goes

through it so fast, that the meaninglessness of the whole activity becomes visible for all those participating. However, she seems to be participating in a different activity than her class-mates: It seems as if struggling with the work-sheet is an integral part of the game, the participants are playing. Thus, Hatice also makes use of the *opportunity to decide not to* participate in the classroom activity. It seems as if this does not remain unnoticed by the second teacher: instead of complimenting her for carrying out (correctly) three times as much calculations as her peers, she *invents* some illusive instructions to slow down Hatice. We assert (and the teacher did also in the interview) that Hatice, as a consequence of truancy, will be one of the many cases of students who leave school without any form of graduation.

### DISCUSSION

1) Positing resistance to a discourse of one's own exclusion as a symptom of the *necessity* of educational failure in the particular context.

The cases of Hatice and Melinda are reports of instances, where students' decisions not to participate in the practice of infantilizing and objectifying activities (Straehler-Pohl & Gellert, 2011) leads to their individual educational failure (expulsion, not graduating, etc.). For the others in the class, these two students function as examples for the apparent fact that educational failure is an individual *contingency*. Both students could have made use of their agency in a very different way. However, given the case that one participates in these infantilizing and objectifying activities, educational failure *actually* is a *necessary outcome* in the long run.

2) Positing the necessity of educational failure in the particular context as a symptom of the institution of schooling.

We do risk a lot when we keep on considering educational failure as the unpleasant obstacle on the didactic road towards salvation. I want to address that it is not only the teacher's didactical inappropriateness of *organizing* meaningless activities, but also the majority of students who actively *participate* in the *game of failure*, however demanding and meaningless the activities are. Together, the teachers and the students create a system, where failure *is* a necessity and a predictable result of the process. The reasons neither lie in the teacher's individual pedagogical ineptitude nor in a lack of professionalism; it is rather, the result of twenty-five years of experience in an institution that constantly fails. The reasons also neither lie in the students' cognitive inability nor in their bad behaviour; it is rather, a result of six years of school, showing them that *they are not the ones who profit from*

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*making use of their agency.* Thus, the *necessity* of educational failure is a *result* of the systematic organization of segregation.

#### REFERENCES

- Lanas, M., & Corbett, M. (2011). Disaggregating student resistances: analyzing what students pursue with challenging agency. *Young: The Journal of Scandinavian Youth Studies* 19 (4), 417-434.
- Pais, A. (2012). A critical approach to equity in mathematics education. In: Skovsmose, O. & Greer, B. (eds): *Opening the Cage: Critique and Politics of Mathematics Education*. Rotterdam: Sense.
- Popkewitz, T. (2007). *Cosmopolitanism and the Age of School Reform: Science, Education, and Making Society by Making the Child*. New York: Routledge.
- Rösner, E. (2008). *Hauptschule am Ende. Ein Nachruf*. Münster: Waxmann.
- Straehler-Pohl, H., & Gellert, U. (2011). Learning mathematics as a "practically able" learner: An instance of institutional denial of access. *Quaderni di Ricerca in Didattica* (paper presented at CIEAEM 63).
- Žižek, S. (1991). *For they know not what they do: Enjoyment as a political factor*. New York: Verso.

**WORKING GROUP 3:  
Democracy in mathematics teacher education  
GROUP DE TRAVAIL 3 :Démocratie en formation  
des enseignants en mathématiques**

**Hellenic Mathematical Society  
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**Reflections on school mathematics experiences:  
Mathematics teaching as a practice of excluding  
students from mathematical culture**

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**Abstract**

This presentation outlines the conceptual framework and reports selected preliminary findings of an ongoing research aimed at tracing main aspects of mathematics teaching practice that is prevailing in Greek secondary schools and spotting characteristics of this practice directly promoting or indirectly reinforcing the exclusion of students from mathematical culture.

Memories of experiences from secondary school mathematics content and the teaching behaviors and pedagogies of their mathematics teachers were collected from 288 prospective kindergarten teachers using a questionnaire.

The conceptual framework for the interpretation of data adopts the Bourdieu's analytic tools of habitus, field and habitant. As a main conclusion, it may be claimed that the habitus of mathematics teaching is a strong contributing factor to the construction of cultural divisions among students so as some students to be included in, and others to be excluded from mathematics culture.

### **1. Background of the study**

This presentation reports the conceptual framework and selected preliminary findings of an ongoing research aimed at, firstly, tracing main aspects of the mathematics teaching practice that is widespread in Greek secondary schools and, secondly, spotting characteristics of this practice which directly promote or indirectly reinforce the inclusion or the exclusion of students from mathematical culture.

Mathematics teaching is understood as a practice that intentionally attends to students' learning of mathematics by attending to the representations of mathematical knowledge, the students' mental processes of knowing, and the instructional media in which teacher and student interact (Cohen, 2011). As exemplified by Herbst et. al. (2010), attending to the representations of mathematical knowledge includes teaching tasks such as selecting embodiments of mathematical ideas, formulating mathematical statements, providing mathematically persuasive explanations, choosing problems for students that promote understanding of target mathematical concepts and more. Attending to the students' mental processes of knowing includes tasks of teaching such as eliciting students' thinking, interpreting students' conceptions and identifying errors, e.t.c., while attending to the instructional media includes a number of diverse tasks of teaching associated with interpersonal dynamics, communication, and the affordances and constraints of the institution where the teaching and learning activities are taking place.

Mathematics teaching practice is widely explored and many research methods have been applied to its investigation. Most of them collect data employing many instruments and techniques, from action research and observation in present time to interviews and questionnaires addressed to teachers or/and students immediately after a sequence of mathematics lessons. Let aside any other validity problems, the data concerning teaching practices which are collected by asking students in current or in the immediate past time may be considered as charged by their momentary and occasional reactions and assessments and thus in some way biased. On the contrary, data collected by students when a long time has been passed which are referring to their memories of mathematics teaching may be considered as more valid being processed and stabilized by personal reflective re-assessment. On this ground, the research reported in this presentation has collect and analyses memories of experiences and reflective assessments of school mathematics teaching as offered by students after two or three years of their graduation from secondary schools and currently being prospective

kindergarten and primary school teachers. Such an approach allows, in my view, not only the tracing of main aspects of the prevailing mathematics teaching practice but also the uncovering of its contribution to the construction of cultural divisions among students so as some of them are included in, and others are excluded from mathematics culture.

In this line of thought, the conceptual framework for the interpretation of our data adopts the Bourdieu's analytic tools of habitus, field and habitant, which offer a theoretical viewpoint to understanding mathematics teaching practices contextualised in educational fields. As described by

Bourdieu, habitus is “*an acquired system of generative schemes objectively adjusted to the particular conditions in which it is constituted*” (1977, p. 95), which, however, it is not only a structuring structure, which organizes practices and the perceptions of practices, but also a structured structure: the principle of division into logical classes which organizes the perception of the social world. (Bourdieu, 1984, p.170). Habitus is “*...necessity internalised and converted into a disposition that generates meaningful practices and meaning-giving perceptions; it is a general, transposable disposition which carries out a systematic, universal application - beyond the limits of what it has actually learnt.*” (Bourdieu, 1984, p. 170). Noyes (2004) argues that “*Bourdieu's central concept of habitus illuminates the mathematics teacher socialisation context because it explains how embodied life history structures classroom practices*”. The concept of habitus is dialectically related to the concept of field, which is the social medium whereby habitus comes about and become reconstituted through experiences. In our case the field is school mathematics teaching. Both these concepts, which can only exist in relation to each other, are closely associated to the concept of habitant. Habitant refers to physical and social space wherein the agents, in our case mathematics teachers, are physically sited. The mathematics teachers being physically located in the habitant of school mathematics practice are leaded either to a process of harmonization of their dispositions to those of the generalized other prevalent in that habitat or in dissonance, depending upon the relative social positions of the various agents involved.

So, the habitat shapes the habitus which in its turn shapes the habitat through the social usages that it makes of it. According to Bourdieu (1974, 1989), habitus, field and habitant are the constitutive elements of a complex process through which schooling system regulates the intergenerational transmission of cultural capital, in a way that reinforces the existing social difference and maintains the established unequal social hierarchies.

## **2. Research methodology**

In the research reported in this presentation they have participated 288 prospective kindergarten teachers. A questionnaire constructed containing 30 items which register their memories of experiences concerning secondary school mathematics content and the teaching behaviors and pedagogies of their mathematics teachers. The items of the questionnaire emanate from a content analysis of 80 answers to the following prompt provided by the researcher during one of his lectures to his students being newly graduated from the secondary school: *“please write in a few words the most pleasant and the most unpleasant experience from your secondary school mathematics that they have been impressed in your memory.”* Initial coding of the reported experiences generated a number of positive and negative experiences which were classified in three categories using as a criterion their direct relation to the mathematics classroom. These categories are: pedagogy experiences associated with instructivist or constructivist features of teaching and learning practices, teacher experiences referring to interactions with teachers during mathematics classes and mathematics content experiences. Experiences related to testing and examinations have been included in one of these three categories, according to the related case. Finally, five positive and five negative statements were selected for each category that represented the most frequently occurring experiences and they properly worded as items of the questionnaire. All items were answered on a 5-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

## **3. Preliminary findings**

Preliminary findings of this ongoing research are briefly presented in the following with no further comments due to the space limitations in this conference paper. The participants of this research report that new math content has usually not been easy to understand and they had usually difficulties in comprehending math content. Frequently during mathematics classes they got lost and had trouble keeping up in their lessons. They generally have had difficulties relating new mathematical concepts to those they had previously learned and this is remembered as a frustration producing situation, as they consider mathematics a sequentially organized progression of concepts, an idea also emphasized by their teachers.

The content experiences from school mathematics are found to be closely related to their memories of teachers' teaching behavior as well as to



their experiences from their teachers' pedagogies. Most of the participants in this research they report that their math teachers were not, in most cases, patient and supportive in their efforts to learn mathematics and many times they remember them to become frustrated or even derogatory with their students. Many of them they are convinced that their teachers did not spend the necessary amount of time helping them to understand concepts and techniques of mathematics. Furthermore, a significant percentage of the respondents acknowledge that many times their mathematics teachers made them feel dumb in class and most of the students remember that they did not feel comfortable seeking help from their mathematics teachers outside class. At the same time, a significant percentage of the respondents acknowledge they believe that many of their mathematics teachers were competent as mathematicians.

Concerning pedagogies of school mathematics most of the participants report experiences of an instructive model employed by their teachers. Their mathematics teachers used, as a rule, a lecture format in their lessons relied on the chalkboard to present concepts and techniques of mathematics taught, demanded from their students to sit quietly and listen while they were assigned several homework problems in each lesson. Most of the teachers are remembered to focus mainly on memorization of facts, formulas and procedures. Very few of the respondents report that they remember mathematics concepts taught to be connected to real world situations and also very few remember their teachers to use mathematics games to reinforce their understanding of mathematics concepts and procedures.

Finally, not significant differences found in school mathematics memories according to the age of the participants. That is, teaching behaviors and pedagogies of Greek secondary school mathematics teachers are well established and time-independent. It seems that the structuring power of dominant school mathematics teaching practice reproduces the predominant models of mathematics teaching philosophy and pedagogy through the reproduction of teachers' dispositions by processes of their harmonization illuminated by Bourdieu (1984).

#### **4. A final comment**

Bishop (1988) has argued that the learning of school mathematics may be seen as a process of "mathematical enculturation" which aims at the initiation of students into the conceptualization, symbolization and values of mathematics culture. Lerman (2006) associates mathematics enculturation to a process of becoming mathematical, and comments that according to re-

search evidence becoming mathematical can mean different things in different modes of teaching.

From the viewpoint of mathematical enculturation, it may be claimed that the habitus of mathematics teaching is a strong contributing factor to the construction of cultural divisions among students so as some students to be included in, and others to be excluded from mathematics culture.

#### References

- Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer
- Bourdieu, P. (1977). *Outline of a Theory of Practice* (Nice, R, Trans.). (1992 ed.). Cambridge MA: Cambridge University Press.
- Bourdieu, P. (1989), How schools help reproduce the social order, *Current Contents: Social and Behavioural Science*, 21(8), 16.
- Bourdieu, P. (1984). *Distinction: A social critique of the judgement of taste*. Cambridge, MA: Harvard University Press.
- Bourdieu, P. (1974), The School as a Conservative Force: Scholastic and Cultural Inequalities. In J. Eggleston (Ed.) *Contemporary Research in the Sociology of Education*. London, Methuen, 32-46.
- Cohen, D. (2011). *Teaching: Practice and its predicaments*. Cambridge, MA: Harvard University Press.
- Herbst, P., Bieda, K., Chazal D. & González, G. (2010) Representations of mathematics teaching and their use in teacher education: What do we need in a pedagogy for the 21st century? Retrieved from <http://hdl.handle.net/2027.42/78158> (27-01-12)
- Lerman, S. (2006). Learning mathematics as developing identity in the classroom. In P. Liljedhal (Ed.) *Proceedings of the Canadian Mathematics Education Study Group*, University of Ottawa: CMESG, 3-13
- Noyes, A. (2004), (Re) Producing Mathematics Educators: A sociological perspective, *Teaching Education*, 15 (3), 243-256

## TEACHERS' REFLECTION ON SELF- REGULATED LEARNING IN MATHEMATICS CLASSROOM: A CASE STUDY

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### *Abstract*

*This paper presents a comparison between a teacher's reflection regarding self-regulatory teaching practices enacted during a mathematics teaching session and researchers' recording of such practices. The different focus of reflection and the difficulties emerged in teacher's perception regarding self-regulation address the need for a special training program for teachers.*

### **Theoretical Background**

A major and constant pursuit in education is the high quality and effectiveness of educational practices through the design of new and powerful teaching-learning environments. Good learning, including mathematics learning, is claimed to be a constructive, cumulative, self-regulated and situated process of knowledge building and meaning construction (DeCorte, Verschaffel, & Op't Eynde, 2000). Current approaches of teaching/learning emphasize the need to develop teachers' and students' skills that contribute to students' active, independent and self-regulated learning.

Research in mathematics education has recently focused on the attributes of self-regulated learning aiming at an active mathematical knowledge building and at skilled mathematical problem solving (De Corte, et. al, 2000). Strategic learning, that is, mastering and applying multiple skills and strategies to effectively monitor and regulate learning and problem solving, is an essential component of self-regulated learning and it

is critical for successful performance (Alexander, Graham, & Harris, 1998). Planning, self-monitoring, and self-evaluation are very important metacognitive skills for mathematics learning and problem solving. Moreover, inducing students' own interest and adaptive motivation towards mathematics learning is a major challenge for educators and can further foster their autonomous activity.

Although promoting students' self-regulatory skills in order to build mathematical knowledge and meaning is a widely accepted point of view, what is still under investigation, particularly in Greece, is the degree to which regular mathematics teaching in primary school adopts the above suggestion. Greek primary school teachers' reports on inducing students' self-regulatory strategy use in mathematics indicated that they tend to facilitate more solution evaluation by the students and less further reflection on the solution produced or constructive collaboration among the students. It should be interesting: to record and analyze teachers' actual discourse and instructional practices during regular mathematics class<sup>1</sup> and to investigate whether mathematics teachers can identify in their teaching practices specific attributes of facilitating students' self-regulated learning.

The present study aimed at recording actual mathematics teaching sessions in primary school and identifying in teachers' discourse the degree to which their teaching promotes students' self-regulatory skills. A second aim was to ask a mathematics teacher to reflect on a recorded teaching session and to identify herself incidents concerning self-regulated learning skills. We aimed at comparing teacher's ability to identify a posteriori traces of inducing students' self-regulated learning and the kind of support she actually offered during class. Teaching session was examined with a focus on the following self-regulatory skills: planning, self-monitoring, self-evaluation, interest and motivation enhancement, and autonomous action. It should be noted that this is a part of a broader research project funded by the Research Committee of the University of Thessaly titled "Self-regulated learning and mathematics in the classroom: Teachers' discourse and students' practices".

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<sup>1</sup> Recording actual instruction would provide realistic information on practices that facilitate students' self-regulation in mathematics in the Greek classroom.

## **Method**

### *Procedure*

The research is conducted in a public primary school (Athens). After obtaining access by the school director, a primary school teacher agreed to videotape a third-grade mathematics class. After that, a verbatim transcription of her discourse and dialogue with the students has been prepared. The teacher was asked to reflect on the transcribed dialogues and to fill in a self-recording worksheet with a focus on the self-regulatory skills examined. Teacher's discourse and dialogues were also analyzed by the authors.

### *The participant teacher*

A female third-grade teacher —21years experience— participated voluntarily in this study. She agreed to reflect on a session of her own mathematics instruction and to record traces of inducing students' self-regulatory skills. She was never educated before on self-regulated learning practices; therefore, she was familiarized with the concept of self-regulated learning with a focus on the skills examined.

### *Instruments*

The worksheet asked the observer (teacher, authors) to identify traces of teacher's discourse that facilitated students' self-regulatory skills. The skills examined were: planning, self-monitoring, self-evaluation, interest and personal motivation enhancement, and autonomous action. Specifically, the worksheet included the following questions:

Stem for each item: "In your mathematics teaching discourse, please identify traces of ..."

- a)... facilitating students' abilities to plan their thinking and acting in order to face a mathematical activity.
- b)... facilitating students' abilities to observe or to monitor themselves during their own attempt to face a mathematical activity.
- c)... facilitating students' abilities to evaluate and assess their outcomes and solution produced or to reflect on it.
- d)... facilitating students' motivation or strengthening their interest to participate in a mathematical activity.
- e)... facilitating students' abilities to act in an autonomous way.

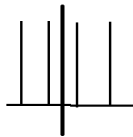
## **Data analysis – Results**

In order to identify the traces in teacher's discourse indicating facilitation of students' self-regulatory skills, two sources for identification

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were addressed: the authors and the teacher. Below, these results are presented.

Regarding traces of facilitating students' abilities to plan their thinking and acting in order to face a mathematical activity, the authors identified three (3) instants or episodes in teacher's discourse out of the 45 minutes session of mathematics instruction. In the two episodes, the teacher asked students to recall a mathematical rule used in arithmetical operations (e.g. "What do we have to pay attention to when we do an addition?") and she induced them to apply the rule. In the third episode, the teacher asked students to analyze the function of the abacus.



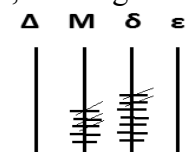
39. Teacher: *Which are the positions in this abacus? Pay attention, please. Which are the positions of the digits that I have to write on?*

In the above episodes, the teacher seems to facilitate students to plan their actions by asking them to recall rules, apply them and to think about the use of abacus. However, the teacher didn't explicitly emphasize that these actions concerned a planning process as the use of a mathematical tool or a rule.

Regarding the teacher's reflection on inducing planning skills, she identified five (5) different episodes. None of them was identical to the authors' judgments. The teacher related the facilitation of students' planning with the suggestions that she offered to them for planning their solutions. Specifically, she identified actions closely to the third episode of the above planning. For example, she considered as facilitation of planning skills the description of an effective strategy to her students. The quotation below is representative.

(The student had to solve the subtraction 5, 70-3,20 using the abacus).

101. Teacher: *...you have to delete 3 euro and 20 cent. You will delete (on the abacus) the euro you pay.*



(The teacher passed through the students' desks and checked their writings).

In addition to the planning skills, we identified episodes of inducing students to enact their prediction of the result skills. An example is the following quotation:

(The students had to solve the addition 4,32+3,25.)

15. Teacher: *Could you make a rough mental calculation? I want to buy two things. The first costs 4 euro and 32 cent and the second costs 3 euro and 25 cent. Can I broadly estimate how much money I need?*

The teacher presented to her students the way to plan their action using a real life context. Either in the above example she didn't explicitly linked her suggestion with the planning process.

Regarding facilitating students' abilities to observe or to monitor themselves during their own attempt to face a mathematical activity, although no trace was found by the authors, the teacher considered the question "*How did you find it?*" as related to the student's self-monitoring process.

Regarding facilitating students' abilities to evaluate and assess their outcomes and solution produced or to reflect on it, episodes were identified by the authors which were mainly related to the development of mathematical justification by the students. In the quotation below the teacher induced a discussion in the classroom on "*Why  $2,50 = 2,5$* ".

116. Teacher: *(she is writing on the blackboard 2,5) Could I write it in this way?*

117. Some students: *Yes.*

118. Teacher: *Yes, I can. Why?*

119. Student18: *You can put a zero.*

120. Teacher: *If I want it. Why is correct to write 50 cent in this way?*

121. Student7: *It can be deleted.*

122. Teacher: *Another opinion? It isn't wrong that you said. Is there another opinion?*

123. Student20: *Because zero has not any value.*

124. Teacher: *The student7 told to me the same with you. Is there something else?*

125. Student13: *5 alone has the same value with 5 tenths.*

126. Teacher: *Exactly. ...*

In this dialogue, the teacher gave to her students the opportunity to reflect on their decision (utterance 117) asking justification. However, as it is obvious, each student's justification was assessed by the teacher (utterances 120,122,124,126) and didn't provoke any discussion among the students. Such teaching instants are critical for enhancing students' reflection and self-assessment skills. The teacher herself didn't point out any

episode that facilitated evaluation of the solution produced by the students.

Regarding facilitating students' motivation or strengthening their interest to participate in a mathematical activity and students' abilities to act in an autonomous way, neither authors nor the teacher pointed out any evidence of using such skills during this specific mathematical teaching session.

### **Discussion**

This paper focused on the importance of self-regulated learning as a major objective of mathematics education and as a crucial characteristic of effective mathematics learning (De Corte et al., 2000). The first aim of this paper was to identify in teacher's discourse traces of promoting students' self-regulatory skills during a session. The focus was on the following self-regulatory skills: planning, self-monitoring, evaluation of the solution, interest and motivation enhancement, and autonomous action. As expected, a few episodes that actually induced students' self-regulatory skills were recorded concerning only planning skills and evaluation of the solution skills. This finding confirms the constant need to better educate mathematics teachers to adopt the self-regulated learning principles in their teaching. Teachers should direct their efforts on facilitating their students to act autonomously, independently in order to construct meaning, to allow self-monitoring to take place, and to motivate themselves in the face of difficulties.

The second aim of this paper was to ask the mathematics teacher to reflect on the recorded teaching session and to identify herself incidents that elicit students' self-regulated learning skills. The comparison between them, showed that:

- It was difficult for the teacher to recognize critical moments regarding students' self-regulatory skills.
- The teacher seemed to connect her directions to the students to plan a mathematical solution in a concrete way with the planning process.
- The teacher connected the students' self assessment with their self monitoring process.
- There were more differentiation and less coincidence of views between the teacher and the researchers in several aspects of students' self-regulation in mathematics.

The above points should be interpreted bearing in mind that there was not previous teacher's training on self-regulated learning in mathematics. In



general, it seems that there is a need for further educating in service teachers: a) on the importance of developing students' self-regulatory skills in mathematics, and b) on identification and improvement of such skills in their students (Mevarech, & Kramarski, 1997). The difficulties that emerged on teacher's perception of facilitating students' self-regulation allow us to further reflect on planning a special training program for teachers in order to shift teaching towards giving students opportunities for a more autonomous, independent, skilled, motivated, and self-regulated learning of mathematics.

### References

- De Corte, E., Verschaffel, L., & Op't Eynde, P. (2000). Self-regulation: A characteristic and a goal of Mathematics Education. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of Self-Regulation* (pp. 687-726). Academic Press.
- Alexander, P., Graham, S., & Harris, K. (1998). A perspective strategy research: progress and prospects. *Educational Psychology Review*, 10, 129-154.
- Dermitzaki, I., Leondari, A., & Goudas, M. (2009). Relations between young students' strategic behaviours, domain-specific self-concept, and performance in a problem-solving situation. *Learning and Instruction*, 19(2), 144-157.
- Mevarech, Z. R. & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal*, 34, 365-394.
- Mousoulides, N., & Philippou, G. (2005). Students' motivational beliefs, self-regulation strategies and mathematics achievement. In H. L. Chick, & J. L. Vincent (Eds.), *29th Conference of the International Group for the Psychology of Mathematics Education: Vol. 3*, pp. 321-328. Melbourne
- Pintrich, P. R., & DeGroot, E. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology*, 82(1), 33-40.

## **Collaborative distance learning mode: an approach to infinity for prospective teachers**

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### **Abstract**

This study reports findings from a distance-learning course named “infinity and its implication for high school math classroom” for prospective teachers in Brazil. Adopting a Vygotskyan perspective for learning, we develop tasks that include problems involving the idea of infinity that are usually taught in secondary schools. Based on Design Research methodology, we developed tasks that were familiar but not usual to these students (prospective teachers) and in order to solve the problems students interact with others in small groups using a platform built for allowing collaborative learning, the VMT. We will present findings related to the use of the platform as well as the approach for teaching in virtual environments.

Key words: distance-learning mode; infinity; secondary school content; teaching approach; interaction

### **Introduction**

Distance learning courses- DLC- proposals usually suggest that their aim is delivering teaching for students who are not able to attend regular classrooms, also that those courses let students to work in their own pace, choose the time that better fit in their schedule and so on. In large countries as Brazil distance learning courses may democratize access to students who live far from universities or those who live in rural areas far from schools.

Analyzing what is implicit in the discourse of many proposed DLC we may say that their instructional design are based on the premise that knowledge is produced by the individual itself rather than in a social

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environment.

We assume a different view, for us knowing is a social action. Different from information that may be transferred, knowledge is grounded in experience. Dictionaries, for example, define a word in terms of other words; however human beings (us) understand and produce meaning to words based on our experiences in the world. We need to interact with people and the world in order to learn words such as red, heavy, up, friend and so on. Abstracts or new words are learnt in this same way (Bolite Frant, 2009). When a child learns what is heavy it is not useful to tell her that heavy is the opposite of light, she needs to experience to hold a light bag and a heavy one, to hold a small cotton ball and a small lead one, in order to understand the difference.

This paper contributes to the conference theme “Mathematics Education and Democracy: learning and teaching practices”. First because distance learning mode may offer a more democratic access to math education and second teachers’ practices in this environment should be discussed since the role of teachers is different if we will provide a democratic and learning environment for understanding school mathematics.

The aim of this study was to develop a distance-learning course for prospective teachers in order to discuss about mathematical infinity and its implications for secondary school, to analyze which aspects of developed tasks facilitate or impede interactions in this virtual environment. Also, to investigate teacher’s role within this new setting.

The study was designed in three cycles; the first included a literature review on the theme, choosing a platform to work, reviewing secondary mathematics school textbooks and elaborating the tasks. The second cycle consisted on a field work: a first course offered to 3 prospective teachers, changes in the designed course and the third phase consisted on a course for 6 prospective teachers and changes and analyses in order to cope with our aims.

Based on our premise about knowledge we looked after an environment where students could share their experiences writing and/or drawing, or sharing other digital devices. We decided to use a platform that was created for collaborative learning in mathematics by a team lead by Gerry Sthal at Drexel University, named Virtual Math Team - VMT. Although the VMT was originally built for extra schoolwork, students, from different schools and countries, who would like to solve problems and discuss with other students we found a project called e-math directed by Arthur Powell from Rutgers University that inspired our work. And we will present, in this

CIEAEM64 conference, the third cycle.

### **Field work**

We developed a synchronous course titled "infinity and its implication for high school math classroom" that was offered by a federal university in Brazil and was opened to prospective teachers. Many students wanted to attend this course but due to time constraints we worked with 6 of them.

Students were in different places in the state of Rio de Janeiro. Following the familiar chat rooms from social networks, two rooms were opened in order to have no more than 4 students each. Because, in a regular chat room with 10 persons or more writing, some at the same time, it is difficult to follow and to participate, sometimes what one said is no longer in the screen.

The tasks were developed after a careful review of researches about student's understand of infinity, and a review of textbooks used in secondary schools, specifically looking at the exercises involving infinity such as sequences, series, geometric progression, cardinality of number sets.

To open a discussion about sequences and series we developed a story based on a folk story known in Brazil as John and Mary, two children who went to a forest and got lost, there they met a witch. Emphasizing the idea of a familiar but not usual task, in our story there were two sides on an infinity road inside the forest, filled with logs on both sides, and the witch offer them a possibility for escaping, the kid who finished to put the logs in his or her bag was free from being eaten by the witch. John and Mary should take the first log and put in each infinity bag, then John will take the second log and cut in two pieces, take one of them and put in his bag, the third log he should cut in four pieces take one of them and put in the bag, the fourth log in eight pieces take one and put in the bag and so on. Mary should take the second log cut in two pieces and take one to put in her bag, the third log she should cut in three, take one piece to her bag, the fourth in four pieces and so on. The students in small groups, 3 in each room, discussed how many pieces each kid carry on her and his bag.

In room 1, one student said that the sequence that John was carrying remembered something she had learned about geometric progression and she found a formula for the infinity sum of g.p. using Google search. The others said that it would never be 2, it will be infinity because the road was infinite. And the discussion took place in different spaces of this platform, on the chat and on the whiteboard. See figure 1.

The screenshot shows a VMT whiteboard interface with a chat window on the right. The whiteboard content includes:

**Whiteboard:**

Material: GeoGebra Summary Topic Our Web

Letamos a seguir, as suas respostas. Agora, leia as respostas dadas por eles, depois diga quem foi a questão e justifique.

Antonio  
O conjunto A tem mais elementos.  
Justificando  
Pela enumeração A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, ...}  
B = {1, 3, 9, 18, 25, 36, ...}  
Em A temos que  
entre 1 e 4 temos 2 elementos que não estão em B  
entre 4 e 9 temos 4 elementos que não estão em B  
entre 9 e 16 temos 6 elementos que não estão em B  
entre 16 e 25 temos 8 elementos que não estão em B  
E assim por diante, portanto A tem mais elementos que B.

Beatriz diz que:  
Os dois conjuntos tem a mesma cardinalidade

Justificando  
Se continuarmos veremos que todo elemento de A tem um correspondente em B

1, 2, 1+1 = 3, (2+1)=5, 4+4 = 9, (2+1) + 9 = 16

correção:  
1, (2, 1+1=3) + 1 = 4, ...

mmmm  
Estou tentando pensar como a Tamy, foi o que entendi!

1 4 9 16 25  
de 1 para quatro acrescento 2 vezes

**Chat (463):**

Current users: soraia

Chat history:

- soraia 18:00:11 15h1min24s BRT: SALA ESTUDO 8
- soraia leaves the room 19:00:11 15h1min26s BRT
- soraia joins the room 19:09:11 15h19min30s BRT
- soraia leaves the room 19:09:11 15h19min50s BRT
- vivi joins the room 02:07:11 15h48min15s BRT
- vivi leaves the room 02:07:11 15h50min00s BRT
- janete joins the room 11:09:11 16h50min00s BRT
- janete leaves the room 11:09:11 16h53min00s BRT
- soraia joins the room 11:13:51min00s BRT

figure 1: VMT whiteboard and chat

The teacher just followed both groups discussion and after the session finished she compiled their answers and elaborated another task based on their answers. She created three different student names so the students in each room would not feel that his or her answers were better or worse than others. And they started the new session discussing the fictitious students' solutions.

The second task, based on Dreyfuss and Tsamir (2004) article, involved comparing infinite sets.

We still have two fictitious students Antony and Beatriz that offer two different answers to same problem and our students should discuss their answers. This approach helps those students who maybe afraid of giving a wrong answer by putting the responsibility of the answer on the fictitious students. The task was:

Given two sets A and B such that:

$A = \{1, 2, 3, 4, 5, \dots\}$  and  $B = \{1, 3, 9, 27, \dots\}$  what can we say about their cardinality Is  $A < B$ ,  $A > B$  or  $A = B$ ?

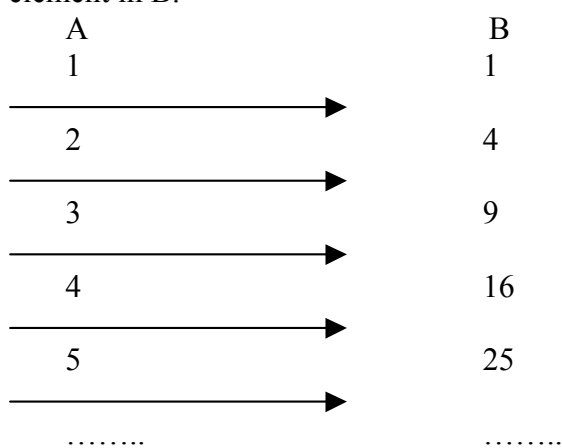
Antony answer:

A has more elements than B because

Between 1 and 3, the 2 is only in A; between 3 and 9 there are 4, 5, 6, 7 and 8 in A nor in B and so on.

Beatriz answer

They have the same cardinality because using the bijection idea we can see on the diagram that for each element in A there is a correspondent element in B.



This task gave place also to a rich discussion among the students and the property of infinite sets was discussed and raised by our students. At the end they said that infinite sets were different from finite sets because they could have a subset that had the same number of elements.

They also said that it was the first time they were thinking about it because during their school years they had fifty percent of chance to give a correct answer to those kind of question, usually they only had to answer true or false, sometimes they score it sometimes not.

Other two tasks and a closing session for comments followed the same approach, a problem was posed, students discussed, teachers interfere always raising more questions rather than giving answers. (for space constraints we will bring them for the WG)

All the sessions were transcripts immediately after one happened in a html format.<sup>2</sup>

### Findings

We found that working in small groups chat was easier to follow and to

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<sup>2</sup> The VMT generates a html transcript of the session and a video of every participant movement, drawing, writing, erasing.

participate. In a regular chat room writing it is difficult to follow and to participate, sometimes what one said is no longer in the screen. In this environment with only 3 students in a room it did not happen, moreover the reference feature allow students to refer to a previous line or to a draw and those who were interacting with him/her easily find the line or draw just following the referred place.

The chat and whiteboard space although familiar to students, who are used to social networks or cellular phone texting, required a more careful posting than in oral speech and it helps to follow discourse while in a distance mode.

On the teacher side, it was difficult not to interfere during "silence time" because in face to face course, according to the teacher, "*we may see them working, or their face thinking but in VMT a second with no posting may lead us to be veeery anxious hahaha. We do not know if they are working, if they leave the computer, we can not see what they are doing*". After analyzing the transcription of a few sessions the teacher found that first of all silence times were not so long as she taught, moreover those silences were more frequent when a task was not usual for the students or when the students were thinking how to approach the problem before sharing.

The tasks in this environment also need to be more explicit because in a face to face mode the teacher may, while writing on the blackboard or handling a worksheet, read aloud the problem, including gestures, may give more explanation than what is written.

If in this kind class one or more students are not working it was easier for the teacher to interact with them, and hear their explanations. We also saw that in this environment words had to be more carefully used by the teacher. If the teacher posted after her statement "right?" students will agree posting "yes" or "ok" no matter what she was conveying.

Our findings in this environment may also inform face to face courses because in those classes teachers also end their utterances with a "right?" or even start talking and stop in the middle of a word and the students complete it not always understand why.

### **References**

- Bolite Frant, J. 2009. As Time Goes By: Technology, Embodiment and Cartesian Graphics. Proceedings of PME35 Thessaloniki, Greece
- Cobb, P. et al.(2003) Design Experiments in Educational Research. Educational Researcher, Vol. 32, No. 1, pp. 9–13

Dreyfus, T., and Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *Journal of Mathematical Behavior*, 23, p271-300

Tsamir, P., and T. Dreyfus (2002). Comparing infinite sets — a process of abstraction - The case of Ben. *Journal of Mathematical Behavior*, 21, 1–23.

Vygotsky, L. (1978) *Mind in Society*. Harvard University Press

Vygotsky, L. (1984) *Thought and Language*. MIT Press

FIGURE 1: VMT – whiteboard and chat spaces

The screenshot displays a web-based interface for a virtual meeting. The main window is titled "TabbedChat: sorala (CID:1303817005723)". It features a menu bar with "File", "Edit", "Chat", and "Whiteboard Zoom". Below the menu is a toolbar with various icons for editing and navigation. The whiteboard area is divided into several sections:

- Text:** "entamos a seguir, as suas respostas. Agora, leia as respostas dadas por eles, depois diga quem foi a questão e justifique."
- Text:** "Antonio  
O conjunto A tem mais elementos.  
Justificando  
Para uma numeração A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, ...}  
B = {1, 4, 9, 16, 25, 36, ...}  
Em A temos que  
entre 1 e 4 temos 2 elementos que não estão em B  
entre 4 e 9 temos 4 elementos que não estão em B  
entre 9 e 16 temos 6 elementos que não estão em B  
entre 16 e 25 temos 9 elementos que não estão em B  
E assim por diante, portanto A tem mais elementos que B.
- Diagram:** A Venn diagram showing two overlapping sets, A and B. Set A is represented by a larger oval on the left, and set B is a smaller oval on the right. The intersection of A and B is shaded. Lines connect the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 to the corresponding elements in the sets.
- Text:** "Beatriz diz que:  
Os dois conjuntos tem a mesma cardinalidade"
- Equation-Block:** 
$$1, 2, 1+1=3, (2, 2+1=5) 4+4=9, (2, 3+1) 9=16$$
- Text:** "correção:  
 $1, (2, 1+1=3) + 1 = 4, \dots$
- Text:** "mmm  
Estou tentando pensar como a Tamy, foi o que entendi"
- Diagram:** A number line showing the sequence 1, 4, 9, 16, 25. Brackets indicate the intervals between these numbers: 1 to 4, 4 to 9, 9 to 16, and 16 to 25. Below the number line, it says "de 1 para quatro acrescento 2vezes".

The chat window on the right shows a list of users and messages. The current users are "sorala". The chat history includes messages from "sorala" and "vivi" regarding room joins and leaves.



## The role of resources in the practice of mathematics teacher

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*This article we document the role of teacher resources in teaching themes related to third-grade algebra in secondary school. The Documental Approach serves as theoretical basis. This research is of qualitative type and the collection of data was carried out by means of non-participative observation in the classroom. Four teachers participated in this study; however, we only present the work of one of them. Our results indicate that the lack of mathematical resources prevent the teacher from providing a correct solution to the problem.*

### 1. Presentation of the problem

Researchers such as Adler (2000) and Gueudet and Trouche (2009) maintain that teacher's resources –e.g., mathematical and didactic knowledge– are essential in the teaching practice. Particularly, Adler (2000) states that it depends on how the teacher uses these resources to promote learning. The growth of research regarding this theme makes us analyze the resources that the teacher uses in class, their nature and the source of their origin; particularly, those resources to teach aspects related to secondary school algebra.

### 2. Conceptual framework

The *documental approach of didactics* is the framework used in this research (Gueudet & Trouche, 2009). For these authors, a resource is everything which is used (*re-sourcer*) in the teaching practice. Thus, for a teacher, the textbook, the discussion with a colleague or a mathematical concept, could be resources. The teacher interacts with resources by means a *documental work* which may be carried out inside class and in which a *documental genesis* is produced; a dynamic and dialectic process through

which resources are transformed into documents by means of *utilization schemes* of the user of such resources. In this way, a document is composed by a set of resources and utilization schemes. Gueudet and Trouche (*ibidem*) assure that, therefore, the documental work of the teacher is the basis of his/her practice.

### 3. Methodology

Four teacher of mathematics of third-grade secondary school participated. The teachers, during several classes and with the aim of not altering their didactic programming, were video-recorded when working on diverse lessons of their textbook; particularly, those related to algebra (SEP, 2006). For the purpose of this document, we only focus on one teacher, whom we have called Pedro (pseudonym). Pedro uses the textbook as a resource (Arriaga, Benítez & Cortés, 2008) for mathematics teaching the concept of slope, trying to comply with another resource relating to the didactic objectives of the current study Program (SEP, 2006). Following on, we expose the analysis Pedro's resources when solving a problem regarding the slope and the angle of inclination of a straight line.

### 4. Data analysis

#### 4.1. Mathematical and didactic analysis of resources used by Pedro

The *textbook* introduces, on the whole, the concept of slope (Figure 1) as the inclination of the straight line regarding the horizontal axis, leaving aside the fact of knowing what happens with the slope when the straight line is either vertical or horizontal. Furthermore, by means of an activity that deals with the covered distance regarding the time spent, the textbook relates slope with the concept of ratio of change; that is, in this activity it is concluded that the quotient of the difference in the distance, regarding the difference in the time "is known as 'ratio of change' or 'slope of a straight line'" (Arriaga et al., 2008, p. 91).

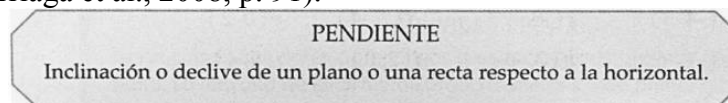


Figure 1. Concept of slope (Arriaga et al., 2008, p. 90).

In this way, the slope of a straight line is linked with the ratio of change as a measure of inclination of the straight line. Similarly, the textbook includes activities focused on building linear graphs and determining "their ratio of change (slope)" (*ibidem*, 2008, p. 91). However, different from this concept, the textbook introduces the angle of inclination of a straight line, but does not define it, even though we consider it may be found intuitively.

#### 4.2. Development of Pedro's practice

With the purpose that students interpret linear graphs, Pedro worked on an activity (Figure 2) from the textbook (Arriaga et al., 2008). This activity deals with a situation in which a moving object covers –at a constant speed (80 km/h)– a distance between two cities, and it is expected that students trace a graph, whose slope is the value of speed.

Suponiendo que un automóvil sale de la ciudad de Querétaro a las 6 de la mañana y va hacia Toluca, a una velocidad constante de 80 km/h, ¿a qué hora llegará a Toluca, si se sabe que la distancia entre esas dos ciudades es de 195 km? Traza la gráfica correspondiente para observar la distancia que recorrió el automóvil conforme avanzaba el tiempo.

Hora de llegada a Toluca: \_\_\_\_\_

Contesten las siguientes preguntas:

a) ¿Qué distancia avanzó a las 2 horas de recorrido? \_\_\_\_\_

b) ¿En cuánto tiempo recorrió 100 km? \_\_\_\_\_

c) ¿En cuánto tiempo recorrió la mitad de la distancia que hay entre las dos ciudades? \_\_\_\_\_

d) A esa velocidad constante, ¿cuánto tiempo habría tardado en llegar si la distancia entre las ciudades fuera de 260 km? \_\_\_\_\_

e) Viajando a mayor velocidad, ¿el ángulo de inclinación de la recta aumenta o disminuye?

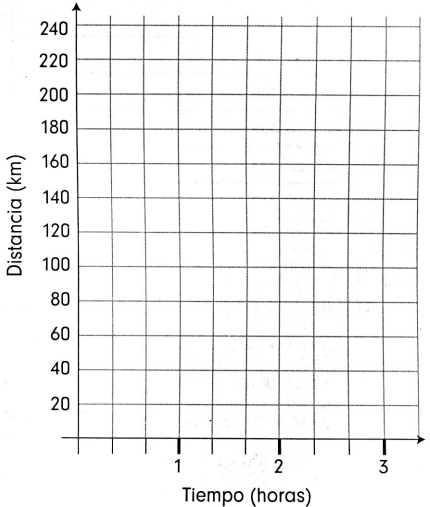


Figure 2. Activity which Pedro used as the basis to discuss slope and the angle of inclination of a straight line (Arriaga et al., 2008, p. 231).

The activity includes a question regarding the discussion of the relationship between the speed of the moving object –that is to say, the slope– and the angle of inclination of a straight line: “Travelling at a greater speed, does the angle of inclination of the straight line increase or decrease?” (íbidem, p. 231). However, the activity does not introduce the meaning of this angle and its relationship with the slope of the straight line.

Pedro decided that students carry out an activity and he chose one of them to expose what he did: “you were chosen to give explain what you did”. The student traced a straight line whose function is  $d=80t$  (Figure 3), taking into account the relationship that exists between the covered distance

and the time spent: “for every 80 kilometres it advanced 1 hour”. Immediately, the student indicated that the angle of inclination of the straight line “increases”.

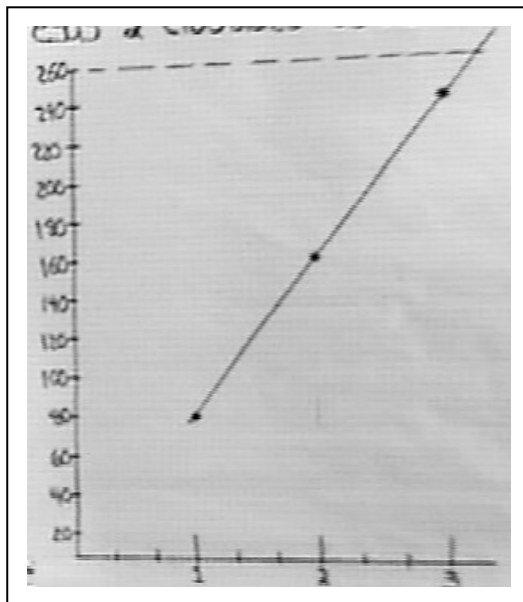


Figure 3. Straight line with a  $d=80t$  function traced by the student.

Student: “Travelling at a greater speed, does the angle of inclination of the straight line increase or decrease?” I would say it increases. Because if at this point, you add 100 kilometres to 1 hour, it reaches this point [*He traces the (1, 100) point*]; if you add that to two hours, it would be like 200 [*he points (2, 200)*], and this it raises, it increases.

In order to justify the answer and given the fact that the covered distance is in function of the spent time, the student increased the distance regarding time (“... if at this point, you add 100 kilometres... [*he traces the (1, 100) point*]”) in such a way that the points which correspond to each relationship are separated as the student takes successive values of time, in increases of one hour (“if you add that to two hours, it would be like 200 [*he points (2, 200)*], and this it raises, it increases”). This separation indicates, for the student, that the angle of inclination of a straight line increases at a greater speed. It is worth noticing that the student, obliquely, makes allusion to the slope of the straight line. As asked by Pedro, the student traced the straight line that passes through (1, 100) y (2, 200) and whose function is  $d=100t$  (Figure 4).

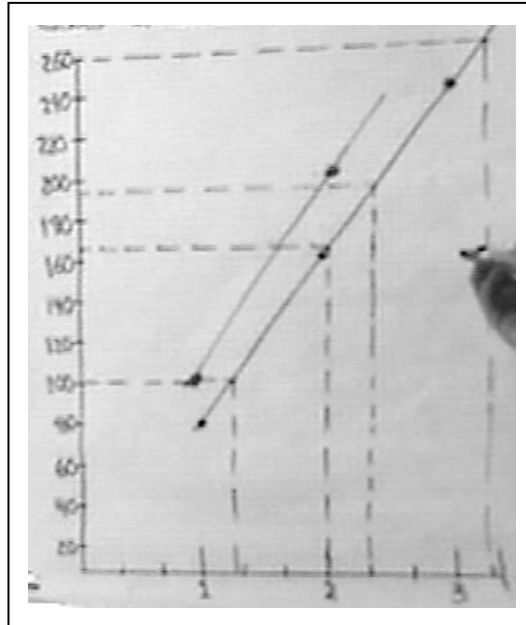


Figure 4. Student's demonstration.

We conjecture that the student's demonstration of his process was not clear to Pedro, given the fact that he does not show, visually, that the angle of the straight line increases when the speed of the moving object is increased. For this reason, Pedro decided to use his own resources considering the straight lines traced by the student as it shown below.

Pedro: How do you realize it increases?

Student: The angle goes up.

Pedro: In order to do that, direct your lines... towards 0 [*plane's origin*]. Upon directing it towards 0 [*he points out the straight line  $d=80t$* ] it results in an angle, and we can measure it. Upon directing the other one [*straight line  $d=100t$* ] towards 0, we obtain the second angle, which we can also measure. Do you agree? Name that one angle  $x$  [*It refers to indicating with an  $x$  the angle of inclination of a straight line  $d=100t$* ]... Trace the other angle, that one being angle  $y$ .

In order to determine whether the angle of inclination of the straight line increases, Pedro asked the student to elongate both lines until the origin of the Cartesian plane and thus obtain the angles that form these two lines with the axis representing time (Figure 5). According to Pedro's request, the student marks the traced angles with the letters  $x$  and  $y$ . The angle of the straight line, whose function is  $d=80t$ , is marked with letter  $y$ , while the

angle of the other straight line ( $d=100t$ ) is marked  $x$ . It is worth noticing that the only justification given by Pedro regarding tracing and identifying the angles, is that with this process he can “measure” and compare both angles, without arguing why there should be two straight lines and why they should pass through the origin  $(0, 0)$ .

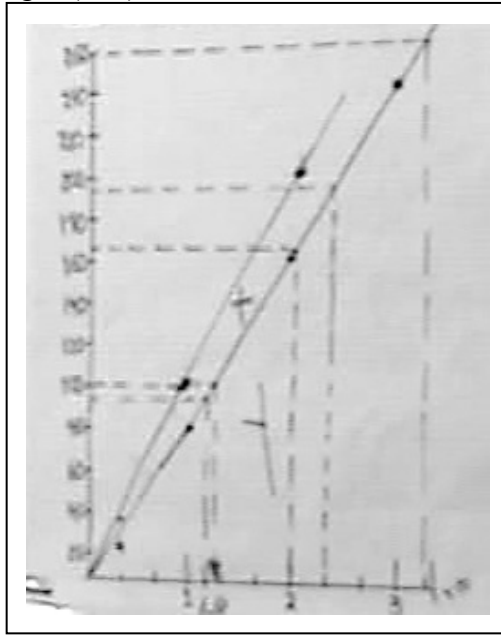


Figure 5. Angles comparison.

Now that Pedro has identified the angles, he posed the next question to all students: “which of the two [*angles*] is greater?” Students assured that angle “ $x$ ” is greater. In order to validate this answer, Pedro compared both angles visually, as it is shown below.

Pedro:  $x$  [*is greater*], because it goes from 0 until up there. This is  $y$  [*In the plane he drags a ruler of the time axis to the straight line  $d=80t$ , and later on it is? [He continues dragging the ruler until the straight line  $d=100t$ ].*

Students:  $x$ .

Pedro: Then, if the speed increases, what?

Student: Increases... [*Pedro interrupts*].

Pedro: The grade... of inclination... increases... We have seen it visually.

The argument that Pedro posed of why angle  $x$  is greater than  $y$ , lies in the fact that this angle is the most open of both angles. However, the language used by Pedro is confusing, since he points out that angle  $x$  “... goes from 0 until up there”; that is to say, it goes from the axis which

represents time to the line with  $d=100t$  function. In order to demonstrate this argument, Pedro measured and compared both angles dragging a graduated ruler from the time axis towards straight line  $d=100t$ , stopping at line  $d=80t$  with the intention of indicate that the way covered until that moment was angle  $y$ . However, due to the language used by Pedro, it could be understood that the way which is still to be covered corresponds to angle  $x$ .

Subsequently, Pedro posed an question to his students looking for them to relate angle  $x$  with the constant speed –slope of the straight line  $d=100t$ –: “But to what speed does  $x$  correspond?” In this way, Pedro concluded that “The grade... of inclination... increases”. In this episode it is worth noticing that Pedro emphasized the need to explicitly see the angle.

### 5. Conclusion

The documental work of Pedro was aimed at answering whether the angle of inclination of a straight line (whose function is  $d=80t$ ) increases when the constant speed of the moving object –that is, the slope of a straight line– is greater. The resources used by Pedro revolve around visually comparing the angles of inclination of the two straight lines with different slope and intercepted in the origin of the plane. In order to do that, Pedro had to elongate the straight lines until the (0, 0) point, identify the angle of inclination of each straight line and slide a ruler from the axis which represents time to the two straight lines. However, Pedro does not give any mathematical arguments of why using two straight lines instead of one, why they have to be elongated and why they should pass through the origin of the plane. We consider Pedro used these resources due to the fact that the student’s arguments were not clear for him or they do not visually show that the angle of inclination increases. Moreover, although the textbook and the study program were also Pedro’s resources, they did not provide him with the necessary help.

### 6. References

- Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Math Teacher Educ*, 3, 205-224.
- Arriaga, A., Benítez, M. M. & Cortés, M. C. (2008). *Matemáticas 3*. México: Pearson.
- Gueudet, G. & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71, 199-218.
- SEP (2006). *Reforma de la Educación Secundaria. Fundamentación Curricular. Matemáticas*. México: SEP.

## THE GREEK PRIMARY SCHOOL TEACHERS' VALUES ABOUT MATHEMATICAL THINKING

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### *Abstract*

*This paper introduces a pilot research as regards the Greek primary school teachers' values when they teach mathematics. The analysis of the results was based on Bishop's theoretical framework of mathematical values. The preliminary results showed an emphasis on the mathematical values of rationalism, objectism and control.*

### **Theoretical Background**

One of the challenges for mathematics educators concerned with issues of democracy is how mathematics education could support the goals of democracy itself (CIEAEM 64). But, "dealing with issues of democracy in mathematics education clearly requires engaging with values" (Bishop 2008a, p.232). Towards this direction, this paper concerns an initial investigation of the Greek primary school teachers' values when they teach mathematics.

There are different perspectives about the definition of the notion "value" and the origin of values, but we could mention that values are generally associated with the affective domain as well as with questions of ethics and moral decision making (Bishop, 2008a; Hannula, 2002). They are connected with "the worth of something" and it is when one must make choices that one's values emerge (Seah & Bishop, 2000). Moreover, they could be regarded as "interpersonal and public agreements about what "ought" to be done by the participants of a community of practice for the social group to function" (Atweh & Seah, 2008, p. 3).

According to Bishop (2008b) values are clearly specific in cultures and the values of mathematical thinking are influenced by the culture of the people, the goals of the social institutions of a society regarding mathematics education, the educational institutions' values, the teachers' values and decisions as well as the individual learners' goals and values regarding



mathematics. He used the notion of mathematical values in order to describe the values which have developed as the knowledge of Mathematics has developed within Western cultures and the notion of mathematics educational values to describe the values related to the pedagogy of the discipline.

He considered three components as to the analysis of mathematical values (Bishop, 2008b): a) the ideological component that concerns the values of rationalism and objectism, b) the sentimental component that includes the values of control and progress and c) the sociological component that concerns the values of openness and mystery. On the other hand, mathematics educational values could be focused on five complementary pairs (Seah & Bishop, 2000): a) formalistic-activist view, b) instrumental-relational understanding/learning, c) relevance-theoretical knowledge (solving daily problems or not), d) accessibility-specialism (mathematics for all or for the elite group) and e) evaluating –reasoning (steps of knowing, applying routine operations, searching problem solving/reasoning and communicating).

One of the most important questions about values is what values do mathematics teachers think they are teaching. The little research that has been done about this issue highlights the effect of culture in the evolution of values (Atweh & Seah, 2008; Leu & Wu, 2004; Sam & Ernest, 1997). Moreover, teachers find it difficult to discuss values and mathematics because they are not used to doing so, although they do hold values about mathematics and mathematics education (Bishop, 2009). The above issues are being focused and examined in this paper through a primary investigation of the Greek primary school teachers' mathematical values.

### Method

48 Greek primary school teachers (14 male, 34 female) participated in this pilot research, realized in Athens, in January 2011. The teachers were teaching in 4 schools with different socio- economical and cultural features (12 teachers participated from each school). The following table 1 shows the characteristics of each school.

Schools	status	
	Socio-economical	Cultural (immigrant students)
A	medium	25%
B	high	-
C	medium	-
D	low	75%

Table 1. The schools' status

Eleven teachers had been teaching 1-10 years, 19 had been teaching 10-

20 years and 18 had been teaching 20-30 years.

The teachers completed a questionnaire that included six open questions. The questions were the following: 1) For which reasons do you consider it is worth for a child to develop mathematical thinking? 2) What teaching practices do you consider effective for the development of a child's mathematical thinking? 3) What students' practices do you consider significant for the development of their mathematical thinking? 4) What parental practices do you consider significant for the development of a child's mathematical thinking? 5) Describe the important elements of a school mathematics textbook which you consider support the development of a child's mathematical thinking. 6) Which state policies for the design of Mathematical Education do you consider important for the development of children's mathematical thinking?

The analysis of the results was based on Bishop's theoretical framework of mathematical values. More specifically, the following table (table 1) shows some keywords of teachers' expected written reports and their connections with concrete mathematical values (Bishop, 2008b).

Mathematical Values	Teachers' expected reports
rationalism	Logical reasoning, inference, proofs, argumentation
objectism	Invention of symbols, use of tools, history of mathematics
control	Analysis of why routine calculations and algorithms work, applying mathematics to solve problems in society
progress	Alternative and non-routine solution strategies, generalization
openness	Public justification and critical analysis of an idea
mystery	Wonder and mystique of mathematical ideas

Table 2. Mathematical values

### Preliminary Results and Discussion

In this paper we will present the results of the first and second question. The following table (table 3) presents the values that emerged from the teachers' written reports on the first question.

Mathematical values about mathematical thinking	Number of teachers			
	School A	School B	School C	School D
rationalism	10	6	12	8
objectism	1	-	-	-
control	9	9	7	6
progress	2	1	5	1
openness	-	-	-	-
mystery	-	-	-	-

Table 3. Analysis of the first question

According to table 3, the majority of the teachers' written reports are associated:

-With the value of rationalism over its complementary value of objectism (e.g. "*(the child)* develops logical processes and secure and completed conclusions", "*(the child)* identifies a problem and its parameters and combines the data in a right way in order to find the solution", "*(the child)* is able to reason with proofs").

- With the value of control (e.g. "*(the mathematical thinking)* helps the management of problematic situations in everyday life, " it helps the child to cope with demands of the society) over its complementary value of progress (e.g. " the child learns to use strategies in order to succeed his/her goals")

- The values of openness and mystery were not reported.

- There were small differences among the teachers that were teaching in schools with different socio-cultural and financial features.

Table 4 presents the values that emerged from the teachers' written reports on the second question and table 5 presents some examples of their reports. We mention the values presented in the teachers' reported practices as to the development of mathematical thinking.

Mathematical Values in teachers' reported practices	Number of teachers			
	School A	School B	School C	School D
rationalism	1	3	2	-
objectism	6	6	10	3
control	6	3	5	6
progress	6	1	2	-
openness	5	-	3	-
mystery	-	-	-	-

Table 4. Analysis of the second question

Values	Examples of reported teaching practices
rationalism	"We should help the student to lead his/ her thinking to hypothesis, reasoning and conclusions." "Using questions like: How did you find it? How did you arrive at this conclusion?"
objectism	"The use of number line for the representation of mathematical problems". "The use of tables and diagrams". "The exploitation and use of new technologies".
control	"The design of activities that they could give solutions in everyday problems". "The use of authentic problematic situations". "Interdisciplinarity and connection with other learning objects".

progress	“To allow the development of improvised pupils’ strategies”. “To pose open problems”. “to promote the development of different children’s ways of the solution of a problem”.
openness	“Working in groups by finding different arguments, controlling, criticizing and concluding”. “Discussion among the pupils for possible solutions and different strategies”.

Table 5. Examples for the second question

According to table 4, the majority of the teachers’ written reports concerning their practices are associated:

-With the value of objectism over its complementary value of rationalism. This finding could be explained by the fact that when the teachers write about their practices, they emphasize the tools that they use in order to promote their goals. That is we are not sure that they interpreted the use of different representations as a value itself, but rather as a means of their teaching.

-With the value of control over its complementary value of progress, although we could find more reports concerning the value of progress in comparison to the first question.

-There were some reports about the value of openness. However as in the case of objectism there is the same problem of interpretation.

- The teachers’ reported practices in school D presented deviations in comparison to the ones from the other schools, as they were connected only to the values of objectism and control.

These preliminary results put an emphasis on the mathematical values of rationalism, objectism and control. However, an analysis of all the questions will give us a better picture of the Greek primary school teachers’ values when they teach mathematics. Furthermore, the analysis of each teacher’s reports will allow us to have a deeper understanding of how mathematical values determine his/her teaching decisions.

In our opinion, the pre-mentioned mathematical values are associated with the notion of democracy, if we assume that democracy is associated with decision making based on argumentation, on the creation of new and alternative tools and strategies in order to cope with the emerging problems in our society, on the participation in public discussions through the mutual respect among the interlocutors. However, further research is needed to be conducted in order to investigate ways of promoting these values to our teachers.

### References

Atweh, B. & Seah, W.T. (2008). Theorising values and their study in

- mathematics education. In Jeffery, Peter L. (ed.) *2007 AARE Conference*.
- Bishop, A. (2008a). Mathematics teaching and values education- an intersection in need of research. In P. Clarkson & N. Presmeg (eds.), *Critical Issues in Mathematics Education* (pp. 231-238). Springer
- Bishop, A. (2008b). Teachers' mathematical values for developing mathematical thinking in classrooms: theory, research and policy. *The Mathematics Educator*, vol.11, 1/2, 79-88.
- Bishop, A. (2009). Researching family mathematical practices: background research and future challenges. *Proceedings of the 3<sup>rd</sup> Conference of GARME* 43-53. Rhodes, Greece
- Hannula, M. (2002). Attitude towards Mathematics: Emotions, Expectations and Values. *Educational Studies in Mathematics*, 49, 25-26.
- Leu, Y-C. & Wu, C-J. (2004). The mathematics pedagogical values delivered by an elementary teacher in her mathematics instruction: attainment of higher education and achievement. *Proceedings of 28<sup>th</sup> PME Conference*, 225-232.
- Sam, L.S. & Ernest, P. (1997). Values in mathematics education: what is planned and what is espoused? *Proceedings of the Day Conference*, BSRLM, University of Nottingham, 37-44
- Seah, W.T. & Bishop, A. (2000, April 24-28). Values in mathematics textbooks: a view through two Australasian regions. *Paper presented at the 81<sup>st</sup> annual meeting of the American Educational Research Association*. New Orleans, LA.

## Are elementary teachers ready to prepare students for their role as critical citizens?

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*Abstract: With the present study we intended to explore the knowledge of in-service teachers with regards to the notion of statistical literacy and its connection with effective citizenship in a democratic society.*

### INTRODUCTION

It is a fact that the understanding of mathematics is crucial for effective citizenship as it enables pupils to act in the framework of a democratic society. Although there is a prominent belief that statistics is not mathematics (Cobb & Moore, 1997) paraphrasing the previous statement we could say that statistics is a source for decision-making and action. Statistics play a key role in shaping policy in a democratic society, so statistical literacy is essential for all citizens in order to keep a democratic government strong (Wallman, 1993). Recognizing the key role that statistics plays in modern society, the National Council of Teachers of Mathematics (NCTM, 2000) recommended that “by the end of high school students should have a sound knowledge of elementary statistics” (p. 48).

Statistical literate students-future citizens- are able to understand and critical evaluate statistical results that permeate daily life-coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions (Wallman, 1993). The fact that statistical literacy is in a direct relation with democracy has been emphasized from many researchers. Over a half century ago, Wishart (1939), commented that the teaching of statistics is important because it protects individuals from the misleading practices of “the propagandists” (p. 549). More than half a century later, Gal (2000) stressed that individuals need to be able to make sense of numerical information in order to avoid falling prey to the influence of data that looks incontrovertible simply

because it is quantitative in nature. In accordance with the above statements a recent (1996) white paper of the European Commission argued that in a society in which the individual will have to understand complex situations and vast quantities of varied information, “*there is a risk of a rift appearing between those who are able to interpret, those who can only use, and those who can do neither*”.

Citizens are constantly incurred with the results of surveys, experiments, and observational studies. In order to be able to interpret the results of these studies, it is vital to have some understanding of basic statistical “tools”-notions that are used in the planning and the conducting of studies. In other words it is crucial for them to be at a certain level statistical literate.

The knowledge of citizens is connected with the knowledge of teachers of compulsory education as teacher knowledge is connected to what and how students (future citizens) learn (Ball & Bass, 2000). Starting from this point of view we designed and conducted a study which its main goal was to measure how able are elementary teachers to understand statistical notions that enables them to act as informed citizens in a democratic society.

### **BRIEF LITERATURE REVIEW**

There are several studies that are related to statistical knowledge of teachers. Studies have been made about teachers’ comprehension of measures of center (Callingham, 1997), about comprehension of graphs (Jacobbe & Horton, 2010) and about teachers beliefs and attitudes towards statistics (Begg & Edwards, 1999). Precisely, for the interpretation of media graphs there is the study of Monteiro & Ainley (2007) whose focus is the critical sense of student teachers. In Greece there aren’t similar studies except of two about the professional development of elementary teachers in statistics (Chadjipadelis, 1999; Pagge, 1999).

### **THEORETICAL FRAMEWORK AND RESEARCH QUESTION**

For the present study we adopted a theoretical framework that it is based on the work of Watson (2004) and it concerns the expectations for statistical literacy of students when they leave school to participate in society. It includes (i) the understanding of basic statistical terminology, (ii) the understanding of terminology when it appears in social contexts and (iii) the ability to question claims that are made in context without proper statistical justification.

Keeping these three lines of research, and taking into consideration the mathematics curriculum for compulsory education, we designed and

conducted a survey on in-service elementary teachers' level of understanding of basic statistical notions.

Our research question was: **How able are elementary teachers to understand statistical notions that are connected to effective citizenship in a democratic society?**

### **METHODOLOGY**

In order to answer the research question a combination of quantitative and qualitative techniques was used. We constructed a questionnaire which we administrated to in-service teachers. After its completion and its assessment we conducted semi-structured interviews with the participants.

#### Questionnaire

The questionnaire items that we used for this study were either adapted from items used in previous research (Aoyama, 2003; Garfield, 2003; PISA, 2003) or constructed by the researchers for the needs of the present study. The questionnaire included ten open-ended items. The analysis was conducted by coding each item's responses utilizing a rubric for levels of correctness (0-4). Reliability of the instrument was tested using a Cronbach's alpha simulation approach and the results showed  $\alpha=0.70$ , which is accepted.

Items contained basic statistical notions that are encountered in everyday life and are connected with effective citizenship in a democratic society: average, proportional thinking, reading charts and graphs and inferring about them.

Items with graphs formed the majority of the questionnaire. For some of them we investigated teachers' competency in reading the data, reading between the data and reading beyond the data (Friel et al, 2001), while for others we used misleading graphs and we asked from the participants to judge them in comparison with given claims.

#### participants and setting

20 teachers participated in this study. All of them were in-service elementary teacher. All had enough years of teaching experience (from 7 to 23 years with a mean of 10 years) and they had participated in programs of professional development.

The research was conducted during November and December of 2011. After the administration and the analysis of the questionnaire a 30-minute semi-structured interview was conducted with each one of the participants. The interview was focused on their answers to the questionnaire and to the way they understand the connection of statistics with effective citizenship.



### KEY RESULTS

Teachers were familiar with the notion of average but many of them had difficulty to understand that it can be a decimal number or that knowing only its value we can't infer about the data set. Although they could make inferences about the data of a table they showed difficulty to correlate rows and columns and use proportional thinking. Surprisingly, they were able to understand a statement that concerned the notion of probability.

As regards to the comprehension of graphs they weren't able to use the correct terminology, to understand the structural components and the specifiers (Friel et al. 2001). Also they weren't able to understand why a graph was misleading as it was constructed with truncated scale in order to emphasize the given claim.

In their interpretation of media-graphs, in-service teachers drew less on technical knowledge about graphs, than on other resources such as their personal opinion or personal knowledge (Monteiro & Ainley, 2004) about the data.

All teachers that participated in the present study were conscious of the importance of the statistical knowledge for the effective citizenship. As Chris, a young teacher, observed: *All these (the items of the questionnaire) are very important as we see them in our everyday life...It is very important for me to be able to recognize and understand misleading claims...I want to be able to judge the truth...*

However our study confirmed the results of previous surveys (Batanero et. al, 1994) about the poor state of statistical literacy in adult population. This is probably due to lack of education at university level-as pre-service teachers- on the statistical literacy.

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### REFERENCES

- Aoyama, K., Stephens, M., (2003). Graph interpretation aspects of statistical literacy: A Japanese Perspective. *Mathematics Education Research Journal*, 15(3), 207-225.

- Ball, D.L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.
- Batanero, C., Godino, J.D., Vallecillos, A., Green, D., & Holmes, P. (1994). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematics, Education, Science and Technology*, 25(4), 527 – 547.
- Beggs, A., & Edwards, R. (1999). Teachers' ideas about teaching statistics. Paper presented at the Joint Conference of the AARE and NZARE, Melbourne.
- Callingham, R. (1997). Teachers' multimodal functioning in relation to the concept of average. *Mathematics Education Research Journal*, 9(2), 205-224.
- Chadjipadelis T. (1999). Teaching teachers to teach statistics. Contributed paper at the International Statistical Institute, 52nd Session, Helsinki.
- Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly*, 104(9), 801-823.
- Friel, S., Curcio, F., Bright, G., (2001). Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications, *Journal for Research in Mathematics Education*. Vol. 32, No. 32, 124-158.
- Gal, I. (2000). Statistical literacy: Conceptual and instructional issues. In D. Coben, J. O' Donoghue, & G.E. Fitzsimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 135-150). Dordrecht: Kluwer.
- Garfield J. B. (1993). An Authentic Assessment of Students' Statistical Knowledge. In N. L. Webb & A. F. Coxford (Eds.) *Assessment in the Mathematics Classroom, 1993 Yearbook of the National Council of Teacher of Mathematics (NCTM)*, (pp.187-96). Reston, VA.
- Jacobbe, T. & Horton, R.(2010). Elementary school teachers comprehension of data displays. *Statistics Education Research Journal*, 9(1), 27-45.
- Monteiro, C., & Ainley, J. (2004). Exploring the complexity of the interpretation of media graphs. *Research in Mathematics Education*, 6:1, 115-128.
- Monteiro, C., & Ainley, J. (2007). Investigating the interpretation of media graphs among student teachers. *International Electronic Journal of Mathematics Education*, 2 (3), 188-207. Retrieved from <http://www.iejme/>.

- NCTM. (2000). Principles and standards for school mathematics. Reston, VA: Author.
- OECD. (2003). PISA Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills. Paris, France: OECD.
- Page, J. (1999). How to teach statistics to a life-long learning group of school teachers. Contributed paper at the International Statistical Institute 52nd Session, Helsinki.
- Wallman, K.K. (1993). Enhancing statistical literacy: Enriching our society. *Journal of the Statistical Association*, 88, No.421, 1-8.
- Watson, J.M. (2004), Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education*, 4, 305-337.
- Wishart, J. (1939). Some aspects of the teaching of statistics. *Journal of the Royal Statistical Society, Part IV*, 532–551.

## **Questions in the Greek mathematics classroom: Issues of equity and democracy**

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### **Abstract**

This paper constitutes a small part of a further study which was carried out within the framework of a professional development program, focused on the pedagogical knowledge of teachers. Interesting findings emerged about issues of equity and democracy in the questioning patterns that Greek teachers use in the mathematics class.

### **Theoretical Framing**

Mathematics instruction is an opportunity for open discussion about mathematics in which pupils' voice has an important role (Boylan, 2002). However, this rarely occurs in real classrooms. The usual patterns of classroom discourse, are ruled by a fundamental asymmetry of power between teachers and pupils (Myhill & Dunkin, 2005). Within this environment, the dominance of the teacher is apparent and is expressed, among other issues, in the way teachers use an important component of mathematical discourse, the questioning.

One issue that defines a democratic classroom is pupils' opportunities for an equal access to learning. This can be affected by teachers' expectations which can lead them to treat pupils differently in the classroom. Some ways in which this can occur in the context of questioning, are possible differences between which pupils are most frequently called upon in class and which are not, or among pupils to whom the easier questions are directed, as well as prejudices in appraisal of pupils' work (Boylan, 2002). Such beliefs and practices cause the lack of access of all pupils to a quality mathematics education and the limitation of their opportunities for an academic achievement (English, 2002a).

The underlying pattern of discourse in the mathematics class, where teacher controls questioning, constantly encourages pupils' passivity (Watts et al., 1997). In usual mathematics classrooms the nature of pupils' participation can be characterised as marginal (Boylan, 2002), since pupils are generally not involved in the production of the practices as teachers talk and pupils just sit and listen (Galton et al., 1999).

This teachers' tendency to control classroom interaction has the consequence of limiting pupils' enquiries resulting in a very low level of pupils' questions (Wragg & Brown, 2001). One main reason for the paucity of pupils' questions is the Initiation-Feedback-Response pattern of classroom discourse, which rarely gives pupils an opportunity 'to fit a question into the ongoing cycle' (Dillon, 1988). An asymmetry also exists regarding the authority and questioning of teachers and pupils. When a teacher asks a question it is an assertion of his/ her knowledge and authority. When pupils ask a question it indicates a lack of knowledge and may diminish their authority (Boylan, 2002).

Teachers often use certain questioning techniques that can affect the susceptible democratic atmosphere of the class. One of them is the use of controlling or norming questions in order to exert social control and to maintain the power structure (Ainley, 1987). This impedes the creation of a conjecturing atmosphere in the classroom which supports the mathematical thinking of all students (Mason & Wilder, 2006).

When pupils do not respond to a question, teachers often start the process of funnelling (Bauersfeld, 1995). The teacher keeps asking more and more precise and detailed questions in an attempt to find something that pupils can answer, making them play one form of the game "guess what is in my mind". In this way, the teacher reveals that s/he is not genuinely interested in pupils thinking and voice but s/he only seeks the answer that the teacher has in his/her mind (Mason & Wilder, 2006).

Another common technique that teachers use in the mathematics class is the «cloze technique» (Mason, 2002) or according to Ainley (1987) the «hovering question», in which, the teacher pauses in a flow of statements and expects students to fill in the missing word. But, in this way the reasoning is done by the teacher and the pupil is called just to fill in the missing word, often without knowing anything of what is going on. This technique teaches students that their own knowledge is subordinate to the teacher's (Kirby, 1996).

### **Methodology**

This work took place on the occasion of a professional development program, carried out during the last two years. Seven primary school teachers and two researchers are the members of this Program. The Program focused on the pedagogical mathematical knowledge, based on teachers' needs, as presented by them at the beginning of and during the Program. One need that emerged during the operation of the Program, was the issue of questioning in the mathematics class. In order to deal with this, we carried out a small research to become aware of the practices that teachers used until then, in the classroom interactions, regarding the issue of questioning, so as to direct the future activities of the Program.

The purpose of the study was to search for issues of equity and democracy at the patterns of questioning in the Greek mathematics class. More particularly, the themes that we searched were the number of teacher-generated questions and pupil-generated questions, and the use of controlling questions and questioning techniques that may impede the democratic atmosphere of the mathematics classroom.

A case study approach was adopted. Researchers observed the mathematical instructions of two teachers of the Program (the first was a man with a 2 year educational experience and the second a woman with 15 years) every day for one month. The selection of teachers was based on their working experience as it was interesting to search possible differences between the most inexperienced and the most experienced teacher of the community. The mathematical instructions were audio-recorded and analysed according to the research purpose. This paper presents data from 3 mathematical instructions of every teacher, which were selected in a random way. Findings from the three instructions are similar to the overall findings.

### **Discussion**

*Which is the number of teacher-generated questions and pupil-generated questions?* It has been found (Table 1.), that teacher-generated questions were outnumbered pupil-generated questions at the instructions of the two teachers. This pattern reveals an imbalance between teacher authority and student voice in the mathematics classroom and gives the answer to the hard question that usually emerges: "Who does the work in the math lesson?" In this case, in which the answer is "the teacher", teachers are heading towards the wrong direction and it is important to search what can be done to restore the balance.

Instructions	Number of teacher-generated questions			Number of pupil-generated questions		
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
1 <sup>st</sup> teacher	120	102	116	3	6	5
2 <sup>nd</sup> teacher	142	125	138	3	4	5

Table 1.

*Do teachers use questions for social control and for exerting authority during the mathematics instruction?* Teachers in their instructions opted for the use of controlling and norming questions, instead of the use of assertions or direct statements, giving them a managerial role of organising and maintaining order in the classroom. More particularly, teachers used such questions each time students did not pay attention to the lesson or make a fuss, intending to restore order and discipline. Examples of these questions are: “*What do we do when we come into the classroom?*” “*We don’t do that now, do we?*”, “*What do I want from you now?*”, “*Are we ok now?*”, “*Does the 4<sup>th</sup> group attend the lesson?*”, “*Can you repeat exactly what I have said?*”. The difference that is recorded between the two teachers (Table 2.) is that the first teacher used less controlling questions than the second teacher (who had 15 years of working experience).

Instructions	Number of controlling and norming questions		
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
1 <sup>st</sup> teacher	15	10	13
2 <sup>nd</sup> teacher	25	29	27

Table 2.

*Do teachers use questioning techniques that impede the democratic atmosphere in the mathematics classroom?* Both teachers used the “process of funneling” when pupils didn’t respond to a question, by asking more detailed questions. In this way, they directed pupils to follow the answer teachers had in their mind, and revealed that they were not really interested in pupils thinking but only sought a right answer. This process was very often found in both teachers.

Teachers also used the “cloze technique”. Classroom discussions often resulted to pupils uttering only elliptical sentences or just the missing word. In this way the reasoning was done by the teacher and pupils often didn’t know anything of what was going on. In fact, the second teacher used the “cloze technique” to a great extent. Some examples are: “*this tells me that I cut the cheese into two pieces and take... what?*”, “*the fractions that tell us*

*that we split something into? ... what pieces? ... (equal) ... are called... ”.*

Enacting democratic practices in the mathematics classrooms is not an easy task. However, it is important to seek a process which allows and fosters teachers' change. The effort to create a more democratic mathematics classroom begins with teacher preparation and the investment in ongoing professional development (Stemhagen, 2011) and results from the work of a committed teacher who believes in the capacity of all people to learn mathematics and creates space for the voice of each pupil.

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### References

- Ainley, J. 1987, Telling Questions, *Mathematics Teaching*, 118, p24-26.
- Bauersfeld, H. 1995, "Language Games" in the mathematics classroom: Their function and their effects. In P. Cobb & H. Bauersfeld (Eds.), *The Emergence of Mathematical Meaning: interaction in classroom cultures*, Lawrence Erlbaum Associates: Hillsdale, NJ p271-291.
- Boylan, Mark. (2002). "Teacher questioning in communities of political practice." A paper delivered at The Third International Mathematics Education and Society Conference (MES3), Helsingor, Denmark: 1, 182-191.
- Dillon, J.T. (1988) *Questioning and Teaching*. Beckenham: Croom Helm.
- English, L. D. (2002a). Preface. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. xi). Mahwah: Lawrence Erlbaum.
- Galton M., Hargreaves L., Comber C., Wall D., and Pell T. (1999) Changes in patterns of classroom interaction in primary classrooms: 1976–1996. *British Educational Research Journal* 25 (1), 23–37.
- Kirby, P. (1996) Teacher questions during story-book readings: Who's building whose building? *Reading* 30 (1) 8–15.
- Mason, J. (2002). Minding Your Qs and Rs: effective questioning and responding in the mathematics classroom, in L. Haggerty (Ed.) *Aspects of Teaching Secondary Mathematics: perspectives on practice*, RoutledgeFalmer, London, p248-258
- Mason, J. & Wilder S. (2006). *Designing and using mathematical tasks*. London: Tarquin.
- Myhill D. & Dunkin F. (2005) «Questioning Learning» *Language and Education*, 19 (5) 415- 427.



- Stemhagen, K. (2011). Democracy and school math: Teacher belief-practice tensions and the problem of empirical research on educational aims. *Democracy & Education*, 19(2), Article 4. Retrieved from <http://democracyeducationjournal.org/home/vol19/iss2/4/>
- Watts, M., Alsop, S., Gould, G., Walsh, A. (1997) Prompting teachers' constructive reflection: Pupils' questions as critical incidents. *International Journal of Science Education* 19 (9), 1025–37
- Wragg, E.C. and Brown, G. (2001) *Questioning in the Primary School*. London: Routledge Falmer.

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## **Mathematics Teacher Education - collaborative work influence in the professional development**

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### **Abstract**

This qualitative research analyzes text records of logs from an Elementary School group of teachers aiming at understanding how collaborative work influences teacher's professional development. The interpretative analysis allowed us to identify categories such as follows: Shared reflection, Learning, Trust and Reflection upon practice, Exchanging experience, Shared goals and Commitment to others, all of which have collaborative work features. Categories were analyzed using CHIC software, which allows the relational analysis among them showing teacher development under this perspective can promote professional development by giving teachers the opportunity to experience themselves as learners and, also, as tutors to others.

**Keywords:** Continued Education, Collaborative Work, Professional Development.

### **Introduction**

Research studies have been presenting indicators for continued education processes that help change teaching practices with an aim to improve the quality of Mathematics instruction. One of these indicators is the creation of situations which explore aspects of daily classroom practices, inte-

grating contextual activities so that teachers can revisit their practice, reflect upon it and rebuild it.(Imbernón, 2010).

During the development process, it is necessary to intentionally develop strategies that benefit collaborative skills as a practice built by the group members. Research has found the existence of signs that collaborative work is essential for teachers' professional development. (Miller, 1990)

Collaborative work, as proposed by Fullan and Hargreaves (2000), has a number of features which essentially are attitudes established in teachers' relationships that reveal trust, commitment, sharing of ideas and experiences, as well as valuing themselves both individually and as a group.

Boavida and Ponte (2002) have pointed collaborative work as one of the possibilities in continued education processes, underscoring that collaboration happens "when a number of intervenientes work together, not in a hierarchical relationship, but under the basis of equality so that mutual help is forged to reach the goals that will benefit all" (p. 45). In collaborative work, there is the advantage of multiple looks over the educational situation which in turn allows for the production of consistent interpretative frames about the issue at stake.

More specifically the research by Lobo da Costa (2004) showed that collaborative work comprised characteristics present during the development process which were taken as categories. They are: (C1)Shared reflection, (C2)Learning/Learning with others, (C3)Teacher's actions, (C4)Development actions, (C5)Research about practice, (C6) Exchanging experience, (C7)Representation of every member's ideas, (C8)Partnership, (C9)Shared goals, (C10)Commitment to Group, (C11)Trust, (C12)Voluntary participation, (C13)Dialog/ Interaction, (C14)Autonomy development, and (C15)Reflection upon action.

When working in groups of a collaborative nature, the developer-developpee relation is overturned. Hence, the often well-established idea that continued education processes consist of one developer who works with a group of teachers to promote their development is now replaced by the idea of forming a team of educators in which they work together in a learning-developing mutual relationship. (Lobo da Costa, 2004).

It is worth noting that setting up a collaborative learning team requires that its members have democratic attitudes which imply openness, sensibility, generosity and humbleness because, when working in groups, the teacher experiences situations that require negotiation, dialog, argumentation and also respect towards other people's ideas, mainly when

the majority's decision and the individual's thoughts do not converge.

Living this formative process based on collaboration and democratic principles can help the teacher develop pedagogical strategies so as to promote democracy in the mathematics classroom.

### **Research scenario**

The research that supports this paper is part of the project "Continued Education for Mathematics Teachers at Elementary and High Schools: Forming Study and Research Groups on Development Processes" (ECPMEFM, in Portuguese), which is linked to the Education Observatory Program. The latter is one initiative by the Brazilian Federal government whose goal is to improve teaching and learning processes in the country's public schools and uses universities as partners.

The selected ECPMEFM Project has been developed at a private university in the city of São Paulo with a group of instructors, doctoral and master degree students who work with and carry out their research with mathematics teachers at public schools. The development and research project has as its aim to develop a methodology for mathematics teachers' continued education, and uses the collaborative professional learning networks.

Continued education is designed on different strategic actions connected with the contextualization of learning and building a collaborative peer network, including the possibility of virtual interactions as one of the ways to allow for written records of the participants' reflective logs. (Prado, 2003; Lobo da Costa et al, 2008). The development actions sought to:

- approach mathematics syllabus contents in the state of São Paulo, starting at Sequences and Plain Geometry.
- learn and analyze the goals of Mathematical Education.
- discuss issues related to practices developed at schools with the students and the studied theories, logs, and studies carried out in the classroom.

This is the scenario in which the present paper was developed, and whose goal was to understand how a collaborative learning network is established in the course of continued education activities and the **implications of such collaborative work for the instructor's professional development**. For that, we analyzed data collected from the group of participants in the project, a total of thirty Elementary teachers working at the public school system of the state of São Paulo.

Data collection was made using reflective logs produced individually by

group members after one semester of interaction in the project.

Logs analysis was interpretative and used the characteristics established by Lobo da Costa (2004) as categories. Besides the interpretative analysis, the categories received statistical treatment using CHIC software (Correspondence and Hierarchical Cluster Classification)<sup>1</sup>, which allows for viewing similarities and variable classes mapped at the levels of a hierarchical tree.

### Results

The interpretative analysis of the logs showed the existence of the following categories of collaborative work characteristics:

Code	Category	Description
C1	Shared reflection	Reports revealing one's thoughts and queries to the group
C2	Learning/ Learning with others	Reports stating one's own learning (specific and syllabus-content)
C3	Teacher's actions	Reports related classroom experiences.
C6	Experience exchange	Reports related to contents and practical activities.
C9	Shared goals	Reports describing search processes to reach common goals.
C10	Commitment to group	Reports stating attitudes of commitment to other persons
C11	Trust	Reports that show a feeling of belonging and comfort.
C13	Dialog-Interaction	Reports that acknowledge the value of group dialogs
C14	Autonomy development	Reports that show more confidence about decision-making.
C15	Reflection upon action	Reports that display reenactments of the pedagogical practices in the classroom.

Figure 1: Categories of Collaborative Work

We noticed that out of the 15 categories listed by Lobo da Costa (2004), ten were identified in this research, and we present a relational analysis based on the view of the similarity trees generated by CHIC. The following figure shows the resulting similarity tree:

<sup>1</sup> For further details on CHIC, see Gras (2000) and Almouloud (1992).

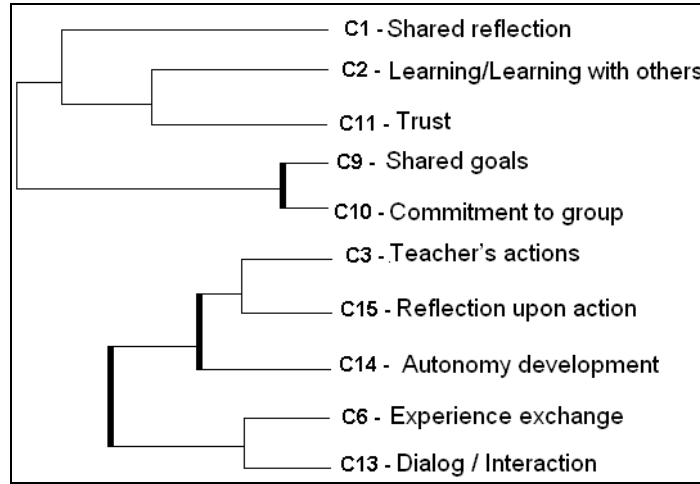


Figure 2: Similarity tree

In figure 2, two classes are found. Class-1, formed by categories (C1) Shared reflection, (C2) Learning/Learning with others, (C11) Trust, (C9) Shared goals, and (C10) Commitment to group, and Class-2, formed by categories (C3) Teacher's actions, (C15) Reflection upon action, (C14) Autonomy development, (C6) Experience exchange and (C13) dialog.

We named Class-1 **Interaction** and it was formed by one sub-class, consisting of categories (C1, (C2, C11) and by a *cluster* formed by categories (C9, C10), as shown in the following figure:

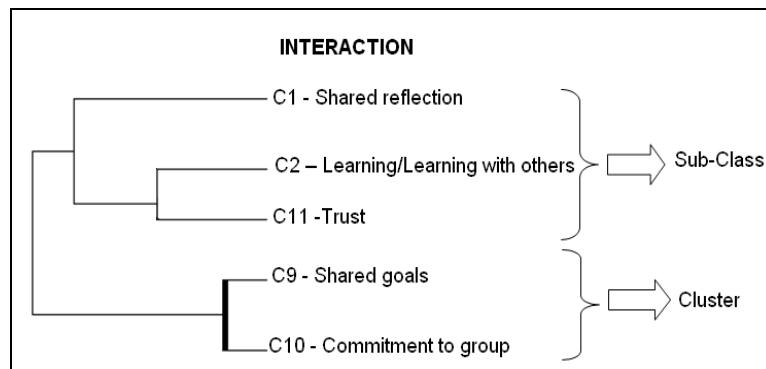


Figure 3: Class-1 Interaction

It can be observed that *cluster* (C9, C10) shows a significant level of similarity which indicates a high probability of interaction happening among

the group of teachers in terms of shared goals, generating attitudes of commitment. This possibility of peer interaction is essential and should be one of developers' purposes since this experience can contribute to establish commitment with one another in the learning context provided by the project.

Sub-class formed by the set of categories (C1 (C2, C11)), shows a discreet level of similarity, and yet it provides signs of teachers' awareness that in order to learn with peers it is necessary to establish an atmosphere of trust for them to expose their conceptual shortcomings. Such trust, according to Freire (1997) is what enables teachers to feel accepted by the group and take stances to learn with other people's experiences and share their reflections about practical and theoretical questions that were studied with the group under developers' pedagogical mediation.

Class-2, named **Teacher's work** is formed by a chain of categories {(C3, C15), C14), (C6, C13)} as shown in the following figure:

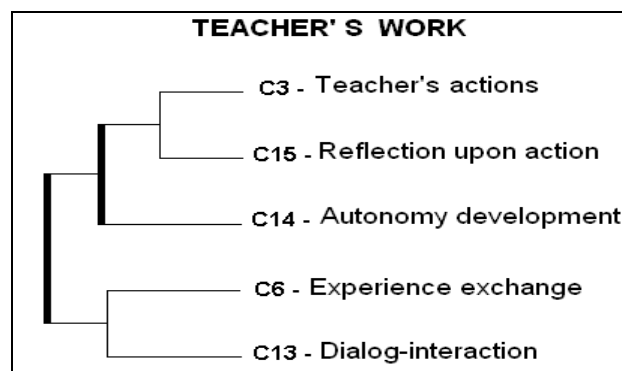


Figure 4: Class-2 Teacher's Work

In this chain of categories it becomes clear the existence of a higher level of similarity between categories (C3, C15), which shows that the experience from teachers participating in the project allowed them to bring up their classroom context to be reported, reflected upon and understood. Thus the importance of encompassing the actions performed in classroom context when working on teachers' development processes. However, such actions should be reflected upon and understood.

This possibly occurred when the teacher exchanged experiences of practices with their peers and through the dialogs established among group

members, as well as the ones between teachers and the studied scholars, who clarify their learning, allowing for better conditions for the development of intellectual autonomy.

Such autonomy can stimulate the Elementary School Mathematics teacher to achieve professional development as this learning process should be continuous and dynamic so that he can work with his students, the future professionals of a new society.

We can see that these characteristics are interrelated; the teacher's professional development is linked to reflecting upon his practice to rebuild it. And, in this process, it is the exchange of experiences that will embolden teachers, as it unveils to others new possibilities and references that allow them to dare to change their teaching practices in relation to learning strategies. Hence the need to acknowledge the dialog and interactions established in the collaborative work during the continued education development. The characteristics that appeared more discreetly in these records were "Shared goals" and "Commitment to others", showing that they become present as group members perceive themselves as having more autonomy to share their goals to achieve knowledge, as well as to develop committed attitudes towards their peers' learning.

We underscore that for teacher's developers, understanding such process is necessary to ensure that formative strategies are developed using dynamic actions, so as to establish a movement between analysis and a deeper understanding of the mathematical content and, almost simultaneously, have such analysis encompass the various aspects of the teachers' pedagogical practices. It is within this movement, between action and reflection, and between mathematical contents and their re-contextualization in the classroom practice that the teacher's praxis is developed towards a learning spiral.

### **Conclusion**

The study showed a connection between the collaborative network as a collective learning environment in the context of continued education and the possibility of providing momentum to the instructor's professional development. The network is created by means of a process where formative actions are developed based on experiences that highlight typical characteristics of collaborative work.

The suggestion is that such networks include the use of contributions from virtual learning environments since they allow for a break from time and space restraints, and also enable dialogs/interactions that are established



by writing using the various communication tools of the virtual environment. This type of interaction, which encompasses the sharing of experiences, knowledge, reflections and questions, helps to build a collaborative learning and reflective environment among teachers. This manner of learning ensures that each participant will simultaneously feel like both, learner and teacher, and continue to learn throughout his life.

### References

- Almouloud, S. (1992). L' Ordinateur, outil d'aide à l'apprentissage de la démonstration et de traitement de données didactiques. **These de Docteur**. U.F.R. de Mathématiques. Rennes, França: Université de Rennes I.
- Freire, P. (1997). **Pedagogia da Autonomia - Saberes necessários à prática educativa**. São Paulo, Brasil: Paz e Terra.
- Fullan e Hargreaves, A. (2000). **A escola como organização aprendente: buscando uma educação de qualidade**. 2ª ed. Porto Alegre: Artes Médicas.
- Gras R. (2000). Les fondements de l'analyse statistique implicative, **Quaderni di Ricerca in Didattica del Gruppo di Ricerca sull'Insegnamento delle Matematiche (G.R.I.M.)**, nº 9, Palermo, 189-209.
- Imbernón, F. (2010). **Formação continuada de professores**. Porto Alegre: Artmed, 2010.
- Lobo da Costa, N.M.(2004) Formação de professores para o ensino da matemática com a informática integrada à prática pedagógica: Exploração e análise de dados em bancos computacionais. **Tese de Doutorado em Educação**. São Paulo: PUSP, SP.
- Lobo da Costa, N. M., Prado, M.E.B.B., Campos, T.M.M. (2008). Formação do professor de Matemática: Uma abordagem pedagógica usando recursos de ambientes virtuais In: **6o Congresso Internacional de Educación Superior Universidad**, Havana, Cuba.
- Miller, J. (1990) **Creating Spaces and Finding Voices: Teachers Collaborating for Empowerment**. State University of New York Press.
- Prado, M.E.B.B. (2003). Educação à distância e formação do professor: redimensionando concepções de aprendizagem. **Tese de Doutorado em Educação**. São Paulo: PUSP, SP.

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## **TEACHER PRACTICE IN AN INQUIRY-BASED MATHEMATICS CLASSROOM**

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### **Abstract**

This paper presents a framework for an inquiry-based approach to mathematics teaching. It was developed by combining theoretical perspectives and case studies of experienced teacher that usually conduct inquiry based teaching of mathematics. This framework describes the actions teachers intentionally perform with two identified purposes: to promote the mathematical learning of the students and to manage the students and the class as a whole.

### **INTRODUCTION**

In the last decade, research in mathematics education has consistently pointed out the need to promote student's learning that goes far beyond the acquisition of mathematical knowledge, but including also the development of mathematical capabilities such as problem solving, reasoning and communication (Ponte, 2011). These recommendations have been reflections in mathematics curricula in many countries of the world, as happened in Portugal with the new math program for 1-9 classes, which began implementation in 2010 (DGIDC, 2007). Promoting these new goals is not compatible

with a type of traditional classroom, based on the exposition of the teacher, who, as stated Sierpinska (1998), the teacher talks and students listen. Instead, this new class model implies new roles for the student and, consequently, for the teacher. This class is based on a new attitude of the students, working with mathematical tasks and discussion of results and ideas (NCTM, 2000; Ponte, 2005; Sullivan et al, 2006). Naturally, the changing role of students in the mathematics classroom involves from the teacher an inquiry-based approach to teaching, with the realization of other instructional actions. It is precisely here and in this context of change in the mathematics classroom, this study fits through, we aim to offer a detailed characterization of the actions of the mathematics teacher in a classroom-based inquiry, from contact with the teachers that usually conducts such classes.

### **THEORETICAL PERSPECTIVES**

In Portugal, like in many countries, the mathematics lessons that follow the standard of teaching: exposition of the theory (by the teacher) and the resolution of exercises (by the students) have been questioned and gave rise to a pattern in which the students are more "attached" in its learning. Ponte (2011) stresses an alternative mode of work in which "teacher introduces a task for the students to work for some period of time and, in a second moment, the students present their solutions to the whole class and discuss the solutions of their classmates" (p. 250). In same direction, others authors say mathematics lesson in inquiry-based approach is generally organized in three or four phases: the "launch phase", the "explore phase", and the "discuss and summarize phase" (Stein, Engle, Smith, & Hughes, 2008). The inquiry-based approach to mathematics teaching demands from the teacher more than the selection of the rich tasks. The selection of a valuable task is, of course, very important because it creates conditions for learning. But after the task selection, and bearing in mind the objectives of the curriculum, is necessary, to the teacher, think about how to propose the exploration of the task in class, or what kind of activities should lead to promote mathematics learning (Stein, Engle, Smith, & Hughes, 2008).

The idea of activity is diffusely defined in the literature. Even e Schwartz (2002) says that activities are chains of events related by the same subject for the same reason. Teachers' practices in the classroom are a series of complex actions, which have their basis in a certain intentionality, which derives from their professional knowledge (Ponte and Chapman, 2006). While in mathematics classes based on the traditional pattern, these actions

are well described and stabilized, the same does not happen in the mathematics classroom lessons in inquiry-based approach.

### METHODOLOGY

In this study, to investigate the activities performed by teachers, in a classroom-based inquiry, we choose an interpretative methodological approach, because we wanted to get the perspectives of the participants.

We selected experienced teachers, who usually develop lessons inspired in an inquiry-based approach to mathematics teaching and work in different mathematical topics that were available to participate in this work. At this point, we collected data from a primary school teacher, named Célia,

Data analysis is inspired by the theory and is based on content analysis of data collected through (i) observation of the teacher in two classes (videotaped), and (ii) two interviews (before and after the classes in which Célia explains her intentions and, in the end, reflects on her action). The analysis of the cases of the teachers led us to identify concrete actions that are present in their practices in each phase of the lesson and the intentions that justify them.

### DEVELOPING A FRAMEWORK FOR INQUIRY-BASED MATHEMATICS CLASSROOM PRACTICE

In an inquiry-based classroom practice, Célia performs a variety of actions that are based on two purposes. On one hand, *promotion of the mathematics learning of the students*;

“That is the idea, to have confrontation. It is not a presentation, it is a learning moment, therefore it can't be a presentation, neither a correction, because it's not that (...) the moment is to confront, to think together about the different resolutions, different representations that have to appear from there, from the presentation... it is a goal.”

And secondly, *Management of students work and class as a whole*: “So I have to select the presentations they want to explore during the discussion and sort according to a more complex sometimes, (...) and put the pair that did the work to present”

We identify in Célia's classes four main phases, according different intentions: Launching the task to students; Supporting students autonomous work on the task; Orchestrating the discussion of the task; and Systematizing mathematical learning. In Table 1 we present our proposal of framework for describing the intentional actions of the teachers in a mathematics inquiry-based classroom practice.

	<b>Promotion of the mathematics learning</b>	<b>Class management</b>
<b>Launching the task to the students</b>	<p><i>Guarantee the appropriation of the task:</i></p> <ul style="list-style-type: none"> <li>- Clarify unfamiliar vocabulary</li> <li>- Mobilize and verify prior knowledge</li> <li>- Set goals</li> </ul> <p><i>Promote adhesion to the task:</i></p> <ul style="list-style-type: none"> <li>- Challenge for work</li> <li>- Request an expected result</li> <li>- Establish connections to student prior experiences</li> </ul>	<p><i>Organize students' work:</i></p> <ul style="list-style-type: none"> <li>- Establish time for each phase of the class</li> <li>- Set forms of work organization (individual, pairs, small groups, whole-class)</li> <li>- Organize the class materials</li> </ul>
<b>Supporting students autonomous work on the task</b>	<p><i>Guarantee the development of the task by the students:</i></p> <ul style="list-style-type: none"> <li>- Focusing on results and ideas</li> <li>- Provide a comparison of ideas</li> <li>- Challenge students to justification</li> <li>- Explain in order to allow students to continue their work</li> <li>- Suggest representations</li> <li>- Request records</li> </ul>	<p><i>Promote the work of students/groups:</i></p> <ul style="list-style-type: none"> <li>- Set interactions between students</li> <li>- Provide materials</li> </ul> <p><i>Guarantee the production of materials for the students presentation:</i></p> <ul style="list-style-type: none"> <li>- Provide appropriate materials</li> <li>- Provide specific time to prepare the presentation</li> </ul> <p><i>Consider the selection and sequencing of the student presentations:</i></p> <ul style="list-style-type: none"> <li>- Identify resolutions less or more comprehensive, less or more complete</li> <li>- Identify resolutions less or more formal</li> <li>- Identify resolutions with common error</li> </ul>
<b>Orchestrating the discussion of the task</b>	<p><i>Promote mathematical quality of the presentations:</i></p> <ul style="list-style-type: none"> <li>- Ask for clear explanations with mathematical evidence</li> <li>- Ask for justifications of outcomes and representation used</li> </ul> <p><i>Promote interactions among students in the discussion of mathematical ideas:</i></p> <ul style="list-style-type: none"> <li>- Encourage questioning for the clarification of ideas</li> <li>- Encourage analysis, debate and comparison of ideas</li> <li>- Identify and make available to discuss questions or errors in the presentation</li> </ul>	<p><i>Create favorable environment for presentation and discussion:</i></p> <ul style="list-style-type: none"> <li>- Put an end to the autonomous work of students</li> <li>- Provide the reorganization of the places to focus on a common space (whiteboard, QI, overhead...)</li> <li>- Promote an attitude of respect and genuine interest on different presentations</li> </ul> <p><i>Manage relationships among students:</i></p> <ul style="list-style-type: none"> <li>- Set the order of presentations</li> <li>- Justify the reasons for not submitting the work of some students (by example, to avoid repetition,...) and ensure rotation of groups in the next task</li> <li>- Promote and manage the participation of students in the discussion</li> </ul>

<b>Systematizing mathematical learning</b>	<p><i>Institutionalize concepts or procedures on mathematical topics:</i></p> <ul style="list-style-type: none"> <li>- Identify key mathematical concept(s) from the task, clarify its definition and explore their multiple representations</li> <li>- Identify key mathematical procedure(s) from the task, clarify the conditions of its implementation and review its use</li> </ul> <p><i>Institutionalize ideas or procedures concerning the development of transversal capabilities:</i></p> <ul style="list-style-type: none"> <li>- Identify and connect the dimensions of transversal capabilities in presence</li> <li>- Enhance the key factors for its development</li> </ul> <p><i>Establish connections with prior learning:</i></p> <ul style="list-style-type: none"> <li>- Highlighting links with mathematical concepts, procedures and transversal capabilities previously worked</li> </ul>	<p><i>Create an appropriate environment for the systematization:</i></p> <ul style="list-style-type: none"> <li>- Focus students at the collective systematization</li> <li>- Promote recognition of the importance of this phase of the class for learning</li> </ul> <p><i>Guarantee written record of the ideas that result from systematization:</i></p> <ul style="list-style-type: none"> <li>- Record in computer or physical resources (boards, interactive boards, transparencies, posters ...) by students or teacher</li> <li>- Request written records in student notebooks</li> </ul>
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Table 1: Intentional actions of the teachers in an inquiry-based classroom practice

This framework is under construction and will be refined through the analysis of other cases of teachers, from other grades and working with different mathematical topics.

## FINAL CONSIDERATIONS

The framework demonstrates complexity and the demands for mathematics teachers to developing an inquiry-based mathematics teaching practice, a fact which contradicts the idea of a teacher less active compared to a class based on exposition. During an inquiry-based class, the teacher needs to pay attention to several aspects that decisively affect the opportunities for mathematical learning of students. It is our expectation that the process we adopted for the elaboration of this framework provides a broad view of the authentic practice of teachers, and can serve as a resource for teacher development based on reflection on his teaching of mathematics.

**REFERENCES**

- Even, R., & Schwartz, B. B. (2002). Implications of competing interpretations of practice and research and theory in mathematics education. In *Proceedings of PME Conference (Vol. 2, pp. 337-344)*.
- DGIDC (2007). *Programa de Matemática do ensino básico*. Lisboa: DGIDC.
- NCTM (2000). *Principles and standards for school mathematics*. Reston: NCTM.
- Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.), *O professor e o desenvolvimento curricular* (pp. 11-34). Lisboa: APM.
- Ponte, J. P. (2011). Using video episodes to reflect on the role of the teacher in mathematical discussions. In O. Zaslavsky & P. Sullivan (Eds.), *Constructing knowledge for teaching second-dary mathematics: Tasks to enhance prospective and practicing teacher learning* (pp. 249-261). New York, NY: Springer.
- Sierspinkska, A. (1998). Three epistemologies, three views of classroom communication: construtivism, sociocultural approaches, interaccionism. In H. Steinbring, M. Bussi & A. Sierpinkska (Eds.), *Language and communication in the mathematics classroom* (pp. 30-62). Reston: NCTM.
- Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning*, 10(4), 313-340.
- Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms. *International Journal of Science and Mathematics Education*, 4, 117-143.

## **Analysing students' definitions of geometrical concepts**

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### **Abstract**

The paper presents a study that took place during a geometry course for future Mathematics teachers. An aim of the study was to analyse the students' concept definitions and their critical thinking. The results show that the students had rich concept images, but their critical thinking abilities were not enough for them to 'transform' these images into acceptable definitions.

### **INTRODUCTION AND THEORETICAL BACKGROUND**

One of the aims of general education and personal development is promoting critical thinking, which, together with maturity and intellectual independence, are the basic characteristics of human self-realization (Heymann, 2003). Living in the complex modern times requires by the individuals – and the collectives – skills that move beyond the mere reproduction of 'successful' techniques and procedures. Additionally, democracy as a form of social organisation requires the active participation of well-informed and reflective citizens. In that aspect, critical thinking is related to evaluating your own reasoning at the moment of its conducting and to evaluating an existing reasoning conducted by you or others (Klakla, 2003). It relies on overcoming the conflict between formal thinking and intuitions, established habits or suggestions for the meaning of terms which in everyday language are similar (but not the same) to those included in a mathematical definition (Klakla, 2003). Such conflicts can be observed in geometry and all fields where the visualisation of mathematical content (concretised on a drawing or a diagram) plays a significant role. Overcoming this conflict can be related to the differentiation between the



concept image and the concept definition (Vinner, 1991). The former refers to the “total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes” (Tall and Vinner, 1981, p. 152) and is built up by the individual through different kinds of experiences with the concept. The latter refers to a formal definition which determines the meaning of the concept. Concept definition and concept image are supposed to be activated simultaneously in the process of concept formation and the processes of problem solving or task performance. However, knowing the concept definition does not guarantee the understanding of the concept. Nevertheless, concept definitions are very important during the preparation of future mathematics teachers, which “should be trained to use the definition as an ultimate criterion in various mathematical tasks” (Vinner, 1991, p. 80).

### **CONTEXT OF THE STUDY**

The research was conducted among 36 future Mathematics teachers in the second year of their studies and consisted of three phases during their ‘School Geometry’ course from October 2011 until January 2012. The aims of the research were to:

- analyse the students’ concept definitions of some basic geometrical concepts;
- diagnose the students’ level of geometrical knowledge, their intuitions, their misconceptions and their ability in defining and using mathematical language;
- examine the effect of the course in the students’ geometrical knowledge, including their concept definitions;
- examine the students’ ability in critical thinking.

In Phase 1 the students were given a test which contained, among others, questions asking for the definition of geometrical concepts known from school: trapezium, inscribed angle, point reflection and chord. In Phase 2 the students were asked to assess some given sentences if they are correct definitions of three geometrical concepts: trapezium, inscribed angle and point reflection. All sentences were taken from the students’ answers from Phase 1 and only one was correct in each case. The answers had to be justified and the false sentences had to be corrected, if possible. In Phase 3 a discussion took place; it was planned to include a negotiation of the necessary conditions and terms of the definitions to be valid. Due to space limitations we will analyse part of the results from Phase 1 and the results of Phase 2 related to the inscribed angle.

## RESULTS

### Phase 1

Table 1 summarises the results of the test given in Phase 1.

Categories of answers	Concepts			
	Trapezium	Chord	Inscribed angle	Point reflection
correct	13	13	6	3
wrong – missing info	19	2	11	3
wrong – wrong info	2	12	11	4
wrong – particular case given	1	1	0	5
lack of answer	1	8	8	18

Table 1. Test results from Phase 1

Point reflection was the most difficult concept for the students to define; some identified it with axial symmetry or with a figure which has one or more lines of symmetry, or with the symmetry axis of a segment. Examples of “wrong-wrong info” answers are:<sup>1</sup> “A line perpendicular to the side which goes through its middle”, “It is this one which by going through the centre of a figure, divides it into two identical parts which are its mirror reflections”. For some students point reflection was connected to the coordinate system and they defined it as point reflection at the point (0, 0). Some “wrong-missing info” answers were: “It is symmetry according to the centre” (2 students), “symmetry according to the point S”. It seems that even if the students had a concept image, they could not express it verbally; eventually, they merely tried to reformulate the concept's name.

Concerning the inscribed angle, in the “wrong-missing info” answers the students wrote only one condition: “it is an angle whose vertex belongs to the circle”; “an angle between two chords”. Their concept image seemed to include all the associated properties and processes, but it was not sufficient for reaching the definition. We also noticed some misconceptions, expressing a problematic concept image: “an angle resting on a circle whose vertex lies in the centre of the circle”, “an angle inscribed in a circle if it rests on a semicircle”, “an angle which constitutes a part of a circle (360°)”.

### Phase 2

Many students (trapezium: 13 students; inscribed angle: 12 students; point reflection: 20 students) assessed the correctness of the given definitions without any justification. Some students tried to correct the definitions

<sup>1</sup> All students' answers were translated from Polish to English; we have tried to preserve grammatical or syntactical mistakes.

by adding or removing words; others used examples and counterexamples, written or drawn.

Table 2 shows the results from Phase 2. “Wrong-wrong” means that a wrong sentence was assessed as wrong, “wrong-correct” means that a wrong sentence was assessed as correct, etc.

Categories of answers	Concept definitions		
	Trapezium (7 sentences, 252 answers)	Inscribed angle (8 sentences, 288 answers)	Point reflection (6 sentences, 216 answers)
Wrong – wrong	105	140	102
Wrong – correct	104	96	62
Correct – wrong	11	5	1
Correct – correct	25	27	30
Lack of answer	7	20	21

Table 2. Task results from Phase 2

Due to space limitations we will analyse only the part related to the inscribed angle. Table 3 below summarizes the students’ answers.

Categories of answers	Inscribed angle							
	1	2	3 (correct)	4	5	6	7	8
Wrong	9	23	5	17	27	17	30	17
Correct	25	12	27	17	6	18	4	14
Lack of answer	2	1	4	2	3	1	2	5

Table 3. Task results from Phase 2, concept: inscribed angle

The given definitions for the inscribed angle were the following:

1. An angle whose vertex lies on the circle.
2. An angle created by two lines lying inside a circle.
3. An angle whose vertex belongs to a circle and its sides are formed by two chords.
4. An angle resting on an arc of a circle, it is twice smaller than the central angle.
5. An angle which constitutes a part of a circle.
6. An angle created by two chords.
7. An angle between two radiuses of a circle.
8. An angle is inscribed in a circle if it is resting on a semicircle and its end is on the edge of the circle (it is twice smaller than the central angle).

Although most students chose sentence 3 as the correct definition, five students answered that it is not correct because there was a lack of precise information about the location of the angle: “if it is inner angle”; “but it has to be an angle between an intersection of these chords”. The most problematic was sentence 1 since 25 students answered that it is correct. The fact that the vertex is lying on the circle was sufficient to accept this definition although it is only one of the necessary conditions. At the same time, sentences 4, 6 and 8 also caused difficulties to the students. In the case of sentence 6 there is again only one condition from the two which are required. Sentences 4 and 8 are related to the theorem of the inscribed and central angles resting on the same arc; but they are only a part of that theorem. The students did not know the definition so they referred to any known information about this concept. It is noteworthy that the students who declared sentence 3 as the correct one also accepted sentences 4 and 8.

Generally, the students used a single example (e.g. a drawing) both in order to ‘prove’ that a sentence is a correct definition and to reject it, thus they were not demonstrating critical thinking. Figure 1 shows a drawing made by a student in order to accept sentence 1 as a correct definition (she also wrote “YES” next to it). Figure 2 shows a drawing made by another student as a counterexample for the incorrectness of sentence 1. Next to it she wrote “because it can be such a case”.



Figure 1

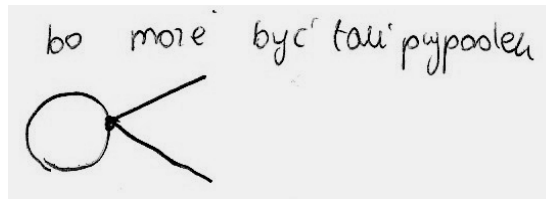


Figure 2

### CONCLUSIONS

Students have had big difficulties with defining the concepts although all of them were known from school. Their concept images included drawings, properties, formulas or particular cases (e.g. isosceles trapezium), but this knowledge was not sufficient to reach the proper concept definitions with all their crucial conditions.

To assess if a given sentence is a correct definition the students were expected to use critical thinking, i.e. analyse the sentence and compare it with a known concept definition. In the case of a conflict between the concept

image and its definition they should assess what is not correct and what is missing in order for the definition to be acceptable. But we have found that most students were based on their concept images rather than on their definitions: if the given sentence consisted of facts related to the concept (e.g. the theorem about the inscribed and central angles) then they were willing to accept it as a correct definition.

Our research has also revealed various misconceptions related to the particular geometrical concepts; what is striking is that some of them (e.g. point reflection as axial symmetry) remained unchanged after the course. It was only at Phase 3 when the students managed to overcome their misconceptions after thorough discussions on the various concept definitions. These discussions have led the students to acknowledge the importance of proper and precise mathematical language, which should give the definition a commonly accepted character. Thus, putting emphasis on language may result in changes of the concept image, either in broadening or in concretizing. In that aspect, the students agreed that the concept definition should contain the crucial features of the concept but not its properties.

Concluding, our study has shown that students' concept images are strongly established and their critical thinking is not adequate to assist them in 'transforming' them into proper definitions. However, by engaging students in focused discussions, many features of concept definition may come to the fore, together with various misconceptions.

#### REFERENCES

- Heymann, H. W. (2003). *Why Teach Mathematics? A Focus on General Education*. The Netherlands: Kluwer Academic Publishers, p. 264.
- Klakla, M. (2003). Dyscyplina i krytycyzm myślenia jako specyficzny rodzaj aktywności matematycznej. *Studia Matematyczne Akademii Świętokrzyskiej*, 10, 89-106.
- Tall, D., & Vinner, S. (1981). Concept images and concept definitions in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169.
- Vinner, S. (1991). The role of definitions in the teaching and learning of Mathematics. In D. Tall (Ed.). *Advanced Mathematical Thinking* (pp.65-81). The Netherlands: Kluwer Academic Publishers.

## How to question in an online forum to promote a democratic mathematical knowledge construction?

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**Abstract:** In this paper we analyse the democratic access to powerful mathematical ideas from a logical and psychological point of view in the arena of the classroom, when participating in an online-forum to construct geometrical knowledge with a cultural and sociologic meaning. From a logical and psychological point of view, conceptual, procedural questions were proposed to analyse the different patterns of participation of 31 students in grade 10 involved in the experience. We conclude the existence of differences of participation in function of the logic of the questions proposed that reinforce a more authoritative or democratic co-construction of geometrical knowledge.

### INTRODUCTION

A forum is a virtual blackboard where the participants can interact without the need to be connected simultaneously. Forums have the cognitive and attitudinal challenge of giving opportunities to the students to create descriptions, about their own experiences with a high level of conversation structure. From a theoretical point of view the analysis of the participation in these forums is immersed in the diversity of meanings that emerge from the social interactions in a conversation that allows creating mathematical knowledge (Skovmose & Penteadó, 2009). The participation in online discussions can improve analytical thinking and problem-solving skills, promote development of deeper understandings of course content, although there is a broad spectrum of effectiveness and there isn't a clear model for meeting specific mathematics course goals (Miller, 2012).

### PARTICIPATION IN A MATHEMATICAL ON LINE DISCUSSION FORUM

The context, arena in words of Skovmose and Valero (2009), is the online discussion forum, where from a sociocultural perspective mathematics

knowledge construction takes place. The construction of knowledge is product, but also process, of the conversation that takes place in the discussion forum. Gordon Pask's Theory of Conversations gives us information about the importance and relevance of the kind of questions needed to open a discussion to construct conceptual and procedural knowledge (Pask, 1975). The kind of questions introduced in the forum is going to be crucial to determine the logical and psychological powerful of the mathematical ideas discussed in the forum.

The analysis of the quality of the learning is understood as a representation of the knowledge acquired directly or indirectly from the forum participation. The forum participation is analysed in terms of the density of clue words, the length of the messages and the quantity of them, the average of the ratio of publications (total of post/total of students), the average of discussion lines length (total of posts/total of lines of discussion) or the topic of every line of discussion. In the case of mathematics, we extend these parameters to analyse the density of mathematical concepts with a strict use of mathematical language or its quotidian use and the density of graphs and images.

Hammond (1999) uses these parameters measuring participation to conclude the existence of three different models of participation: *non-participant*, where subjects keep completely out of the forum for a large period of time; *quite-learner*, where the subjects read the posts but usually they do not send messages; *communicative participation*, where subjects that participate in the forum articulate interesting aspects and answer other posts. "*Communicative learners not only had a commitment to 'learning by doing' they had a strong sense of responsibility for their groups which obliged them to take part*" (Hammond, 1999, pp. 361).

A communicative participation in a Analytical Geometry online forum gives the opportunity to the students for a democratic access at least to sociological and psicological powerful ideas. The topics more o less powerful culturally speaking, provide opportunities to envision a desirable range of future possibilities, and the logic of the ideas discussed consequence of the kind of questions proposed. According to Li et al. (2009), we consider four levels of questions that present an increase in the logically speaking democratic access to mathematical powerful ideas. *Concept level questions*, which mainly concerns the explanation or definition of a concept as well as some attributes of a concept. *Axiom level questions*, which is relevant to some common sense and the relationship between concepts. *Rules level questions*, which indicate the logical relationship between axioms. And, *method level questions*, which are related

with how-term procedural construction of knowledge.

### METHODOLOGY

We designed two different case studies (online forums) for grade 10 in mathematics' subject. In total we had 31 students (Group A, 14; group B, 17). Both subgroups had been attending to the same face-to-face group, but work separately in the forum. When participating in the forum, each student of both subgroups was only able to read, publish, actualize information of his/her own subgroup, but they hadn't information of the other group.

Students were asked to participate in a forum titled: "*Analytic geometry and every day life*". The forum had a length of three moths. During the first two moths, students learned simultaneously traditional Analytic Geometry concepts and process in a face-to-face context. In the third month, using exclusively the online forum of a Moodle platform in the Web 2.0, students were asked to analyse the applications that the Analytic Geometry have in the everyday life. They were involved in four lines of discussion with four questions to answer (table 1).

Table. 1. Discussion forum

<i>Line</i>	<i>Group A</i>	<i>Group B</i>
<i>F21</i>	Which are the applications of the Analytic Geometry? You should describe why is useful the analytic geometry looking for examples and describing them. You should be careful that one of your classmates hasn't previously argued like you.	Which are the applications of the Analytic Geometry? Put examples. Accompany the examples with graphics and images and explanations about why you chose this example.
<i>F22</i>	Why wheels are cylindrical? Reason why the wheels are cylindrical.	Should be the wheels cylindrical? The aim of this forum is that you reason, using Analytic Geometry, if the wheels should be cylindrical.
<i>F23</i>	What applications have the conics when a satellite is launched to the space? Reason with the Internet information and what you have learned previously at school.	
<i>F24</i>	What are the loci of points? Previously we have defined straight line, circumference, ellipse, hyperbola and parabola as locus of points. Reason: what are and what they are useful for the loci of points.	

Quantitative and qualitative content analysis of every line was applied to count the number of readings, posts and actualizations done by the students, and the number of words, graphs and images published. The data analysed was classified for every line of discussion and subgroup to interpret the pattern of participation.



### COMMUNICATIVE-PARTICIPATION IN THE “ANALYTIC GEOMETRY AND EVERYDAY LIFE” ONLINE FORUM

The participation in the forum has been mainly communicative because 30 of the 31 students have been reading, publishing and actualizing publications with writings, graphics and images. For example, students of group A where asked in forum F22 to answer: “Why wheels are cylindrical?” They argue presenting their opinion about their scientific and quotidian knowledge:

*Ignacio: “I think that the wheels are cylindrical because, in first place, to reduce friction, and the vehicle could drive quickly and more lightly if it where cubes without the need of applying so much force, and to waste less energy. And I think that the cylindrical will be better that the spherical ones...”*

Ignacio explanation is the most deliberative and reasoned, so the other classmates give their opinion around these ideas.

*Lourdes: “I think that the wheels are cylindrical because, they give more stability that with another shape, and in the bends it is easier to turn and overturn in a roundabout, and the force of friction is minor. The conclusion of Ignacio, I think is very good.”*

Students of group B where asked to answer the question: “Should be the wheels cylindrical?” Firstly, students argue affirmatively using reasons similar to those introduced by group A. Then they use the geometrical knowledge to give reasons:

*Manuel: “The wheel should be a cylinder, in order that the module of the segment perpendicular to the plane in which it is the cylinder should be always the same. Because in other geometrical figures, the module should vary, and its segment not always should be tangent to the plane”.*

The question gives the chance of answering negatively and presenting other situations, as squared or elliptic wheels. The differences in the answers give us information about how the same concept asked in a different manner provides an increase level of access to the powerful mathematical ideas. The analysis of those questions provides different characteristics of the communication synthetized in the table 2.

Table 2. Characteristics of the forum

Cod.	Group.	Question	Aim	Level question	Readings	Actions	Words	Graph	
F21	A	Why?	Descriptions Examples	Axiom	7	New lines	177	10	
	B	Why?	Examples Graphics	Axiom	2	Publish Update			
F22	A	Why?	Reason why	Method	4	Answer others	96	8	
	B	Open	Reason if it is necessary	Rules		Publish			
F23		What?	Reason Relation	Rules			Publish Update	96	4
F24		What?	Description	Concept				192	12

The forum F21, subgroup A and B, asked the students the description and publish examples about which are the applications of the Analytic Geometry. In this forum, 10 students publish graphic examples of the applications. These students were of both subgroups, although only students of subgroup A were explicitly asked to introduce graphic examples. We also have found differences in the number of readings done by the students of both subgroups. For every two readings of subgroup B, subgroup A should read seven times. The hadn't take in consideration the indication, "be careful that another student didn't argue previously like you want to reason). Students have opened new lines of discussion instead of updating others students proposals. We think that this performance hadn't promote a socially speaking democratic access to mathematical ideas proposed by others, because there was no interaction between them for a co-construction of knowledge.

We also find this situation in forum F22 in which students' action is only publishing their information in the same line of discussion, but without connexion with the others information. They have been reading the previous information, with the aim of not doubling it, but there is no real interaction between them. In the forum F24, students publish and up date others publications. In this updating they add new information to previous one introduced by another member of the subgroup. Although there is some level of participation, this is no a discussion of ideas. The differences are found in the F22A forum, where students answer others ideas, in a large interaction and discussion of ideas related to a method level question.

If instead of interpreting the participation related to the actions that students do in the forum, we analyse the numbers of words per average in every line of discussion, we find important differences. Concept and axiom level questions (F21, F24), asking for a description, use in average two times more of words than the rules and method level questions (F22, F23). The density of mathematical terms is bigger in the concept and axiom level questions, because most of the students have copy and paste the information directly from Internet, using pages with an accurately use of language. But, the answers for the rules and method questions cannot be found directly from Internet, decreasing in this case the accuracy of the terminology and the syntax used.

### **CONCLUSION**

In conclusion, the students' participation in the online forum "Analytic Geometry and Everyday life" gives them the possibility of accessing powerful mathematical ideas. The potential power can be explained, in words of

Skovsmose and Valero (2002), culturally speaking because it gives the students the change to establish relationships between the traditional mathematical knowledge and the everyday mathematical use. Psychologically speaking, the method level questions promote the metacognition of mathematical concept and procedures, but also their developing of the communicative competence. However, socially speaking the democratic access to powerful mathematical ideas is limited to the kind of level questions proposed. The concept and axiom level question promotes the publication, and update of posts, but it doesn't promote a real interaction between students. Only the method level questions proposed promote a discussion between students and the co-construction of the geometrical knowledge involved.

#### REFERENCES

- Hammond, M. (1999). Issues associated with participation in on line forums- the case of the communicative learner. *Education and Information Technologies* , 4 (4), 353-367.
- Li, Y., Dong, M., & Huang, R. (2009). Toward a Semantic Forum for Active Collaboration Learning. *Educational Technology & Society* , 12 (4), 71-86.
- Miller, T. K. (2012). A Model for Asynchronous Discussions in a Mathematics Content Course. In A. Juan, M. Huertas, S. Trenholm, & C. Steegman, *Teaching Mathematics Online: Emergent Technologies and Methodologies* (pp. 1-21). doi: 10.4018/978-1-60960-875-0.ch001.
- Pask, G. (1975). *Conversation, cognition, and learning*. New York: Elsevier.
- Skovsmose, O., & Penteado, M. G. (2009). How to drag with a worn-out mouse? Searching for social justice through collaboration. *Journal of Mathematics Teacher Education* , 12 (3), 217-230.
- Skovsmose, O., & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English, *International Research in Mathematics Education* (pp. 383-408). LEA.

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## **Assessing pre-service teachers' works in realistic Mathematics**

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### **Abstract**

In this paper we study a part of a teaching experiment aimed to improve pre-service teachers' ability in creating realistic tasks and open-ended questions. The results show that although the students were able to utilise familiar contexts in order to produce mathematical questions, they usually did not manage to go beyond questions that required mere calculations.

### **INTRODUCTION AND THEORETICAL CONSIDERATIONS**

The aims and the content of Mathematics education have undergone a considerable change in the last decades, mostly due to the technological advances. NCTM (2000) claims that "We owe our children no less than a high degree of quantitative literacy and mathematical knowledge that prepares them for citizenship, work, and further study" (p. 289). Quantitative literacy includes "confidence with mathematics; a cultural appreciation of mathematics; the ability to interpret data, to think logically, to make decisions thoughtfully, to make use of mathematics in context; and more" (Schoenfeld, 2001, p. 51).

The usefulness of Mathematics is clearly manifested in the Realistic Mathematics Education (RME) approach (Freudenthal, 1973), according to which, Mathematics is a human activity and the basic processes involved in it are *horizontal mathematization*, in which mathematical tools are used to organize and solve an initial problem situated in daily life and *vertical mathematization* in which the students are re-organizing and operating

within the mathematical system itself (Treffers, 1987).

The present study is based on a teacher preparation course, whose basic aim was to inform the students about RME and enhance their mathematical competences. We will focus on the final part of the course, which consisted of tasks created by the students. Particularly, we aim to analyse students' abilities in:

- a) linking mathematics to their everyday lives;
- b) utilizing these links in ways that can be useful for teaching;
- c) creating tasks that include particular mathematical activities related to RME.

### **CONTEXT OF THE STUDY AND METHODOLOGY**

The teacher preparation course entitled "Didactics of Mathematics" was obligatory for students in the third (in a total of four) year of their studies in the Department of Primary Education of the University of Ioannina in Greece. Our aim was to engage students in activities consistent with the RME approach and, at the same time, to assess them and modify our teaching experiment (Confrey & Lachance, 2000). The course included an initial diagnostic test, a series of lectures accompanied by problem solving and an assignment in which the students were asked to create 10-12 mathematical questions on any topic with their answers. The questions should be closed (one answer, but not necessarily one solution strategy), inquiry (one answer, but further investigation and seeking for information is needed) and open (many possible answers which usually require further investigation and seeking for information). Our data consists of 51 works, produced by 129 students working in groups of two to four. The analysis was done according to our aims; de Lange's (1999) assessment framework was our starting point. According to it, four categories are suggested for context: "zero-order": fake context, not considered for the solution; "first-order": relevant context, needed for the solution; "second-order": relevant context, mathematization needed for the solution; "third-order": the context serves for constructing or reinventing mathematical concepts. Concerning competence, three levels are suggested: (1) reproduction, definitions, computations, (2) connections and integration for problem solving and (3) mathematization, mathematical thinking, generalization, and insight. We initially used de Lange's context categories to "filter" our data: we excluded works which did not contain any first, second or third-order questions. For the remaining 22 works we decided to modify de Lange's categorization, since we noticed that most closed questions were referring to the context,

but, at the same time, context was not necessary for the solution or its validation. These could not belong to either zero- or first-order categories. For the remaining part of the paper, they will be referred to as zero-order. The other two categories (first and second-order context) were implemented in de Lange's sense. We did not find any third-order questions, so we skipped that category. An example follows of two questions together with their associated categories:

Context: Sports (basketball) (data: top view of a basketball stadium; information about its dimensions)

Q5: What is the area of the basketball court? (closed, zero-order context, Level 1: computations)

Q8: Each basketball team can have 24 seconds to complete their attack. If there is 1 minute left for the end of the game, how many attacks can be completed by each team? (open, first-order context, Level 2: connections and integration)<sup>1</sup>

Proposed answer: One possibility is that three attacks can take place, two full and one half. Another possibility is that both teams will not need the whole time, so more attacks could take place.

Our initial aims assisted us in formulating more categories and enhancing the existing ones:

- a) Concerning the links between mathematics and everyday life, we examined the contexts chosen and their relation to those presented during the course.
- b) Concerning the utilization of these links in ways useful for teaching we examined the extent to which the students' alleged categories of questions were consistent with the proposed ones (closed, inquiry and open).
- c) Concerning the involvement of mathematical activities related to RME, like modelling, mathematizing, interpreting complex and dynamic situations we located these activities in the questions and the students' proposed solutions.

## RESULTS

Table 1 refers to five works which contained at least one first (or higher)-order question.

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<sup>1</sup> This question could qualify for Level 3 second-order context, assuming that it could be solved by mathematizing, e.g. by rephrasing the question: How many attacks can be completed in a one-minute period if the maximum duration of an attack is 24 seconds and the minimum is 1 second?

N	Question type						Context			Level		
							Order				Type	Familiar
	CC	AC	CI	AI	CO	AO	0	1	2			
2	4	8	4	0	1	1	8	1	0	Transports	N	1
4	3	4	4	3	3	1	3	5	0	Environment	N	2
4	-	7	-	1	-	0	7	1	0	Health	Y	2
4	-	8	-	0	-	2	8	0	2	Transports	N	3
2	6	10	4	1	2	1	10	2	0	Media	N	2

Table 1. Categorisation of students' works

The abbreviations used in Table 1 are: N: number of group members, CC: Claimed closed questions, AC: Actual closed questions, CI: Claimed inquiry questions, AI: Actual inquiry questions, CO: Claimed open questions, AO: Actual open questions. The Context columns show the number of questions belonging to each order, the type of context and whether the context was familiar to the students (Y) or not (N). The Level column refers to the highest level of all questions included in the work. In two shown cases the students did not label their questions; this happened in five cases in total.

The first observation is that there is a gap between the claimed and the actual inquiry (and in some cases also the open) questions. Also, the number of actual open questions was small. In most cases open questions belonged to Level 3, requiring some kind of mathematization (cf. Q8 presented before). The small amount of open questions is in line with de Lange's (1999) representation of the competence levels as the height of his "assessment pyramid": the higher the level, the smaller the amount of respective questions. The other dimensions of the pyramid represent the mathematical domains and the difficulty level. In our case most Level 3 questions were of low difficulty:

Context: Transports (toll company) (data: 2,50 euros for cars, 4,50 euros for trucks)

Q10: If the company expects an annual income of 468000 euros, (a) how much should be the daily income? (closed, zero-order context, Level 1) (b) How many cars and how many trucks should pass from the tolls daily? (open, second-order context, Level 3)

Proposed answers: (a)  $648000:360=1800$  euros (b) If  $a$  stands for cars and  $b$  for trucks then:  $2,5 \times a + 4,5 \times b = 1800$ . In that case we have many integer answers. For example, if there are 360 cars, the trucks should be 200, since  $(360 \times 2,5) + (400 \times 4,5) = 900 + 900 = 1800$ . If there are 504 cars, the trucks should be 120, since  $(504 \times 2,5) + (120 \times 4,5) = 1.260 + 540 = 1800$

There were cases in which the mathematization process itself was

questionable:

Context: Health (population growth) (data: two latest censuses, population of Cyprus 686000 people)

Q11: Estimate the population of Cyprus in the next census. (open, second-order context, Level 1)

Proposed answer: Considering the fact that the population of Cyprus has increased by 71000 people from 1992 to 2001, one could claim that the population of Cyprus in the next census will be 757000. On the other hand, one could say that the population of Cyprus will not overcome 700000.

The contexts were usually not explicitly based on those discussed during the course; they were usually taken from the internet and included situations from transports, environment, health, media, sports, management, technology, cooking, etc.

### Conclusions

Our teaching experiment aimed to improve our students' ability in creating meaningful realistic tasks that may involve a variety of mathematical processes and, eventually a variety of solutions. Our results show a considerable difficulty on the students' part in creating tasks that go beyond mere calculations. However, the tasks were related to the context chosen (which justifies our decision to insert a different category for context than that proposed by de Lange), but in some cases the chosen context was actually necessary for solving or validating the solution. The situations that constituted the contexts of the works were usually novel, in the sense of not having previously been addressed in the course; this shows that the students were able to identify – and partially utilise – relevant contexts.

The assessment of the students' work and our teaching experiment has proved a highly demanding task. Only by looking into each work and each question we were able to perform our analysis. Concerning the implementation of our approach, we agree with de Lange (1999) that:

... when we emphasize mathematics education that will prepare our citizens to be intelligent and informed citizens, we have to deal with all kinds of real contexts. We have to deal with pollution problems, with traffic safety, with population growth. But does this mean that we have to exclude artificial and virtual contexts? The answer is no, but we need to be aware of the differences for students (p. 29).

To the above we would add mathematical contexts, thus we should be aware of the importance of engaging students in processes of abstraction and generalisation. Moreover, we should be careful in not letting the context



‘overcome’ its function; in our study, most of the claimed open questions were rejected as such because their proposed solutions did not involve any mathematical process.

Concerning the generalizability of our assessment scheme, our study is situated: the students knew that they would be assessed, which made them more cautious and finally less creative, not only in their choice of contexts, but, mostly, in asking interesting questions, which are at the heart of mathematics teaching.

### References

- Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231-265). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.
- De Lange, J. (1999). *Framework for classroom assessment in mathematics*. Utrecht, The Netherlands: Freudenthal Institute & University of Madison, WI, National Centre for Improving Student Learning and Achievement in Mathematics and Science.
- Freudenthal, H. (1973). *Mathematics as an educational task*. Utrecht, The Netherlands: Reidel.
- National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Schoenfeld, A. H. (2001). Reflections on an Impoverished Education. In L.A. Steen (Ed.), *Mathematics and Democracy: The Case for Quantitative Literacy* (pp. 49-54). Princeton, NJ: National Council on Education and the Disciplines.
- Treffers, A. (1987). *Three Dimensions. A Model of Goal and Theory Description in Mathematics Instruction – The Wiskobas Project*. Dordrecht, The Netherlands: Reidel Publishing Company.

## **Pre-service mathematics teachers' pedagogical content knowledge in the area of interdimensional geometry**

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### **Abstract**

The paper discusses the extent in which pre-service teachers are able to introduce new topics and teaching aids of which they have no prior experience into their teaching. Using modelling in dynamic geometry software, they were looking for relations between 2D and 3D objects and constructions. First they solved and later also posed problems that should help their pupils tell a 3D solid from the dynamically changing cross-section of the solid with the intersecting plane. The pre-service teachers' ability to find, pose and solve problems training interdimensional relations among geometrical objects, i.e. a brand new curricular topic, were studied. The results clearly show that the problems posed within the topic of which the pre-service teachers have no experience were not adequate to the pupils' abilities.

### **Overview**

The contribution that mathematics makes to development of an individual and to his/her success in democratic society can be perceived from various points of view. Mathematics can develop a specific type of thinking that enables us to perceive and distinguish phenomena of different degree of similitude, to look for relations among them and to classify the differences. In everyday life, this can apply to intercultural or sociological issues but also to the area of cyberspace and social networks.

Mathematics offers to teenagers topics that go far beyond computational and practical skills needed for life; they open their eyes and guide their discovery of the world. The young are attracted to topics they find mystical, for example extra-terrestrial civilizations, infinity, the speed of light or more-

dimensional spaces. Many of these issues are of mathematical nature and it is very beneficial if the teacher is able to make use of them for refinement of his/her pupils' critical thinking and scientific perception of the world.

One of the potentially attractive topics is the topic of space and its dimensions. Many real-life problems have more dimensions and it is practical to be able to discern these dimensions and to look in which of the dimensions there exist some analogical problems and analogical solutions. For example the media often refer to different "dimensions" of the topics discussed. Which difference is so substantial that it must be perceived as a new dimension of the problem? For example if we consider assessment of a student project according to the three C principle (3C's of Assessment: complete, correct, and comprehensive) (Kovalik, 1993), can this criterion-based assessment be summed up or is it a multi-dimensional matter?

It also is very useful to analyse situations whose models are not at hand and which at the same time provide a comprehensive insight into the issue. The pupil tries to understand a problem with more dimensions than can be met in ordinary world. For example Roberti (1988) deals with interdimensional relations among 2D and 3D objects.

For a long time there was no convenient environment for 3D construction geometry and it was predominantly taught through projection of the spatial situation to one or more planes. The situation has changed and the existence of such environments, e.g. Cabri 3D (Mackrell, 2008), enables creation of mathematical design and enquiry into e.g. analogies of relations between 2D and 3D objects by observing these objects in interactive figures. When moving from one dimension to another, it is possible to use analogies between properties of the objects, relations between these objects, construction steps and finally to get at least rough idea of 4D space.

### **Research objectives**

The question is to what extent teachers of mathematics are able to pose problems with interdimensional relations. Are teachers able to make this generalization? Are they able to pose suitable problems in the given educational environment, training the pupils in perception of analogies between dimensions?

It is new for Czech teachers that they are expected to plan their own curricula and also include in it new technological and pedagogical features. What is teachers' pedagogical content knowledge according to Shulman (1986) in the areas which involve new topics, new aids, new approaches and that make it impossible to use traditional approaches handed over from one

---

generation to another? Are teachers able to incorporate new topics into their curricula as is demanded in the official state educational documents?

### **Methodology and findings**

The research was carried out with 20 pre-service secondary school teachers of mathematics, who were in the final year. The respondents' work was analysed qualitatively, the teaching experiment was accompanied by involved observation and discussion in the work group.

A curriculum, promising a glimpse into the 4<sup>th</sup> dimension on the basis of analogy between 2D and 3D objects and operations was developed. In the first part the students worked with 3D dynamic geometrical software, solved and later posed problems in 3D analogical to problems in plane. Not only the assignments but also the constructions were meant to be analogic. In the second part of the research, the pre-service teachers were preparing materials that would train pupils' ability to tell a 3D solid from its 2D cross-sections.

Examples of such analogical assignments can be found in (Oldknow, 2004). An example of this is for instance the construction of a circumscribed sphere of a tetrahedron as analogy to the construction of a circumscribed circle of a triangle. The task was to look for such assignments for which there would be an analogical construction problem in 4D or problems with analogical construction procedures for transition 2D  $\rightarrow$  3D. We often came across the respondents' wrong use of analogy, for example in case of the problem Construct the orthocentre of a triangle, for which the students erroneously posed as analogical the problem Construct the orthocentre of a tetrahedron or when looking for analogical construction to the construction of the inscribed circle of a triangle in 3D (what does the in-centre correspond to?). This part of research is described in detail in (Vaniček, 2011).

Analogy is very methodically used in a fascinating 9-sequel French video Dimensions (Dimensions, 2008). The 2<sup>nd</sup> sequel of the 1<sup>st</sup> set "La dimension trois" presents an experiment of a solid passing through a "lizard plane" inspired by E.A. Abbott's book "Flatland" (Abbott, 1884). This video recording first presents passing of a solid through a plane and the creation of a dynamic section (Fig. 1 on the left), consequently the watcher's task is to tell the original solid passing through the plane from the dynamically changing section (Fig. 1 on the right).

The point of this video is to make the watcher realize how difficult it is for two-dimensional beings to perceive 3D objects (and to prepare the watcher for the situation in which they will have to reconstruct 4D bodies from their 3D sections in the following sequel).

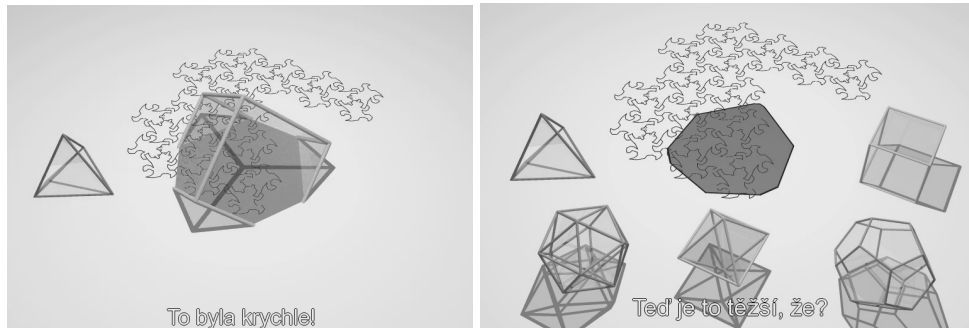


Fig. 1 - snapshot from the video *Dimensions*, telling the solid passing through the plane from its section. Adapted from (*Dimensions*, 2008)

The members of the experimental group first watched the video, then an interactive model created in Cabri 3D. In this model the plane passes through the solid and it is possible to watch the dynamically changing shape of the cross-section (Fig. 2 on the left). It is also possible to look at the situation from the side and see the position of the section in the body (Fig. 2 on the right). The pre-service teachers' task was to tell the solid from the picture on the left.

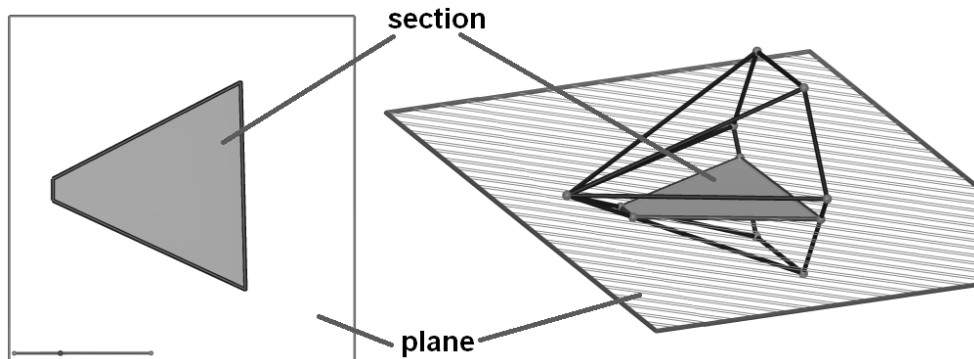
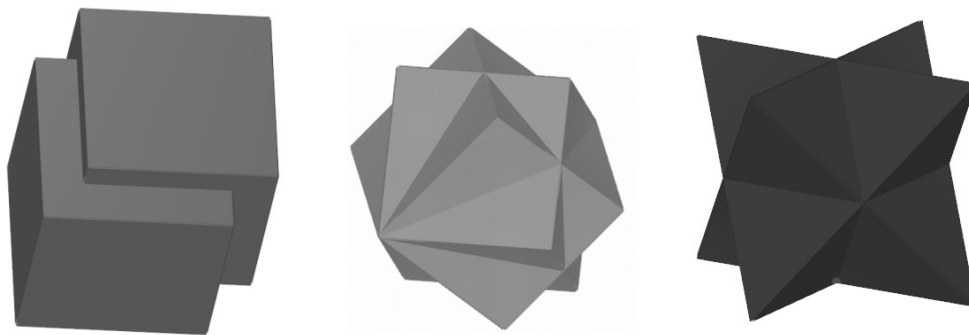


Fig. 2 – Interactive figure made in Cabri 3D for the activities of telling the solid passing through a plane from its dynamic section. The picture on the left shows the same figure from above (without showing the outlines of the solid).

Their reactions suggest that they were surprised to see how difficult it was to tell the solid unless it was regular. The worst results were achieved in a set of problems in which it was a regular hexagonal pyramid in different rotations that was passing through the plane.

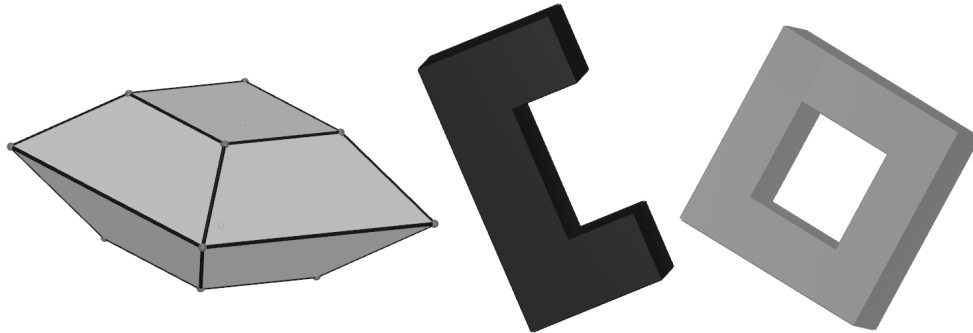
The pre-service teachers were then asked to create such models for their pupils that would develop the pupils' ability to tell a solid from a set of its sections. It turned out that most of the students constructed too complicated problems, i.e. with solids that no other member of the experimental group was able to tell. They did not hesitate to use compound solids which were moreover at different angles with respect to the plane of the section. They created e.g. a cuboctahedron or a star tetrahedron, solids that are difficult to describe even if recognized.

It seems that the respondents lost touch with pedagogical reality. Either they found their problem easy having constructed it themselves, or they intentionally tried to create a very complex problem. It can be assumed that as long as the environment offered construction of more complex solids, they did not hesitate to use them, even though the way to understanding with the help of the used method was more than knotty.



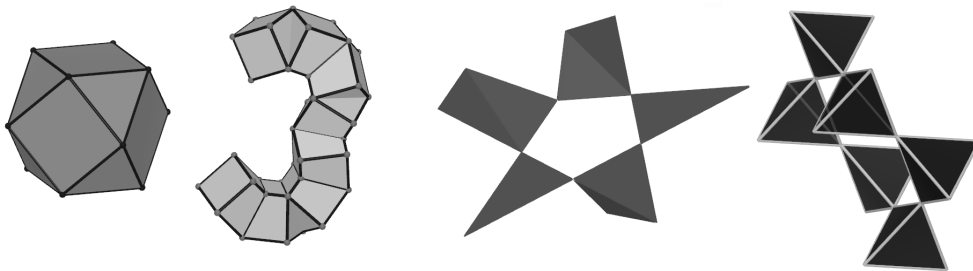
*Fig. 3 – solids made by pre-service teachers, constructed by union of two solids offered by the environment of the computer application (cubes or tetrahedrons).*

Another finding is that the respondents clearly prefer more complex solids offered by the environment of the computer application to simpler solids that they would have to construct on their own. They often constructed solids that were unions of two solids, a solid and its image in translation (see Fig. 3, 4). Out of these the simplest (and also the most suitable for the purpose of this activity) were solids assembled from more cubes.



*Fig. 4 – the most easily recognizable constructed solids – double frustum or solids constructed from cubes*

The pre-service teachers' work also included figures that cannot be included in teaching because of their undue complexity (Fig. 5 on the left) or because they do not satisfy the definition of a solid (Fig. 5 on the right).



*Fig. 5 – absolutely unsuitable work, complex solids (on the left) or figures that are not solids (on the right).*

### **Conclusions**

Pre-service teachers are able to:

- perceive analogy between planar and spatial construction
- pose problems in 3D analogical to problems in plane
- in a limited extent find analogical solutions to 2D constructions in 3D

When constructing models for telling solids from their dynamic sections

- they are technically able to construct such model in Cabri 3D
- they are creative and there are no mistakes in mathematics in their work

- they choose complicated assignments without being successful in telling the solids themselves
- solids that can be recognized on the basis of some analogy to 2D objects are constructed very rarely

Computer tools may model situations supporting development of making generalizations on objects with different dimensions. They provide the space needed to address issues included in the school curricula and thus give the teacher the chance to perceive didactical problems without any bias due to stereotypical perception.

Posing of too difficult instead of methodologically suitable problems by teachers signals their insufficient pedagogical content knowledge in the topics that have been introduced to school practice only recently and of which teachers have no experience from their own school years. One must therefore ask whether in these cases curricular changes may be left to the teacher. The research was supported by the grant GAJU 089/2010/S.

### References

- ABBOTT, E. A. *Flatland, a romance of many dimensions*. [online]. Sealy a Co., 1884. [cit. 2010-06-22] Retrieved from: <<http://www.ibiblio.org/eldritch/eaa/FL.HTM>>
- KOVALIK, S., OLSEN, K. *ITI: The Model*. Kovalik, Susan & Associates, 1993. ISBN 978-1878631183
- LEYS, J., GHYS, É., ALVAREZ, A. *Dimensions*. Educational video [online]. 2008. Retrieved from: <<http://www.dimensions-math.org>>
- MACKRELL, K. Cabri 3D : An Environment for Creative Mathematical Design. In Liljedahl, P. (Ed). *Proceedings 2007 Annual Meeting*. Frederickton: University of Frederickton, 2008.
- OLDKNOW, A. Solid geometry and Cabri 3D [online]. In: *CY maths ICT*. Chartwell-Yorke Ltd., 2004. [cit. 2010-06-22] Retrieved from: <<http://www.chartwellyorke.com/cabri3d/oldknow/solidgeometry.doc>>
- ROBERTI, J.V. Interdimensional Relationships. *Mathematics Teacher*, Vol. 81, No. 2 p. 96 – 100, 1988. ISSN 0025-5769.
- SHULMAN, L. S. Those who understand: Knowledge growth in teaching. *Educational Researcher*. 1986, vol. 15, no. 2, p. 4-14.
- VANIČEK, J. Using analogy in generalization and conceptual learning in computer assisted learning in geometry. *The Electronic Journal of Mathematics and Technology*. Vol. 5, No. 2, 2011. ISSN 1933-2823



**WORKING GROUP 4:**  
**Democracy in research on mathematics education**  
**GROUP DE TRAVAIL 4 : Démocratie en recherche**  
**en éducation mathématique**

Hellenic Mathematical Society  
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**The use of Chat as a pedagogical space to interact  
and learn mathematics democratically**

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**Abstract**

The working with geometry in school should not be restricted to memorizing names, definitions and application of formulas. The study on the conceptual understanding and development must take a central role, even when using new technologies. In this workshop we will discuss the concept (of quadrilaterals and polyhedron) by interactions of future mathematics teachers in two virtual environment of learning: the Chat of Gepeticem (UFRRJ, Brazil) and the VMT-chat (Drexel University, USA). It also promotes a discussion about the nature of tasks and interactions in each scenario visited.

Keywords: polyhedron, quadrilaterals, conceptualization, virtual environment, chat interaction.

**Theoretical Perspective**

In mathematics education has increased the research interested on the use of virtual environments as a setting for study and learn mathematics for both students and teachers. This workshop will address two areas of research in mathematics education: virtual environment of learning and geometry.

The virtual environments of learning are mediated by different

technologies, including informatics (Bairral, 2007). Learning is understood as immersive forms of interaction and participation within environment established (Bairral, 2011). In this constitution there is a transformation in the discourse (Sfard, 2008) and the different ways of appropriating mathematical concepts are shared in the community constituted.

This workshop is part of a longitudinal study<sup>1</sup> where some previous results have been discussed in CIEAEM61. In this workshop we will present results on the nature of discourse in each virtual scenario. These results address reflections on Democracy in mathematics curriculum, particularly, how can the new technologies (for instance, virtual environments) help future mathematics teacher for the design of a Democratic mathematics curriculum and how does ICT contribute to critical thinking and decision-making in the society?

In virtual environments, democratically, individuals can exchange ideas and develop their mathematics concepts, without hierarchy, and domination from one participant to another. In this process is important to pay attention to the nature of each task so that it can promote the democratic construction of mathematical knowledge.

### **Main goals of the Workshop**

- Illustrates the use of two virtual environments as a way to learn mathematics.
- Carry out two activities: one discussing the concept of polyhedron and one of the quadrilaterals.
- Exemplify strategies for analysis of interactions observed in each chat with a focus on the concept studied.
- Reflect on the discursive nature of each scenario communication and its importance in the development of geometric thinking.

### **Environments that will be used in workshop**

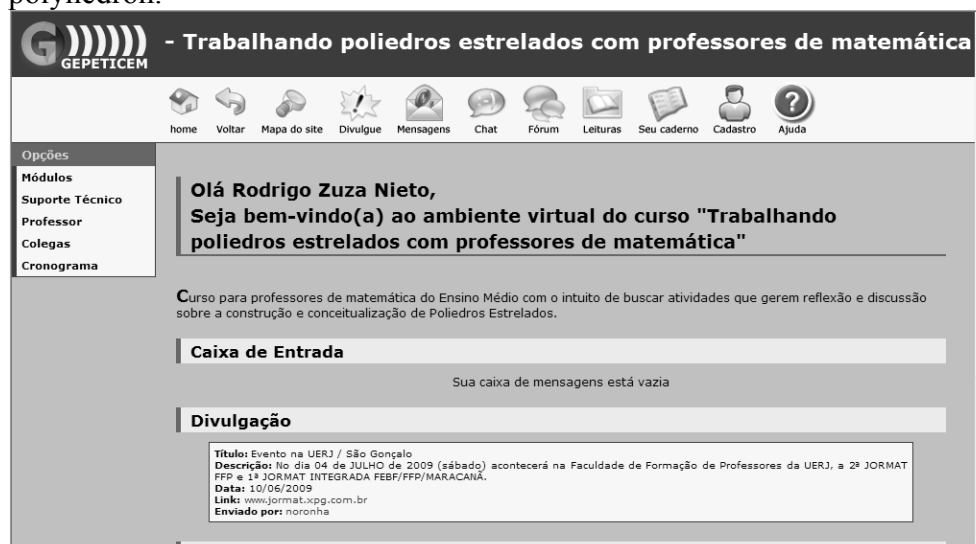
#### **The *Gepeticem environment***

(<http://www.gepeticem.ufrj.br/cursos.php>) is structured around a vision of work that breaks with the axiomatic approach and the memorization of formulas in geometry classes. Developing school geometry is many more than decorative and practical algorithms. Working with geometry enables the development of skills such as experience, to represent and reason, and stir the imagination and creativity. Below, we illustrate the chat screen

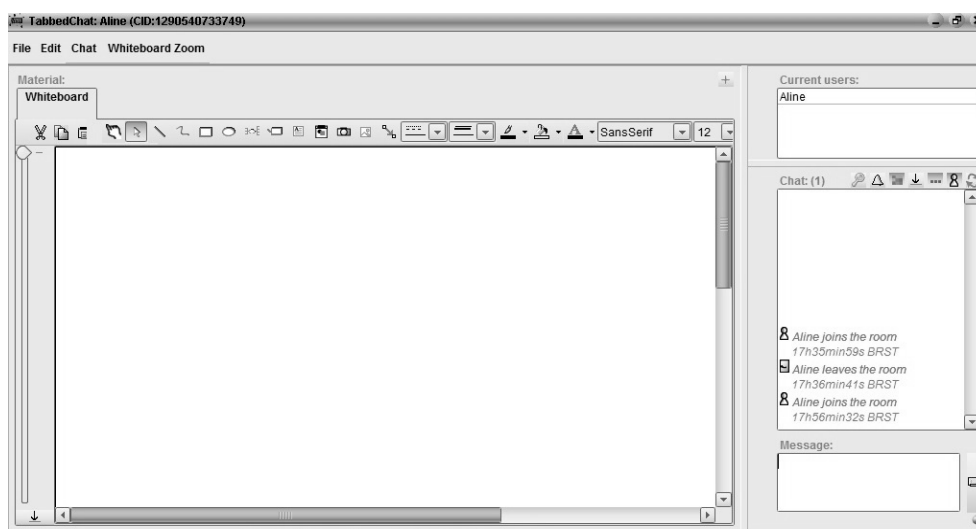
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<sup>1</sup> In Brazil, the research project is supported by CNPq and Faperj.

virtual environment, which has been implemented for the study of polyhedron.



The second virtual environment is the **Virtual Math Teams (VMT-Chat)**. It is composed of three spaces of interaction: the whiteboard, chat (writing) and wiki. In this scenario all inscriptions are recorded. Whiteboard participants can insert pictures and drawings. The environment interface is shown below:



In this environment of chat messages and illustrations on the whiteboard happen concurrently. There is not a predominance or importance of space in relation to each other. For this reason, we believe that the interactive process should be seen as a conjunction. The interrelationship of spaces and the dual nature of the environment are important for the development of mathematical reasoning, as we pointed Cakir (2009). The VMT-Chat is an environment developed by Drexel University (Philadelphia) and it available on <http://vmt.mathforum.org/VMTLobby/>.

The participants are accessing the environment and work in the following learning situations.

### **Exemplifying one Task**

Each virtual learning environment has its specificities and constrains. These singularities imply, for example, in planning the nature of activities to be proposed. Below we illustrate one activity that will be used in one virtual scenario.

#### ***Task in the Gepeticem Chat***

#### **Task 1** – Conceptualization of regular polyhedron

See below how four future teachers characterize regular polyhedron.

Student	Regular polyhedron
1	Regular polyhedron is formed by regular polygons
2	Regular is when it has equals sides and angles
3	Convex regular polyhedron are those in which it faces are regular polygons
4	Is regular when it faces are regular polygons and has the same angle

Analyze and discuss with your colleagues:

- a) The definition of regular polyhedron expressed by each one.
- b) From the responses given develop (in a group) a definition for regular polyhedron.

The other task will be about the concept of quadrilaterals and will be proposed in the VMT-Chat environment. After the realization of its previous activities in virtual environments we will promote a discussion on the proposed type of task in each scenario and the analysis that can be done in

each chat. In the next session we show briefly as we have developed this type of analysis.

### **Analysis from the Chats**

Interactions within Gepeticem chat are writing instant messaging. In the VMT chat users can make different inscriptions (written messages, draw pictures, copy and paste and insert pictures, make references etc.). In both cases the analysis of chat is done with the help of the transcript of the messages and records. Each environment provides a kind of editing and transcription of the messages. In our investigations we are identifying and analyzing the types of interaction, different forms of participation and collaboration, and the elements that illustrate the development of mathematical reasoning emerging in this process.

Let's see how we develop the analysis of interactions in chat Gepeticem (Bairral & Nieto, submitted). We used the following procedures for data reduction: chat transcript (file provided by the platform itself), numbering (in lines) interactions, removal of lines with no idea focused on the analysis (concepts), re-reading interactions and organizations in turns. These moments of revisiting the interactions were four, namely: (1) the collective interactions, (2) interactions of a student and (3) graduating in this collective, and (4) consideration (in the chat of the moment 3) just words or sentences mathematically related to some definition of polyhedron and contrast with the proposal of the chat as to elucidate the process of reflecting on the definition. This analysis allowed us to focus on the process of reflecting on aspects of the definition. Therefore, the fragment that is can be illustrated as follows (moment 4).

mary (21/05/2010 - 10:18:59): in view of a first contact with the **concept of polyhedron** I found the ideas 3 and 4 simpler to understand  
 rodrigozuza (21/05/2010 - 10:31:05): So, how would this new definition using **geometric solids**  
 erj2010 (21/05/2010 - 10:36:06): Another thing .. I think we should define polyhedron without citing previously its elements (**faces, vertices and edges**) and then identify them.  
 alinets (21/05/2010 - 10:36:41): pow people, we live in **3D** why you guys think that they will not be able to understand the **three dimensions** ... I think it's easier then **flat!** \* kkk

Analyzing the fragment before we can identify the group reflecting (according task 1 pages 4-5) and trying to define polyhedron in three domains: definition associated with geometric solids; definition focused on

the elements (faces, vertices and edges), and definition focused on the number of dimensions. Highlighting the words in the definitions contained in the proposal we observed: solid three-dimensional geometric figure of three dimensions, polygons, polygonal regions, and the space limited by them, solid surface, finite number of faces (polygons).

### **Final Remarks**

The Chat can be another kind of scenario to democratize the processes of teaching and learning mathematics. Even though the teacher has his/her role, implementing chat where users interact as equals. Everyone can express and publicize his/her idea for the collective. With access and work in both environments in the course participants intend to develop the motivation to use Chat as a new space of interaction. In addition to meeting and working in two virtual environments for learning mathematics, we expect participants to develop skills for analyzing interactions in this type of scenario. The discussion will also problematize the type of task that should be used in each environment and, consequently, the interactions established.

### **References**

- Bairral, M. A. (2011). Interagindo, ouvindo o silêncio e refletindo sobre o papel do formador em chat com professores de matemática (Interacting, listening the silence and reflecting about the role of trainer in chat with mathematics teachers). *Educar em Revista*, n. 1, 173-190.
- Bairral, M. A. (2007). *Discurso, interação e aprendizagem matemática em ambientes virtuais a distância (Discourse, interaction and learning in virtual environment at distance)*. Rio de Janeiro: Edur.
- Çakir, M. P., Zemel, A.M & Stahl, G. (2009) The joint organization of interaction within a multimodal CSCL medium. *International Journal of Computer-Supported Collaborative Learning*, 4(2), 115-149.
- Nieto, R. Z., & Bairral, M. A. (submitted). “Poliedro é um sólido, correto?” Um estudo com graduandos interagindo em um chat sobre a definição de poliedro (“Polyhedron is a solid, right?” A study with preservice teachers interacting in a chat about the definition of polyhedron).
- Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge: Cambridge University Press.

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## **Playing Robots: Doing Mathematics and Doing Gender**

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### **Abstract**

*The learning of mathematics through contextual themes of a playful character seems to be nowadays the vogue in educational practices and curricula reformations. The present paper explores how learning and subjectivity (and gender subjectivity in particular) evolve together in complex ways when children play together in small groups. Focusing on the case of 'Playing Robots' in a small playtime group we present and discuss data concerning female positioning at a phase in the activity work where the doing of mathematics seems to necessitate the doing of gender from both boys and girls. At this phase, we have observed that whilst male dominance may become the crescendo of expertise, female withdrawal (as compliance or frustration) is not simply the crescendo of in-expertise, but the resistance to dominance.*

### **Playing Robots: mathematical learning and subjectivity**

Seymour Papert has strongly argued, and articulated as part of the theory of constructionism, how 'new' scientific knowledge becomes constructed and meaningful when children are involved in the construction of objects and problem solving in complex tasks that are of personal interest and importance for them. From the beginnings of his research in the 60's, Papert's orientation was towards the design and educational implementation of innovative tools and environments (such as Logo Language and Logo Turtle) that could be used autonomously by children in the early ages. Even though, critique does exist, Logo-like environments have gradually become established and at the same time have inspired (especially within the research environment of MIT Media Lab) the design of robot-tools, machines or epistemic toys, such as Roamer, BeeBot, Lego WeDo, Lego Mindstorms NXT, Pico Cricket, TORTIS, Algblock and later on, languages such as Scratch, Quetzal, Tern etc.

The opportunities for mathematical learning involved on contexts where such epistemic toys are being employed have been outlined at various studies. Specifically, logico-mathematical thinking and programming as part of control-technology come together with the application of specific mathematical subject matter related to number, space and measuring. Mathematical content and skills seem to be simultaneously present in complex tasks in which children-with-adults are asked to confront and resolve. Such complex tasks often come under the theme of 'playing ro-

bots' in projects largely addressed as educational robotics. Recently, the majority of reformed curricula guidelines tend to incorporate in their standards certain goals for attainment placing a direct emphasis on the exploitation of such epistemic toys or machines and their deliberative association with teaching and learning aims and objectives. However, their most celebrated presence comes in the context of playful activity that exists beyond (or at) the boundaries of formal school settings.

Playing robots, as children's activity, is closely related to children's playing digital games or video games but also with making constructions such as Lego bricks. Children have to make a construction with manipulative brick type material and then they have to find ways to control its behavior. Most literature reviews tend to connect children's engagement with 'playing robots' with aspects of learning and specifically the learning of logico-mathematical skills as the ones outlined above, but also with programming skills, the following of guidelines, the making of imaginative constructions and the analyzing of physical activity (e.g. motion, movement). In addition to this, recent theoretical studies address the need to highlight that, *first*, children as participants in the activity are not alone but act within a relational context where they connect with other children, adults, tools, ideas and discourses, and, *second*, children as they play robots enact and embody subjectivities which are deeply and primarily gendered (Walkerdine, 2007). This is evident in ways they choose to represent and express their ideas about robots, the ways they relate to robots themselves and the ways they enact their subject positions as part of the encountered tasks. As such, playing robots becomes a gendered space where boys and girls do mathematics as they do gender.

#### **Research setting, focus and methodology**

This paper is part of a broader research project with our student teachers (at both graduate and undergraduate levels) in the context of specific courses and research seminars where the purpose is to exploit the learning affordances of epistemic toys such as 'robots' and to research how young children relate to them. The particular 'robots' used over the years have been Roamer, Bee-bot, LegoNXT and related virtual manipulatives such as the ladybug, ladyleaf in the NVLM platform (see Chronaki, under review, for a detailed account on a number of studies). Over the years, a number of teaching experiments have been developed and employed as a means of facilitating the ethnographic study of children's activity. The present paper focuses on a single case study of organizing a playtime activity for a group of five children who are close friends, are in different ages (from 5 to 9 year olds) and have varied abilities in technology use. Three of them are boys and two girls of which Angelos and Spyros (boys around 9 year olds) had previous (after school) experience with Lego Mindstorms, Christos (5 year old) a competent follower of the elder boys, Rafaela (8 year old) and Anna (9 year old) with limited experience in Lego construction kits but good with technology. We met four times (i.e. 10-12 hours duration in total) over the span of two weeks. Within the friendly and relaxed context of playtime we aimed to explore how these five children would share knowledge and interest amongst them about 'playing



robots' and how would they relate amongst themselves and with the robot 'creature'. Taking into consideration that children are in early ages, we focused more on participant observation and recording (by means of tape recording and videotaping) utterances and interactions amongst them and much less on interviews (at last for this case study). The teaching experiment was organized around the scope of exploring the idea of making a robot and exploring how such a 'creature' could be assembled and programmed (by means of a simple movement) based on the Lego Mindstorms v1.1 software that came with Lego NXT Kit.

### Playing Robots: setting the activity space

The playing robots activity was organized along three roughly distinct phases; a) the familiarization phase aiming to engage children with robots (via selected visual material) and with the specific Lego Mindstorms software and constructive material, and b) the construction phase aiming to acquaint children with technical details concerning the construction of a robot. As part of the familiarization phase, an open discussion over visual material (i.e. relative photographs, video projection, websites) was held and a brainstorming session concerning the type of 'creature' robots might be (i.e. what is a robot, where do we find them and what they are used for, which characteristics and features they has, what are their limits and possibilities and how do they function) was moderated. At its closing, children were asked to represent graphically, by drawing a robot, what kind of automated / programmable robot they would like to create if they could and what should it be able to do. Their drawings (see Table 1) reveal their attempts to assign features to robots borrowed from their toys that were gendered specific. For examples the girls choose their robot to be a doll that would bring their clothes from the drawer, whilst the boys draw creatures of a soldier in flames.

The construction phase consisted of two interrelated processes, first the constructional-exploration process where we handed out worksheets and a 'manual of construction and use' of a Lego Kit (enhanced by selected explanatory photos) asking children to construct a robot by following the guidelines to assemble the pieces, and second the programming-creation process where children had to make the robot move, or, in other words, to control its movement by simple programming.

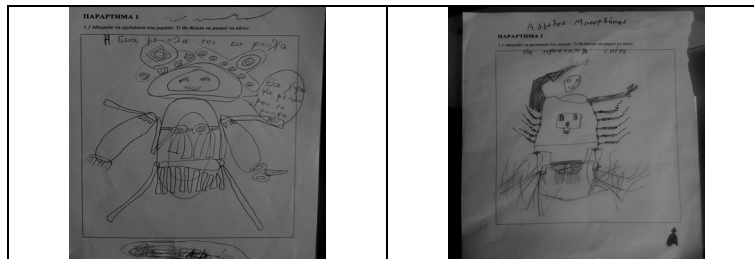


Table 1. Doll in the closet VS Soldier in flames

As can be seen in table 2, children were engaged and motivated to move

through both processes in this phase. However, it was also here that doing gender was evident in a number of cases that will be outlined in the next section.



Table 2. Children work/play in making a robot

### **Doing Gender: dominance vs compliance or/and frustration**

Through our observations, doing gender was a primary resort for children either to confront any difficulties that appear with the tasks in hand (mainly girls' positioning in this group), or to dominate and express enthusiasm and knowledge over specific instrumental stages in the construction phase (mainly boys' positioning in this group). As explained earlier, both Angelos and Spyros, the two experienced boys, tend to dominate the group taking advantage of being eldest and more experienced in Lego constructions and technology. They seemed to get any opportunity for contributing loudly their ideas, leading the activity during the constructional stage, and essentially making vital decisions about roles and responsibilities within the group. The boys dominant subject positioning resulted into girls' subversion either towards compliance (e.g. in Anna's case) or frustration (e.g. in Rafaela's case). But, how this had happened?

Whilst, the two girls, Anna and Rafaela participated actively in the familiarization phase (i.e. expressing interest in playing with robot pieces and designing how their robot would look like in imaginative ways) they fall behind within the constructional phase as compared to the two boys. It would be easy to assume that the technical and instrumental requirements that the actual construction demanded did not allow them to participate actively -although they seemed engaged and motivated. However, by examining closely the dialogues emerging amongst boys and girls we could recount a different story. By the time, during the construction phase, that the two girls realized boys' dominant positioning, they expressed a preference to restrict the collaboration amongst themselves (i.e. *Can we make our own group?*), but when they realized that groups had to be mixed ability, Anna exclaimed: *'Rafaela and I will be secretaries'*. In the meantime, Rafaela is in a group with Spyros and observes him to assemble the pieces. In parallel, she is busy collecting all bits and pieces that are needed for the Lego construction and hand them over to Spyros. At some stage, Rafaela expresses her dissatisfaction:

Rafaela (talking to the researcher): *Sir I'm bored! Only Spyros makes ....*

Researcher: *Yes ... as I can see you only watch. So you don't really help.*

Rafaela: *Yes but Spiros won't let us do anything [...]*

Spyros (talking to Rafaela, asking for a Lego piece): *Give it to me! Let me take a look!*

Rafaela: *Oh come on!!! Why don't you let me try? [talking to Spyros...]..... I did it!!! ... I finished it ... and you said I couldn't do it!*

### **As a way of conclusion**

Taking into account boys' hegemonic behavior in the small group work, the secretary role adopted by Anna was not only familiar to her, but also a safe place to be. As such, Anna's compliance could be seen as a survival (or resistance) strategy and not as withdrawal. At the same time, Rafaela expresses directly her frustration and articulates what exactly makes her feel marginalized within the context of the activity. Both strategies (compliance to the secretary role, expressing frustration) denote subject agency and disclose resistance to a competitive power relations context. The above episodes denote clearly how learning opportunities are closely tied with subject positioning and the performing of gender subjectivity (Butler, 1999).

As said at the beginning, the learning of mathematics through contextual themes of a playful character seems today the most innovative reformation in educational practices. Focusing on Playing Robots' in a small playtime group, we suggest that children's work cannot be taken un-problematically as the 'happy' place for doing mathematics whilst playing with toys. Instead, we emphasize how the doing of mathematics is relationally tied with the doing of subjectivity and gender subjectivity in particular when boys and girls come together around the need to solve a problem or make a complex construction. In these cases, although male dominance may become the crescendo of expertise, female withdrawal (as compliance or frustration) is not simply the crescendo of in-expertise, but the resistance to dominance.

### **References**

- Chronaki, A. under review. Playing Robots: doing mathematics and doing subjectivity in the curriculum of the early ages.
- Walkerdine, V. 2007. Children, Gender, Video Games. New York. Palgrave MacMillan.
- Butler, J. 1999. Gender Trouble: Feminism and the Subversion of Identity. London. Routledge.

## **Following (or not) mathematics related paths as a gendered choice**

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### **Abstract**

*Despite recent efforts for closing the gender gap in the areas of science, mathematics and technology, mathematics related careers are still a marginal choice for many young students. Detailed analysis of men and women accounts on what drives their choices for studying, working and developing a career up growth reveals how discourses concerning 'choice' are highly gendered and co-constructed as subjectivities in the context of their narratives. The present study focuses to discuss this issue by firstly, considering how two female teachers narrate their experiences of school mathematics and secondly, exploring how they opt out not to follow a maths related career path. In doing so, we are dealing with how they these two women appropriate the socially, culturally and historically constructed ideals (and hegemonic discourses) about maths and gender, and, in addition, how it is that these ideals are internalized or embodied, and felt not as external constraints or impositions, but as own affordances or deficiencies.*

### **Choosing school mathematics as an autonomous social agent**

Current research related to choices in studying and working in mathematical related fields (Mendick, 2006, Walshaw, 2005, Murray, 2001) have brought into the fore perspectives that do not locate issues of 'choosing' maths merely with an ideal 'autonomous' individual but, instead, refocus our attention to the social, cultural and political complexities where men and women weave lives along with career paths. Autonomous choice and subject agency have been challenged as core concepts not only towards understanding but also explaining and pursuing our relation to varied layers of a social reality where we live as gendered subjectivities as we strive to become learners and educators. Discourses concerning agency, autonomy and choice, along with rationalism, active participation or collaboration are central to a neoliberal agenda of politics. The publication of the book 'Changing the Subject' in 1984 was amongst the first systematic and coherent attempts towards articulating a critique of the 'autonomous' and self-regulated subject ideal that mainstream psychology discourses were producing and promoting (Henriques et al. 1984). It certainly paved the way for more studies to unravel the multiple relational complexities amongst psychological and sociological analysis and, in fact,

creating the space for psychosocial theoretical studies to advance. However, still the discourse of ‘free choice’ is mobilised as the hegemonic theorisation to capture and explain behaviour, motive and change in certain settings.

From this perspective, the notion of ‘free’ choice to study (or not) mathematics or to develop a mathematics related career is firmly attributed to a personal trait (mainly due to personality, intelligence or brain). Following (or not) a maths related career becomes a matter of having a talent or being gifted and is, often inscribed, as something that only a few people possess (either by virtue or hard training). We believe that any attempt to theorise subjectivity (or subject identity) in relation to mathematics as an esoteric issue, offers little understanding of the complex lived experiences that girls and boys, men and women construct in relation to mathematics education in our contemporary societies. In the context of this paper, we wish to argue for the need to develop an understanding of subjectivity in ways that do not confine individuality in terms of ‘interiority’ metaphors that steam from a neoliberal view of subject agency –where the subject is assumed self-control and self-responsibility (Gill, 2008). Instead of abandoning the social, cultural, political constraints upon subject’s agency, we would need to espouse a theoretical optic that could allow us to see subjectivity in relational terms. Towards this end, we have mobilised theoretical attempts promoted by Foucault (1977/1980, 1984) on how subject positions relate to hegemonic discourses, as well as, by Laclau and Mouffe (1985) on distinguishing and relating amongst subject positions and political subjectivity within the context of discourse(s). This line of thinking proves to be helpful and useful for the type of research (and our research) that strives to recognize human struggle in everyday conditions, and in educational practices, where social action and subject agency is enacted (see also Pechtelidis, 2012).

On these premises, the present study focuses to discuss female choice in following (or not) mathematics related paths. It explores this issue by firstly, considering how two female teachers narrate their experiences of school mathematics and secondly, exploring how they opt out not to follow a maths related career path. In doing so, we are dealing with how they these two women appropriate the socially, culturally and historically constructed ideals (and hegemonic discourses) about maths and gender, and, in addition, how it is that these ideals are internalized or embodied, and felt not as external constraints or impositions, but as own affordances or deficiencies.

#### **Research setting, focus and methodology**

This paper is part of a broader research project concerning the gendered dimensions of mathematics and technology use at the basic levels of the Greek educational system\*. A part of the project was the interviewing of 24 male and female

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\*The research reported here is part of the project ‘Mathematics, Technology, Education and Gender’ (short title) funded by EPEAEK Pythagoras I [Greek Ministry of Education Research Grand for Supporting Research Teams] during the period 2004-2007.

teachers aged between 36 and 47, who attended a biennial academic course aiming to offer in-service training for teachers (Didaskalio) in affiliation to a Greek University. As explained above, our focus was to explore how these teachers negotiate and construct subjectivity through their narratives. In other words, our focus was to unravel narrations for lived embodied experiences about mathematics education and to identify the interrelations between representations about mathematics, education and life. We intended to explore through their narratives the discourses in which these representations were inscribed; the subject positions those discourses made possible; and how these discourses might be related to their subjectivity (Foucault, 1989). Rosalind Gill (2008:433) claims that the focus on subjectivity, although essential, is generally limited in social research: *‘There is very little understanding of how discourses relate to subjectivity, identity or lived embodied experiences of selfhood. We know almost nothing about how the social or cultural “gets inside”, and transforms and re-shapes our relations to ourselves and others’*.

#### **Irene: ‘I had a gift for maths’ and ‘I just wanted to leave home’**

Irene is 42 years old and comes from a rural area in Northern Greece. At school she was very good at maths and, indeed, she expressed passion and ability for top grades. Although she wanted to study architecture (she was very good in geometry) she ended up studying and working as librarian for some years. She, then, studied pedagogy and followed a teaching career and recently completed her dissertation for her master’s degree in Pedagogy. Currently, Irene is satisfied with her academic and professional career, and further aspires to engage in research in the field of special education; possibly at the level of a PhD. She claimed that her choice not to follow a maths related path was, more or less, random. Although, her first choice, as she said, was architecture, mostly because of her aptitude in mathematics as geometry, her drive to leave home was so deep that by the time she had secured an enrolment in librarian studies she could not think of the extra effort to repeat exams.

*Irene: At school I was really good at math. [...] In high-school I had top grades in mathematics and writing. [...] Really good grades! [...] I had a gift for math. My first choice was architecture [...]; I didn’t pass the admission exams... Eh... I studied to be a librarian, which was my 20<sup>th</sup> choice, but I liked it along the way. But my first choice was architecture.*

*Researcher: And why didn’t you insist in order to study architecture or something similar?*

*Irene: At eighteen I just wanted to leave home, yes. I was accepted at university in Athens; I had friends and acquaintances there, so I went and I never had any regrets. I worked as a librarian for eight years and liked it a lot. I liked the structure of this field. It was something completely new to me.*

Reflecting on her own case, Irene generalises on the importance of motivation in subjects such as math - implying that girls need to special encouragement. Irene’s explanation probably draws from popular psychology and pedagogy (Boaler, 1997), where emphasis is placed on ‘woman’ or ‘girl’ as a distinctive and generalised category with certain characteristics and ways of knowing and behaving. This optic tends to account, in consequence, for female deficiency and otherness in certain fields, like math and technology. Instead of focusing on the social,

cultural and institutional practices that tend to prevent or produce inequity, marginalisation and diversity, Irene is mobilised towards escaping a difficult subject position (to be a girl a few decades ago in a very conservative community) and chooses anything that can allow her to escape from an environment that is restrictive and oppressive (mainly in patriarchal terms). As such, the dominant masculine orientated structures in maths related fields become naturalised (i.e. this is the way they are) and remain largely unchallenged.

### **Georgia: ‘I was zealous about math’ and ‘Motherhood affects professional development’**

Georgia is 46 years old, single, and she, also, originates from a rural area of Northern Greece. Initially she studied chemistry in Belgrade, but she had to break off her studies due to financial and family reasons. As a result, she returned to her hometown where she studied pedagogy, worked as a teacher and currently pursues her postgraduate studies. Georgia states that she is happy with her choice to become a teacher because of the professional perspectives it offers. She also expresses her fondness for math and science in general, considering herself as top learner and teacher in this subject.

Researcher: *Were you good at it (implies maths and science)?*

Georgia: *Yes, very good. I had always top grades. But it wasn't just the grades. I was zealous about math.*

Researcher: *Could you expand?*

Georgia: *The more difficult the problems I had to solve, the greater my satisfaction when I succeeded.*

Researcher: *So was it like a game for you?*

Georgia: *Yes. In junior-high during summer holidays I remember myself solving exercises from the math book of the following year every afternoon, when, in fact, I was supposed to be resting.*

Georgia admits that although good at maths she had to return to education as a safer choice and she argues *‘I believe that a woman chooses her studies mainly because of her position in society: being a girl she is encouraged to choose a profession and a position that is less exigent, so she will not tire herself too much; something that is easy. At pedagogical departments we see that the majority of students are girls; maybe it's the nature of the profession that suits them more. [...] Motherhood affects professional development. It rearranges a woman's priorities. Only if she is an extremely organised person can she arrange her time and space in such a manner, so as to be able to manage everything’*.

### **As a way of conclusion**

Georgia and Irene uphold mathematics to be an absolute, objective and abstract form of knowledge. For example, Irene argues how maths is related to *‘organisation, order, method, eh .... one step above, structured thought and affection’* (Irene's interview), and Georgia says that mathematics *‘...gives them (pupils) a method of thinking that is completely different from the usual. They learn to think beyond what is obvious’* (Georgia's interview). Through their expressed passion and success with mathematics they perform a masculine subjectivity that allows them to differentiate. Moreover, it

seems that by drawing on certain discourses about how mathematics is produced at both the academic level and the popular culture (i.e. in literature, cinema, science fiction) they invoke prevailing discourses in order to construct a dominant subjectivity and to exhibit their own skill in a field that is taken to entail social power (Mendick, 2006:84). Power and status stemming from significant dexterity in math is not natural, definite or taken for granted but it is a possible effect owing to those discourses. From this perspective, Georgia's and Irene's narratives do not demystify mathematics and math education related practices as discourses that demand deconstruction. Instead, they return to their own situated needs (I wanted to escape home, I was a mother) so that to justify their choices for not following a mathematics related career as a matter of making a safer choice for life –in which the discipline of mathematics cannot be easily accommodated. As such they, both, fail to place mathematical knowledge in its social, cultural and historical context in material terms and they don't disclose the fact that mathematics (as well as gender) is constantly constructed and reconstructed from and within discourse(s) and social relations (Appelbaum, 1995, Restivo, 1992)

### References

- Appelbaum, P. 1995. *Popular Culture, Educational Discourse and Mathematics*. NY. SUNY press.
- Boaler, J. (1997). *Experiencing school mathematics: teaching styles, sex and setting*. Buckingham. Open University Press.
- Chronaki, A. (2011). Disrupting Development as the quality/equity discourse: cyborgs and subaltners in school technoscience. In B. Atweh et al. (Eds.), *Mapping equity and quality in mathematics education*. Springer.
- Gill, R. (2007). Culture and Subjectivity in Neoliberal and Postfeminist times, *Subjectivity*, 25, 2008, pp.432-445.
- Henriques, J., Hollway, W., Urwin, C., Venn, C. and Walkerdine, V. (1984). (eds.), *Changing the Subject: Psychology, Social Regulation and Subjectivity*, London, Methuen.
- Foucault, M. (1977/1980). *Discipline and Punish: The Birth of the Prison*. New York: Pantheon.
- Foucault, M. (1984). What is enlightenment? In P. Rabinow (Ed.), *The Foucault reader*. New York: Pantheon Books.
- Laclau and Mouffe, C. (1985). *Hegemony and socialist strategy. Towards a Radical Democratic Politics*. London: Verso.
- Mendick, H. (2006). *Masculinities in Mathematics*, Open University Press, 2006.
- Mendick, H. (2005). A beautiful myth? The gendering of being/doing 'good at maths. in *Gender and Education*, Vol.17, No.2
- Murray, A.M. (2000). *Women Becoming Mathematicians: Creating a Professional Identity in Post-World War II America*. The MIT Press.
- Pechtelidis, Y. (2012). *Sovereignty and Resistance: Poststructuralist approaches of education*. Ekkremes. Athens.
- Restivo, S. (1992). *Mathematics in Society and History*. Kluwer Academic Publishers.
- Turkle, S. (2008). *Falling for Science: Objects in Mind*. The MIT Press.
- Walshaw, M. 2005. Getting Political and Unraveling layers of gendered mathematical identifications. In *Cambridge Journal of Education*. Vol. 35. No.1 pp. 19-34.



## **A simple discrete simulation model to explore in the classroom some rules involved in the decision-making process**

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### **Abstract**

The use of the simulations or computational models to understand and explain social phenomena is well established, giving us several opportunities to explore this kind of system. They make it easy to introduce certain ideas and skills. The aim of this contribution is to present teaching material elaborated to work with a very simple agent-based model, a cellular automata called “Voting”, which is free from the social science section of Net-Logo library. It facilitates the exploration in class of several rules involved in the decision-making process. It is a model in which an agent changes its state (“vote” or decision) according to the joint states of all of its neighbours. This teaching material contains an explanation of the design of this model and a collection of exercises and queries to investigate how the simulator works and what we can learn from it. In addition, the results achieved with its implementation in the classroom will be also discussed.

### **Introduction**

The creation of appropriate learning conditions in mathematics classroom practices, where all the students can develop their abilities to solve and understand increasingly challenging problems is of great interest in the education arena in general. Everybody agrees that students have to become critical thinkers and decision makers both inside and outside the classroom. On the other hand, it is nearly impossible to reach a consensus on the most significant students’ mathematical knowledge, skills and understanding to

cope with problems in our society. As to what kinds of learning environments are needed to promote a democratic access to mathematical ideas for all students, or about the nature of the critical process into which learners must be initiated, many answers could be collected. In this contribution I propose one attractive response, from my personal point of view and teaching experience, to deal with these items, which will be illustrated with a detailed example to be used in classroom practice.

Nowadays, the exploration and understanding of some social and economic issues through simulation is a fact (Gilbert and Troitzsch, 2005). The use of simulations or computational models to understand and explain social phenomena gives us some opportunities to do this. Computer simulations in the social sciences are a rather new idea (because simulation only began to be used widely in the 1990s) and it is generating huge potential in mathematics classes in order to accomplish the ideas and questions pointed out above. The introduction of certain ideas and skills is made easy.

The agent-based models (ABMs), with a different philosophy and perspective from the classic and continuous models, are computational methods that enable an academic to create, analyse and experiment with models composed of “agents”, elements or parts that make up a system and which are treated as autonomous and discrete entities, within an “environment”, domain or space with its own characteristics (Gilbert 2008, Grimm et al. 2006). These models focus on the characterization of the entities by means of rules of empirical behaviour, which allow the said elements to interact among themselves and with the environment in which they are found. ABMs, as discrete simulations, introduce the possibility of a new way of thinking about processes based on ideas about the emergence of complex behaviour from relatively simple activities. This kind of models with the corresponding simulators could be used successfully in classroom (Ginovart et al. 2011, 2012; Railsback and Grimm 2011; Wilensky and Rand 2012). Building this kind of models is a well-recognized way of understanding the world, something that everybody tries to do all the time. It is not very difficult to build simulation systems that include heterogeneous agents, for instance to simulate people with different perspectives on their social worlds, different knowledge, different priorities, different economic levels... Each simulation has “inputs” entered by the modeller (attributes needed to make the model match up with some specific social setting) and “outputs” (behaviours of the model through time) which are observed as the simulation runs (Gilbert and Troitzsch 2005).

NetLogo is a multi-agent programmable modelling environment, and it

was created by Uri Wilensky in 1999. It is particularly appropriate to use in teaching, as it provides interesting elements to illustrate ABMs in the classroom, as well as to interact and work with them (Ginovart et al. 2011, 2012; Railsback and Grimm 2011; Wilensky and Rand 2012). Netlogo is continually being updated by the Center for Connected Learning and Computer-Based Modeling in the United States (Wilensky, 1999). NetLogo can coordinate all the instructions given to a number of agents or individuals so that they all operate independently among themselves and with the environment. The NetLogo web page <http://ccl.northwestern.edu/netlogo/> contains abundant documentation and tutorials. This platform also includes a gallery of sample models ('Models Library') of diverse themes of application ready to be executed and they can also be modified or adapted. Among these samples of ABMs, there can be found some rather sophisticated models in social science, like for instance: "AIDS", "Cooperation", "Ethnocentrism", "Rebellion", "Segregation", "Wealth distribution"...

The NetLogo software, models and documentation are distributed free of charge for use by the public to explore and construct models. Permission to copy or modify the NetLogo software, models and documentation for educational and research purposes only and without fee has been granted (Copyright 1999-2011 by Uri Wilensky).

### **Aim**

The general aim is to make the characteristics and possibilities of ABMs better known in a teaching and learning context, so that this modelling methodology may be progressively incorporated into academic curricula, complementing other existing modelling strategies and gaining more use in the classroom. To this end, I establish one specific objective for this contribution which is to present the teaching material elaborated on a very simple ABM, a cellular automata chosen from the social science section of NetLogo library called "Voting". It is a model in which an agent changes its state ("vote" or decision) according to the joint states of all of its neighbours. This teaching material contains a detailed explanation of the design of this model and a collection of exercises and queries to investigate how it works and what we can learn from it. In addition, the results achieved in the classroom with its implementation will be discussed. Although, the participants in this study are a group of students at the Universitat Politècnica de Catalunya (UPC), the teaching material elaborated could also be used in the last years of secondary or high school.

### **Material and methods**

It is possible to install free NetLogo in a computer from <http://ccl.northwestern.edu/netlogo/> (it takes only a few minutes), and it runs on almost any current computer.

Working with an already developed simple and unsophisticated example like “Voting” model (Wilensky 1998), chosen from the NetLogo ‘Models Library’ in the social science section, enables the introduction of this type of computational model in a classroom in an effective way. Also, it allows us to learn how to deal with NetLogo and how to analyse and discuss the results of several simulations. If you have NetLogo installed on your own computer, to open the “Voting” model you can use the following instructions: a) open NetLogo, b) go to File, c) go to the Models Library, d) choose Social Science, e) double click to select and open “Voting”. When we open “voting” model, we find the “Interface” window (Fig. 1). At the top of this NetLogo's main window are three tabs labelled “Interface” (the current), “Information” and “Procedures”. The “Information” window contains some information regarding the model, and in the “Procedures” window can be found the corresponding computer code developed for the implementation of this model (Fig.2).

The “Voting” model was described in Rudy Rucker's "Artificial Life Lab", published in 1993 by Waite Group Press. It is a model in which an agent changes its state according to the joint states of all of its neighbours. It is a simple cellular automata that consists of a number of identical cells arranged in a regular grid. The spatial cells (the agents) are placed in a squared bi-dimensional array. In this social representation they stand for individuals. Each spatial cell can be in one of two states, “off” or 0 (green patch) or “on” or 1 (blue patch). It simulates voting distribution by having each patch take a “vote” from its eight surrounding neighbours, then perhaps its own “vote” changes according to the outcome with the rules assumed. We can adapt the situation of this model, the “vote”, to examples in which the states represent opinions (such as supporting or nor a political party or an initiative), individual characteristics (to have or not to have it) or attitudes (cooperating or not cooperating with others). For instance, people might adopt a fashion only if the majority of their friends (neighbours) have already adopted it. At each time step during the evolution of the system, the state of each cell may change. The state of a cell after any time step is determined by a set of rules which specify how that state depends on the previous state of the cell and the states of the cells' immediate neighbours. The same rules are used to update the state of every cell in the grid. It is a simple

discrete simulation model that enables investigation in the classroom of diverse rules involved in the decision-making process.

Fig. 3 shows an initial distribution of votes on the grid, distribution of green and blue cells after the “setup” button has been pressed, and the distribution at the end of a run, the distribution achieved after some time steps using the default values and rules for the “Voting” model of NetLogo. In this case, once the cells have achieved this speckled pattern, there is no longer any opportunity for change.

Some indications to understand how this computational model works are suggested through a set of exercises. These exercises, with the corresponding queries to be answered and discussed, can assist the examination and analysis of the “Voting” model and its simulation results. These tasks are as follows.

- 1) Carry out, for instance, 5 successive simulations with the same set of parameters which are fixed in the program by default (both switches “change-vote-if-tied” and “award-close-calls-to-loser” are off). What do you see? Why don’t the evolutions and final distributions in the space of the two sub-populations, green cells and blue cells, look exactly the same? What tendencies or patterns of behaviour are observed in the system?
- 2) You can open up the Model Settings, by clicking on the “Settings...” button at the top of the “Interface” window. You will notice that “max-pxcor”, “max-pycor” are fixed to 75. Change these two values to 1 and also modify the “Patch size” from 3 to 140. Setup the new conditions for the simulation, and perform the default rules over this initial configuration of the “Voting” model by hand. You can apply them to the central cell. Compare the simulation results with those that you achieve by hand. Try different initial conditions to repeat these calculations.
- 3) Carry out different simulations by modifying the values of “max-pxcor”, “max-pycor” and “Patch size”. For instance, you can use the following set of values: 1, 1 and 140; 2, 2 and 70, 4, 4 and 35; 8, 8 and 18; 16, 16 and 9; 32, 32 and 5; 64, 64 and 3; and finally, 128, 128 and 2. Examine the simulations that you have obtained. Watch how any arrangement settles to a static state. Approximately, how many time steps are required to achieve these final static states in the different cases evaluated? How can you explain this behaviour?
- 4) Read and check carefully the documentation of the “Information” window and try to understand the computer code of the “Procedures” win-

- do to identify the rule behind the option “change-vote-if-tied”. Watch what happens when only the “change-vote-if-tied” switch is on. How is the result different from those achieved in Exercise 1?
- 5) Read and check carefully the documentation of the “Information” window and try to understand the computer code of the “Procedures” window to identify the rule behind the option “award-close-calls-to-loser”. Watch what happens when only the “award-close-calls-to-loser” switch is on. How is the result different from those achieved in Exercises 1 and 4?
  - 6) What happens when both switches “change-vote-if-tied” and “award-close-calls-to-loser” are on?
  - 7) Can you imagine any other rules to change the status of the cells? Could you describe an application for a real context in your on life where this type of model would be useful?

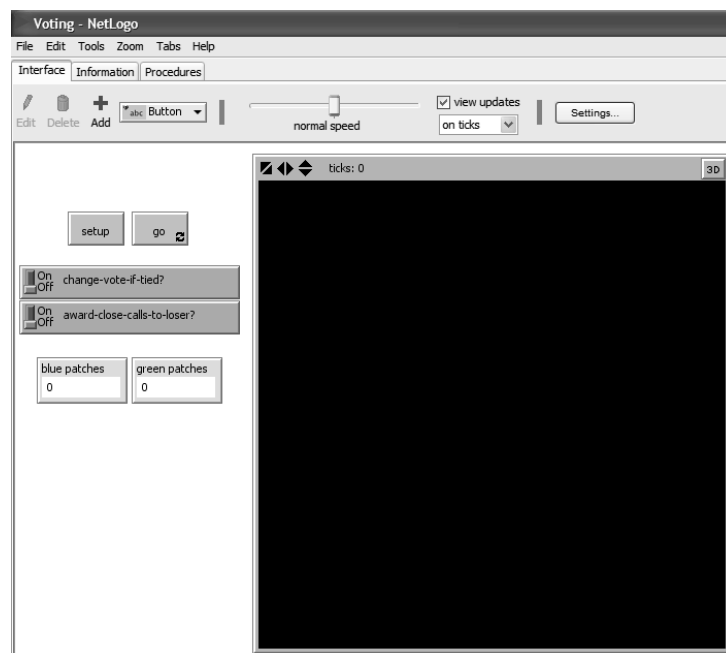


Fig. 1: The “Voting” model from the NetLogo Models Library: “Interface” window.

```

Voting - NetLogo
File Edit Tools Zoom Tabs Help
Interface Information Procedures
Find... Check | Procedures |  Indent automatically

patches-own
[
  vote ;; my vote (0 or 1)
  total ;; sum of votes around me
]

to setup
  clear-all
  ask patches
    [ set vote random 2
      recolor-patch ]
end

to go
  ask patches
    [ set total (sum [vote] of neighbors) ]
  ;; use two ask patches blocks so all patches compute "total"
  ;; before any patches change their votes
  ask patches
    [ if total > 5 [ set vote 1 ]
      if total < 3 [ set vote 0 ]
      if total = 4
        [ if change-vote-if-tied?
          [ set vote (1 - vote) ] ]
      if total = 5
        [ ifelse award-close-calls-to-loser?
          [ set vote 0 ]
          [ set vote 1 ] ]
      if total = 3
        [ ifelse award-close-calls-to-loser?
          [ set vote 1 ]
          [ set vote 0 ] ]
      recolor-patch ]
  tick
end

to recolor-patch ;; patch procedure
  ifelse vote = 0
    [ set pcolor green ]
    [ set pcolor blue ]
end

; Copyright 1998 Uri Wilensky. All rights reserved.
; The full copyright notice is in the Information tab.

```

Fig. 2: The computer code of “Voting” model as shown in the “Procedures” window of NetLogo.

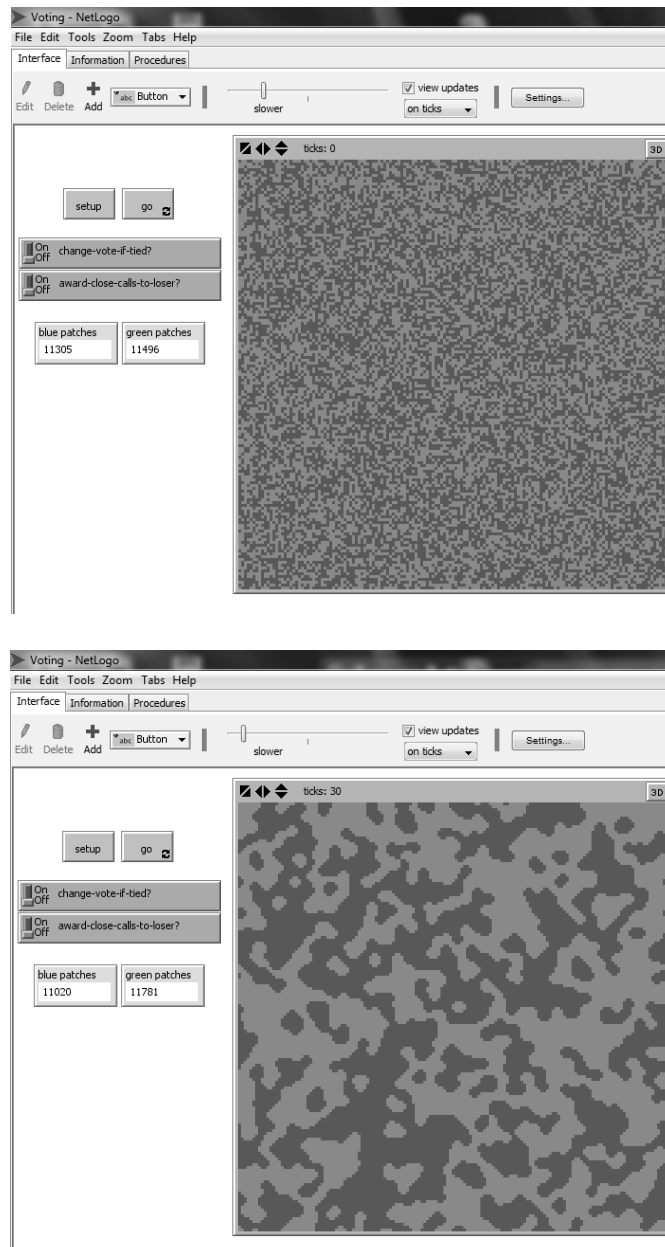


Fig. 3: An initial distribution of green and blue cells after the “setup” button has been pressed (top), and the distribution at the end of a run (bottom) using the default values and rules of the “Voting” model.



**Results and discussion**

The results of the implementation of this teaching material in the classroom will be shown and discussed in detail in the extended presentation of this contribution.

**REFERENCES**

- Gilbert, N. (2008). *Agent-based models*. Sage Publications.
- Gilbert N., Troitzsch K.G. (2005). *Simulation for the social scientist*. Open University Press.
- Ginovart, M., Portell, X., Ferrer-Closas, P., Blanco, M. (2011). Modelos basados en el individuo y la plataforma NetLogo. *UNIÓN-Revista Iberoamericana de Educación Matemática* 27, pp. 131-150.
- Ginovart M., Portell X., Ferrer-Closas P., Blanco M. (2012). Modelos basados en el individuo: una metodología alternativa y atractiva para el estudio de biosistemas. *Enseñanza de las ciencias* (in press).
- Grimm V., Berger U., Bastianen F., Sigrun E., Ginot V., et al. (2006). A standard protocol for describing individual-based and agent-based models. *Ecological Modelling* 198, 115-126.
- Railsback S. F., Grimm V. (2011). *Agent-Based and Individual-Based Modeling: A Practical Introduction*. Princeton University Press.
- Wilensky U. (1998). NetLogo Voting model. <http://ccl.northwestern.edu/netlogo/models/Voting>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
- Wilensky U. (1999). NetLogo. <http://ccl.northwestern.edu/netlogo/>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
- Wilensky U., Rand W. (2012). *An introduction to agent-based modeling: Modeling natural, social and engineered complex systems with NetLogo*. MIT Press. (in press)

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## **Unveiling the flaws of a reception plan for immigrant students**

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*Using different snapshots coming from interviews with immigrant students in Catalonia, we discuss how the Catalan educational system, despite its aim for cohesion and its claims of equity of access and opportunities, may fail to provide immigrant students with the same opportunities to learn mathematics as the rest of the students. Teachers' everyday practice is constrained by how the school system is organized and, perhaps too often, both teachers and educational institutions base their decisions on dominant social representations, thus limiting immigrant children's opportunities to learn mathematics.*

### **THE CONTEXT**

In the last two decades, Catalonia (with Barcelona as capital city), has seen a significant increase in immigrant population. Family regrouping processes have contributed to a change in schools' ethnic and social make-up to the point that, in some neighbourhoods in Barcelona or in cities like Badalona, there are schools where the number of immigrant children may reach 95% of the total school population. In this contribution, we discuss how –in the Catalan educational system– immigrant children's access to mathematical knowledge is actually limited by an educational priority established around (Catalan) language proficiency.

Catalonia is one of the 17 autonomous communities into which the Spanish state is organized. In Spain, and therefore in Catalonia, education is compulsory and free from age 6 to age 16. Primary school corresponds to ages 6-12, and compulsory secondary school to ages 12-16. Catalonia has

two official languages, Catalan and Castilian (Spanish). Because language is one of the defining features of Catalan culture and identity, it is the official language of instruction, even though Spanish is often used as well. In Catalonia, many immigrant schoolchildren, even those who have learned to speak Catalan well and have been partially educated in Catalan primary schools, experience difficulty later on in compulsory secondary school. Moreover, immigrant students, or students born in Catalonia to immigrant families, are rarely found pursuing studies at baccalaureate level (ages 16 to 18), and to an even lesser extent at university. That is especially true when their academic path includes mathematics.

The data we use here to start our discussion come from a broader ongoing study aimed at understanding the transition processes of immigrant students learning mathematics in Catalan schools (for more details, see Costanzi, Gorgorió, and Prat, 2012). Our corpus of data is constituted by the narratives obtained in semi-structured interviews where we explicitly ask immigrant students about their past and present experiences and their expectations about the future, with relation to mathematics learning. To obtain complementary data we interview some of their teachers and observe some of their mathematics lessons.

All the students that we have interviewed have attended school during the application of one of the two consecutive reception plans established by the Catalan educational administration to attend to immigrant students on their arrival: the “Incorporació tardana d'alumnes” (*late incorporation of students*) plan (Generalitat de Catalunya, 1999) or, the present one, “Pla per a la Llengua i la Cohesió Social” (*plan for the language and social cohesion*) (Generalitat de Catalunya, 2004).

There are clear differences between the two plans in their organization, their structure and in how the responsibility of actions is transferred to schools. However, the clearest difference is that the first one mainly focuses on immigrant students learning the (Catalan) language, while the second addresses not only on language learning issues but also emotional, relational and social cohesion aspects. While in the first plan language was essentially regarded as playing an instrumental role in the learning process, in the second one language was considered to have an instrumental role as well as an integrational one.

However, both plans had similar procedures for ascribing immigrant students to groups, according to their age, and similarly considered that the learning of Catalan should take place during school hours. During the implementation of the first plan, immigrant students in the later years of Pri-

mary Education (10 to 12 years of age) and in Compulsory Secondary Education had to attend “language workshops” for 12 hours every week, while their classmates were taking regular courses. The rest of the week, they attended lessons with the whole group. The second plan establishes that children arriving after they are about 10 years old should attend the “aula d’acollida” (reception class), for less than half the school day, until they achieve a sufficient proficiency of Catalan language.

Despite our description of both plans being very limited, it illustrates how both plans provide language learning resources for immigrant students even if it has to be to the detriment of the student's progress in some other subjects.

### SCRUTINIZING SOME CASES

When analysing the transcriptions of the interviews in which students tell us about their transition processes, we identify which are the ruptures and key moments to change students' trajectories in ways that are significant for them (Zittoun, 2007) and we see how teachers are regarded by the students as significant others in the reconstruction of situations, to make transitions out of ruptures. Teachers appear in students' narratives as social resources, as people in the students' social networks that can be asked for, or who may offer, expert or relational support.

Thus Hina, a Pakistani girl, has told us: “*they [the teachers] helped us to connect with the way that we had learned in Pakistan as these teachers had taught Pakistani students before*”. Later on, when talking about the mathematics teacher she insisted on the fact that “*the teacher was aware of the way we learned mathematics in Pakistan, because before us the school had Pakistani children*”. In Hina's school, language learning and social cohesion are priorities. By the end of their compulsory secondary school most students are able to speak Catalan reasonable well. The school also manages to keep youngsters within the educational system, by providing them with occupational workshops when needed.

However, despite the great task accomplished by all the teachers in Hina's school, the percentage of their students that reaches baccalaureate level is much lower than the average in Barcelona. In fact, teachers' efforts are constrained by the school's make-up, which is determined by the assignment of students to schools according to where their families live. The concentration of immigrant population in certain areas leads to a “natural” rise of ghetto schools. There is little teachers can do when there is no regulation to counteract this effect. Even if they manage to achieve social cohesion within the school, when leaving the school, their students will be disadvan-

taged when searching for a job, not because of their language but because of their lack of mathematical knowledge.

In some narratives, teachers are even referred to as inhibiting the student's opportunities, but then the term "teachers" does not refer to individual teachers, but to the institution they represent. For example, when Paola, a Colombian girl, explains her experience when she arrived in Barcelona, she says: "*when I arrived they held me back one year because of Catalan*". "They" refers to the teachers of her school, who cannot decide by themselves, and are only agents of the educational institution that regulates the adscription of students to groups. We see here how Paola's progress is, at least momentarily, interrupted because language is a priority within the educational system.

Sometimes, even when students account for their teachers to have been of great help to them, what we see is that the teacher limits the student's real opportunities to learn mathematics. Ronnie, a boy from Ecuador, tells us: "*since I'm not good enough at math, I won't be able to pass the entrance exam to go to university*". However, he describes his mathematics teachers as having been of great help to him since he arrived. Ronnie is one of the cases that illustrate how teachers, without being conscious of it, through poor classroom practices, restrict their students' future possibilities. Ronnie's teacher tells us that, with an honest intention to make them feel that they succeed, she only proposes routine exercises, because of her perception of immigrant students to be low achievers.

In other cases, it is not the teacher's perception of the student having a low level in mathematics, but the student really having a low achievement because of different reasons. This would be the case of Patricia, born in Honduras. She had left school when she was 12 years old, and when she arrived in Barcelona, aged 14, she was allocated to a group of students according to age. She was telling us that she had a teacher that was particularly following her in Catalan lessons and that "*In mathematics, since I did not pass the examinations, they gave me a portfolio with all the activities that I have to complete. It's like an exam, but much easier*". She is happy with how her teachers support her and she does not yet realize that it will be extremely hard for her to fill the gap in her mathematics learning.

### CONCLUSION

Some schools find themselves in a different situation from others in terms of the background of the students they provide for. There are teachers working at institutions with a high number of immigrant students per group, while there are others working with only a very limited number. There are

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several constraints in the practice of teachers having students from many nationalities, but at the same time, there is the risk that they become invisible when there are only a few immigrant students in a class.

We see how the individual and institutional role of the teacher cannot be separated and too often the system's priorities take over the teacher's disposition. The concern is on how much immigrant children's opportunities to learn mathematics depend on individual teachers' will to become a social resource for them and on how the constraints coming from the educational context may limit teachers' actions.

In our examples we have seen how the Catalan educational system, even when reaching its goal of providing language competence to all children, may fail on the more global aim of school providing equal opportunities. Because of its focus on language, the system may narrow immigrant children's opportunities to learn mathematics. Mathematics is not a priority in the system when planning for immigrant children.

However, in our study we have also found many immigrant children that fail their mathematics, even after learning to speak Catalan well. Therefore, the issue is not only on the consequences of language being a priority in the school system, but also the reasons why language is considered to be a priority.

Both teachers and institutions are impregnated with social representations and valorisations that shape their practices. The 'unreflective use' of existing dominant social representations gives rise to an attempt to assimilate the immigrant student into 'normality', often expressed in terms of reducing obstacles to learning by improving language competence and explaining learning difficulties as student deficits (Gorgorió & Abreu, 2009).

In a democratic society, ALL citizens should be able to make informed decisions based on critical thinking. Since mathematical knowledge is vital for critical thinking, the school system should provide equal opportunities to learn mathematics to all children. To achieve this goal, pre-service and in-service teacher education should contribute to create discussion spaces that fostered critical awareness of, and reflection on, social representations that may affect school practices.

**REFERENCES**

- Costanzi, M., Gorgorió, N., & Prat, M. (2012). Pre-service teachers' representations of school mathematics and immigrant children. In E. Hjørne, G. van der Aalsvoort & G. de Abreu (Eds.), *Learning, social interaction and diversity – exploring school practices* (pp. 203-221) SENSE publishers
- Generalitat de Catalunya (1999). Incorporació tardana d'alumnes. Programa 1999-2000.
- Generalitat de Catalunya (2004). *Pla per a la Llengua i la Cohesió Social* (<http://www.xtec.es/acollida>).
- Gorgorió, N. & Abreu, G. de (2009). Social representations mediating practices in multicultural mathematics classrooms. *Educational Studies in Mathematics*, 72(1), 61-76.
- Zittoun, T. (2007). Symbolic resources and responsibility in transition. *Young*, 15(2), 193-211.

## Évolution d'un cadre théorique sur les représentations

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**Résumé :** Les représentations ont été traitées depuis plusieurs décennies comme thème de réflexion théorique sur l'apprentissage des mathématiques. C'est singulièrement dans les années 80 que plusieurs chercheurs en didactique des mathématiques ont pris comme paradigme le développement « d'un » cadre théorique autour des représentations (voir p.e. Janvier 1987 ; Duval, 1988, Goldin, 1998 ; Hitt, 2002). Dans une expérimentation au début de ce siècle, en prenant en compte différents aspects théoriques sur les représentations et en liaison avec une méthodologie centrée dans un apprentissage collaboratif, de débat scientifique et auto - réflexion (ACODESA), les différentes approches théoriques avant signalées n'ont pas donné une explication satisfaisant des constructions conceptuelles des étudiants (Hitt 2003, 2004). Ainsi, il fallait décider sur la possibilité d'une évolution de ces cadres théoriques ; ou bien, un changement de voie vers un cadre théorique qui permettrait de donner de meilleures explications aux constructions conceptuelles des élèves dans un milieu plus démocratique d'apprentissage collaboratif. En plus, de donner l'opportunité aux élèves de participer à la construction de leurs connaissances d'un point de vue opposé à une construction passive. Nouvelles expérimentations autour de la notion de situation problème (Passaro, 2007 ; González-Martín, Hitt & Morasse, 2008 ; Hitt et Morasse, 2009 ; Hitt et González-Martín, en préparation) nous signalent un chemin à suivre dans la construction d'un cadre théorique que prendre en compte les représentations fonctionnelles des élèves (en bref représentations spontanées) et les représentations institutionnelles (celles que l'on trouve dans les manuels, celles que l'enseignant utilise dans la classe, celles que l'on trouve dans les écrans des ordinateurs).

### **Introduction**

Dans le XX siècle le paradigme principal sur l'apprentissage a été le



constructivisme et constructivisme radical (Piaget, VonGlasesferd, entre autres). Cette approche théorique générale sur l'apprentissage (dans le sens de diSessa, 1994) a marqué les lignes directrices pour l'expérimentation dans le milieu de la didactique des mathématiques. Théories locales ont été développés autour du paradigme constructiviste en donnant lieu à théories sur les représentations (Janvier, 1987 ; Duval, 1988 ; 1993, 1995 ; Goldin, 1998). Dans une approche socioconstructiviste, nous pouvons nommer les travaux de Brousseau (1987) entre autres, mais celui-ci ne prene pas comme sujet principal le rôle des représentations dans la construction des connaissances. Par contre, les étudiants avaient l'opportunité d'exprimer leurs opinions pour construire une connaissance, disons plus démocratique parce qu'elle n'est pas imposée par le professeur et/ou l'institution.

Les théories sur les représentations sont basés sur le fait que tout objet mathématique peut être représenté partiellement seulement. Alors, la promotion de l'articulation entre représentations est la colonne vertébrale de celles-ci, et en conséquence les tâches de conversion d'une représentation dans un registre à une autre représentation dans un autre registre sont prioritaires pour la construction de l'articulation entre représentations.

Un des problèmes principaux de ces théories locales autour des représentations est, d'un côté, qu'elles ont été centrées sur l'explication des constructions individuelles des connaissances mathématiques. D'un autre côté, leur approche théorique a été fortement marquée par les représentations institutionnelles (celles que les enseignants utilisent au tableau, celles que l'on trouve dans les manuels scolaires, ou celles que l'on trouve dans les écrans des ordinateurs). Alors, on voit premièrement que ces théories sont centrées exclusivement sur la construction individuelle des connaissances sans mentionner l'importance des constructions dans un milieu d'interaction sociale des connaissances. Deuxièmement, ces théories ne prennent en compte les productions non institutionnelles qui émergent de l'intuition dans des processus de résolution de situations problèmes ou de la modélisation mathématique à l'intérieur de la communication libre dans la classe de mathématique,. L'approche classique ne permet pas l'ouverture à un type différent d'apprentissage plus démocratique qui pourrait être lié à une construction sociale des connaissances.

Les productions que régulièrement ne sont pas des représentations institutionnelles nous les avons appelés représentations fonctionnelles (Hitt et González-Martín, en préparation), représentations qui vont dans le même sens de ce que diSessa et al (1991) ont appelé meta-représentations. Ces représentations émergent dans l'activité mathématique dans un milieu d'interaction social comme celui signalé par Engeström et al. (1999) autour de la théorie de

l'activité de Lontev (1981).

Tout semble indiquer un changement de paradigme, on essaye de laisser de côté le constructivisme pour se plonger dans une construction sociale des connaissances. Wertsch (1985) signale le suivant autour des fonctions mentales supérieures :

A fundamental distinction that underlies Vygotsky's line of reasoning about qualitative transitions and the role of mediation is the distinction between « elementary » and « higher mental functions (1978, p. 39). Vygotsky's general strategy was to examine how mental functions, such as memory, attention, perception, and thinking, first appear in an elementary form and then are changed into a higher form. In his approach a related distinction is that between the « natural » and the « social » (or « cultural ») lines of development (1960, p. 47). Natural development produces functions in their elementary forms, whereas cultural development converts elementary into higher mental processes. It is the transformation of elementary into higher functions that Vygotsky usually had in mind when he spoke of how the nature of development changes. (p. 24)

De plus en plus il y a de recherches autour de l'apprentissage dans un milieu d'interaction sociale. Un nouveau paradigme émerge entre les chercheurs qui désirent l'évolution d'une théorie des constructions sociales qui permettrait expliquer tant les aspects individuels de l'apprentissage dans un milieu de construction sociale des connaissances plus démocratique.

Ce que je veux dire c'est que l'on peut préciser autour d'un apprentissage collaboratif quand les élèves travaillent en équipe, mais nous devons aussi préciser ce que l'élève a retenu après ce travail, une dialectique entre les connaissances individuelles et connaissances coconstruites dans un milieu plus démocratique.

Alors, à l'époque actuelle, nous avons besoin de construire une théorie qui pourrait prendre en compte les aspects de construction sociale des connaissances et en même temps d'expliquer l'aspect individuel de construction de concepts mathématiques. Aussi, nous avons besoin d'une méthode d'enseignement qui permettrait d'expliquer les constructions individuelles et en équipe des connaissances mathématiques.

Notre méthode nous l'avons appelé ACODESA (Hitt, 2007). Cette méthode essaye de donner un équilibre entre les interventions individuelles et en équipe pour mieux faciliter la construction sociale des connaissances mathématiques. Cette approche démocratique évite l'imposition des représentations institutionnelles sans réflexion de la part des élèves. Voici un résumé

de la méthode d'enseignement :

- Travail individuel : production de représentations fonctionnelles pour comprendre la tâche ;
- Travail en équipe sur la même tâche. Processus de discussion et validation. Raffinement des représentations fonctionnelles ;
- Débat (qui peut devenir un débat scientifique). Processus de discussion et validation (raffinement des représentations fonctionnelles) ;
- Retour individuel sur la tâche (travail individuel: reconstruction et auto - réflexion) ;
- Institutionnalisation. Processus d'institutionnalisation et utilisation des représentations.

Comme nous l'avons montré dans plusieurs articles (voir bibliographie) la méthode ACODESA nous permet de donner une explication de l'évolution d'une représentation d'un élève dans le processus de travail individuel, travail en équipe, travail en grand groupe et retour au travail individuel. Même si en principe nous proposons des étapes pour équilibrer et/ou réguler les interventions des élèves, dans une approche comme telle, c'est l'ambiance créée par eux même (d'un point de vue démocratique) qui facilitera ou pas leur interventions. L'étape de reconstruction (auto - réflexion) alors est très importante.

Étant donné à l'élève une tâche mathématique non routinière, l'élève régulièrement commence avec la production d'une représentation fonctionnelle (Hitt et Passaro, 2007 ; González-Martín, Hitt & Morasse, 2008 ; Hitt et Morasse, 2009 ; Hitt et González-Martín, en préparation), représentation produite de l'interaction de l'élève avec la tâche non routinière demandée. Usuellement cette représentation n'est pas une représentation institutionnelle. Le travail en équipe va permettre à l'élève de confronter sa représentation avec les productions de ses coéquipiers, c'est dans ce milieu démocratique d'interaction sociale des connaissances que l'élève progresse avec ses coéquipiers. Il est probable que l'élève soit confronté à un conflit cognitif en la discussion avec ses coéquipiers. Encore dans ce milieu, mais en grand groupe, dans une discussion libre, la confrontation naturelle des idées des équipes, place l'élève, pensé comme entité individuel immergé dans une micro société, en position de conflit cognitif. Si la position de l'enseignant n'est pas autoritaire et laisse les élèves discuter, dans une ambiance démocratique, en promouvant un débat scientifique (dans le sens de Legrand, 2001), l'enseignant donne l'opportunité aux élèves de résoudre leurs contradictions et de construire une connaissance solide. La majorité des méthodes

d'enseignement dans un milieu d'interaction sociale s'arrêtent à cette étape. Ils ne prennent pas en compte que le consensus, si celui-ci est arrivé dans la classe, est éphémère (Thompson, 2002). Alors, dans notre méthode, un travail de reconstruction à la maison est demandé à l'élève. Comme nous l'avons montré en Hitt et Morasse (2009) et Hitt et González-Martín, en préparation) le processus de reconstruction de la part des élèves n'est pas aussi facile de ce que l'on pouvait croire. L'étape individuelle de reconstruction a besoin de beaucoup plus de réflexion pour assimiler ce que ses compagnons ont exprimé dans la discussion en grand groupe.

En accord avec les résultats d'expérimentation reportés dans les documents avant signalés, nous avons montré sur l'importance des représentations fonctionnelles dans les processus de modélisation mathématique en liaison avec la méthode ACODESA :

Étape I	Étape II	Étape III	Étape IV	Étape V
Émergence des représentations fonctionnelles	Confrontation des représentations	Confrontation des représentations	Reconstruction des productions faites en classe	Processus institutionnel
Travail individuel	Travail en équipe	Débat en grand groupe	Travail individuel à la maison	Présentation des représentations institutionnelles de la part de l'enseignant

De ce point de vue, nous pouvons nous rendre compte sur l'importance des représentations fonctionnelles (diSessa *et al.*, les appellent meta-représentations, 1991) est l'importance de créer un milieu démocratique d'interaction sociale des connaissances pour promouvoir leur évolution. À différence des théories sur les représentations avant signalées, nous pouvons voir que les quatre premières étapes de notre méthodologie fait appel à une théorie sur les représentations fonctionnelles (ou meta-représentations dans le cas de diSessa, 1991) et c'est plutôt dans les étapes IV et V que s'avèrent importantes les théories sur les représentations qui ont été développées dans le passé (p.e. la théorie de Duval sur les représentations).

Étant donné que le travail papier crayon est très important dans notre approche méthodologique, nous avons étendu notre réflexion autour du rôle des activités papier crayon dans une ambiance technologique. En Hitt et Kieran (2009) nous avons montré l'importance du travail papier crayon et le

travail avec une calculatrice de manipulation symbolique avec l'analyse d'une entrevue fait à deux élèves (travail en équipe dans une construction sociale des connaissances) dans un projet à long terme. Récemment, en utilisant la méthode d'enseignement ACODESA, dans son mémoire de maîtrise, Lazli (2011) a montré que les représentations sur papier, la manipulation des objets physiques et l'utilisation de la technologie sont très importantes pour essayer de trouver un modèle mathématique. Ceci nous confirme que même dans un milieu de travail collaboratif avec la technologie, les productions sur papier des élèves continuent à être importantes.

### Discussion

Notre approche théorique sur les représentations prend de nouvelles considérations que non pas été prises en compte dans les théories locales classiques sur les représentations. Un des éléments principaux dans cette nouvelle approche théorique est les représentations fonctionnelles (schèmes qui construisent l'élève quand lui est face à une tâche non routinière) qui permettent la compréhension et structurent l'action vers la résolution de la tâche. Leur évolution nous l'avons considéré dans un milieu démocratique d'interaction sociale en prenant en compte aussi à la fin du processus un travail de réflexion individuel de la part de l'élève, avant que l'enseignant puisse commencer un processus d'institutionnalisation des connaissances.

Notre approche théorique sur les représentations donne le support méthodologique à la méthode d'enseignement ACODESA. C'est ainsi que les productions des élèves sous la méthode ACODESA devraient être analysées (voir Hitt et Morasse, 2009 ; Hitt et González-Martín, en préparation). Notre proposition essaye d'éviter l'imposition des savoirs institutionnels en donnant aux élèves la parole pour mieux construire les connaissances mathématiques dans un milieu démocratique de confrontation des idées dans la classe de mathématiques.

### Références

- Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. 1970-1990. In N. Balacheff, M. Cooper, R. Sutherland & V. Warfield, V. (Eds. and Trans.) Dordrecht, Kluwer.
- diSessa A., Hammer D., Sherin B. & Kolpakowski, T. (1991). Inventing Graphing : Meta-Representational Expertise in Children. *Journal of Mathematical Behavior*, 10, 117-160.
- diSessa A. (1994). Comments on Ed Dubninsky's Chapter. In A. Schoenfeld (Ed.), *Mathematical Thinking and Problem Solving* (pp. 248-256).

- Lawrence Erlbaum Associates, Inc.
- Duval, R. (1988). Graphiques et équations: l'Articulation de deux registres. *Annales de Didactique et de Sciences Cognitives*, 1, 235-253.
- Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. *Annales de Didactique et de Sciences Cognitives* 5, 37-65.
- Duval, R. (1995). Sémosis et pensée humaine: Registres sémiotiques et apprentissage intellectuels. Peter Lang, Suisse.
- Engeström Y., Miettinen R. & Punamäki R-L. (1999). Perspectives on activity theory. London : Cambridge University Press.
- Goldin, G. (1998). Observing Mathematical Problem Solving Through Task-Based Interviews. *Journal for Research in Mathematics Education*. Monograph 9, NCTM.
- González-Martín, A. S., Hitt, F. & Morasse, C. (2008). The introduction of the graphic representation of functions through the concept of covariation and spontaneous representations. A case study. In O. Figueras & A. Sepúlveda (Eds.). *Proceedings of the Joint Meeting of the 32<sup>nd</sup> Conference of the International Group for the Psychology of Mathematics Education and the 30<sup>th</sup> PME-NA*, Vol. 3, pp. 89-97. Morelia, Michoacán, México: PME.
- Hitt F. (2003). Le caractère fonctionnel des représentations. *Annales de Didactique et des Sciences Cognitives*, 8, 255-271.
- Hitt F. (2007). Utilisation de calculatrices symboliques dans le cadre d'une méthode d'apprentissage collaboratif, de débat scientifique et d'auto-réflexion. In M. Baron, D. Guin & L. Trouche (Eds.), *Environnements informatisés et ressources numériques pour l'apprentissage. Conception et usages, regards croisés* (pp. 65-88). Editorial Hermes.
- Hitt F. & Kieran C. 2009. Constructing knowledge via a peer interaction in a CAS environment with tasks designed from a Task-Technique-Theory perspective. *International Journal of Computers for Mathematical Learning*. Vol 14, pp. 121-152.
- Hitt, F. & Morasse, C. (2009). Développement du concept de covariation et de fonction en 3<sup>ème</sup> secondaire dans un contexte de modélisation mathématique et de résolution de situations problèmes. Proceedings CIEAEM 61 – Montréal, Québec, Canada, July 26-31, 2009. "Quaderni di Ricerca in Didattica (Matematica)", *Supplemento n. 2, 2009*. G.R.I.M. (Department of Mathematics, University of Palermo, Italy),  
[http://math.unipa.it/~grim/cieaem/quaderno19\\_suppl\\_2.htm](http://math.unipa.it/~grim/cieaem/quaderno19_suppl_2.htm)

- Hitt, F. & Passaro, V. (2007). De la résolution de problèmes à la résolution de situations problèmes : le rôle des représentations spontanés. In *Actes de la 59<sup>e</sup> Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM59)*, Dobogókö (Hungary), electronic format, 117-123.
- Janvier, C. (Editor). (1987). *Problems of Representation in the Teaching and Learning of Mathematics*. London: Lawrence Erlbaum Associates.
- Lazli, S. (2011). Enseignement de la fonction sinus au deuxième cycle du secondaire par le biais de la modélisation et de l'outil technologique. Mémoire de maîtrise, UQAM, Montréal.
- Legrand, M. (2001). Scientific debate in mathematics courses. In D. Holton (Ed.), *The teaching and learning of mathematics at university level : An ICMI Study* (pp. 127-135), Dordrecht : Kluwer Academic Publishers.
- Leont'ev, A. N. (1981). *Problems of the development of the mind*. Moscow: Progress.
- Passaro V. (2007). *Étude expérimentale sur le développement du concept de covariation entre deux grandeurs révélé par une analyse des représentations spontanées d'élèves du premier cycle du secondaire*. Master Thesis. Université du Québec à Montréal. Montréal, Québec, Canada, 283 pages.
- Thompson, P. (2002). Some remarks on conventions and representations. In F. Hitt (Ed.), *Mathematics Visualisation and Representations* (pp. 199-206). Psychology of Mathematics Education North American Chapter and Cinvestav-IPN. Mexico.
- von Glasersfeld E. (2001). Questions et réponses au sujet du constructivisme radical. In P. Jonnaert & D. Masciotra (Eds.), *Constructivisme choix contemporains. Hommage à Ernst von Glasersfeld* (pp. 291-323). Presses de l'Université du Québec, Canada.
- Wertsch J-V. (1985). *Vygotsky and the social formation of mind*. Harvard University Press.

## LINKING DEMOCRACY AND METACOGNITION: THE CASE OF OPEN - ENDED PROBLEMS

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### Abstract

*The purpose of this study was to investigate the spontaneous appearance of the metacognitive functions of monitoring and control, in fifth grade students working in pairs, while they were solving an open ended problem. Data collection was based on “think aloud” method and the analysis of the data was based on the method which Goos and Galbraith (1996) used for their research. According to our results, the students appeared metacognitive functions spontaneously when they were working in pairs.*

### Theoretical background

Across the world, the correlation between education and democracy is extremely high. The relationship between education and democracy has been at the heart of both academic and political debate of the last decade (Dee, 2004). Mathematics teaching can support the development of democratic goals by the skills and norms embedded in mathematical practice itself. Mathematics instruction can offer a shared experience with understanding, respecting, and using difference for productive collective work. Alternative interpretations and representations of a problem can often serve to open a path to its solution. At the same time the difference in the solving procedure or in the mathematical thought generally is structured and supported by common disciplinary norms and practices, which must be precisely defined and used.

However, although public schooling could support democratic aims, such as free expression and autonomy in thinking, these aims are often



strangely absent in mathematics class (Stemhagen & Smith, 2008). Democratic mathematics education seeks to connect math class to these broader aims and claims that, in doing so both mathematics class and the democratic dimensions of schooling can be strengthened. There are a number of ways researchers have attempted to make the mathematics education–democracy link, such as disagreements in a problem solving procedure must be resolved by the students not by shouting to each other but by reasoned arguments whose construction can be taught and learned. So mathematics instruction can deliberately help young people learn the value of others' perspectives and ideas, as well as how to engage in and reconcile disagreements.

In this paper, we will try to link metacognition and democracy as the students solve mathematical open-ended problems when working in small teams.

The concept of metacognition has gained a lot of interest in mathematical education research and practice (cf. Schoenfeld, 1985; Mevarech & Kramarski, 1997; NCTM, 2000; Ku & Ho, 2010). John Flavell, in 1976, defined metacognition as follows:

"In any kind of cognitive transaction with the human or non-human environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective." (p. 232).

More recently, Nelson and Narens (1990) managed to organize and compose almost the whole existing research on metacognition (Schraw & Moshman, 1995). This model focuses on the interaction between two metacognitive functions: monitoring and control. Nelson and Narens proposed a theoretical mechanism, which is necessary so as to have a metacognitive system, and is composed of two structures: the meta-level and the object-level, and also the flow of information relationship between the two levels. In this model, information flows with the meta-level acquiring information from the object-level (monitoring) and the meta-level sending information to and thereby changing the object-level (control) (Dunlosky & Bjork, 2008).

Mathematics education through metacognitive reinforcement can establish capabilities that can promote democratic behaviours to the students who are involved in the learning process, as metacognitive strategies make them act responsibly and consciously in the context of mathematics instruction and this helps to democracy itself e.g. towards the development of the autonomy of thought. According to Schoenfeld (1990), we could

mention that metacognition helps students to become more effective problem solvers, because they are capable of defining their targets, monitoring their thoughts and assessing whether their actions lead to the target.

On the other hand, democratic procedures are promoted when the students work in pairs, as they have the opportunity to think, discuss and change the reasoning and the argumentation of each other (Goos et al., 1996). Inferentially reinforcement in students' collaboration can help them to develop democratic abilities, which are important in their everyday life.

Generally in the researches that have been held for the scrutiny of the relationship between mathematics and meacognition, open – ended mathematical problems have been used as this kind of problems seem to be more suitable for the trace of metacognitive behavior (Kapa, 2001; Biryukov, 2002; Kramarski et al., 2002). Additionally the nature of an open ended problem is such, that the student is trying to find as many solutions as he can and we could assume that the procedure itself strengthens and emerges democratic practices. Students would work to seek agreement on meanings and solutions, drawing on past shared experiences, definitions, ideas, and agreements about meaning, and they would use and contribute to one another's ideas in a collective effort to solve and understand the mathematics and the problems on which they are working.

### **Method**

In this study 20 pupils of a fifth grade classroom of a Greek primary school of Athens participated. The students worked in pairs and they solved an open ended problem. The trace of the metacognitive functions of control and monitoring was made by the "thinking aloud" method (Goos & Galbraith, 1996; Hartman, 2001). According to Ericson and Simon (1980), during the talk/think aloud method, the subjects declare every thought they make. They denote loudly their thoughts during an activity without the researcher's intervention. In a case of silence the researcher just says "please continue thinking aloud" or "please keep on talking". So the subject has to explain loudly why took into consideration some data or how he solved the problem.

The open ended problem was the following:

A car parking has double parking places for motorcycles than the cars. Each car holds 10 sq meters and each motorcycle holds 4 sq meters. What is the total surface of the parking area?

For the analysis of the data we used an analysis protocol for the “thinking aloud” method and we based on the method Goos and Galbraith (1996) used in their research. By this method we can recognize three types of verbal reports which indicate metacognitive functions. The first type of the verbal report is the New Information, which can be subdivided to New Ideas (NI) or New Procedures (NP). The second type is Local Assessments (LA), referring to Knowledge, task difficulty, procedure, result. The third type is Global Assessments (GA), which refers to the general state of the solutions that were made. New Ideas (NI), New Procedures (NP), Local Assessments (LA) and Global Assessments (GA) denote the metacognitive function of monitoring, while the Transitions (T), happening between the episodes during the problem solving, denote the metacognitive function of control. In Transitions (T), control actions are taken by the solvers, so as to impulse the solution to the right direction. Transitions (T) can be denoted by New Ideas (NI), New Procedures (NP), Local Assessments (LA) or Global Assessments (GA). Below there is an example of verbal reports that denote metacognitive functions during problem solving in small teams of two students.

[5] Student 1: The parking has double places for motorcycles, than for the cars. ... lets say that the cars occupy two places. So the motorcycles will be 4 and I think that if the cars will be 10 then the cycles will be ..... (New Procedure).

[10] Student 2: Good. There may be 80, as 2 times 40 equals to 80 (Local Assessment).

[11] Student 1: Wait a minute. We have to find a solution, but if the cars are more, because the problem does not say the number of the cars .... Then this problem has a lot of possible solutions (New Idea).

### **Analysis and discussion**

During the problem solving procedure, as the students working in pairs, they appeared the chance to reveal metacognitive strategies so as to reach the metacognitive functions of control and monitoring when solving the open-ended problem.

Team	Local Assessments	New Ideas	New Procedures	Global Assessments	Transitions
1	4	-	1	1	1
2	2	-	1	1	1
3	7	3	4	-	1
4	3	1	-	-	-
5	3	1	1	1	1
6	11	5	4	1	2
7	2	1	-	-	1
8	7	3	-	-	1
9	-	-	2	1	-
10	4	4	1	-	1
Total	43	18	14	5	9

Table 1. Metacognitive functions in the open ended problem

According to the above table (Table 1), the students made 43 Local Assessments, 16 New Ideas, 14 New Procedures, 5 Global Assessments (monitoring function) and 9 Transitions (control function). We notice that the metacognitive monitoring function exceeds the metacognitive control function.

The dominance of the verbal reports that reveal the metacognitive function of monitoring shows that each student not only monitors his or her thought but also monitors the thought of the partner in the team. So, the metacognitive monitoring function gave them the opportunity to assess the value of others' perspectives and ideas, as well as to reconcile disagreements.

Metacognitive mathematics instruction can be designed to help students learn that differences can be valuable in joint work and that diversity in experience, knowledge and collaboration can enrich and strengthen collective capacity and effectiveness. How mathematical tasks are used is crucial in determining whether or not their potential is realized in classrooms. If not carefully structured and guided, cognitively complex tasks can degrade to simple routine problems, and problems ripe with opportunity for reasoning and representation can become algorithmic. Similar attention is needed in order for tasks to serve as contexts for the development of democratic skills and dispositions. Such attention is centred on cultivating respect for others' mathematical ideas. Students would need to develop a consistent stance of civility with one another, an intellectual interest and respect, not mere social politeness. This would require learning

to listen carefully to others' ideas, and checking for understanding before disagreeing. Moreover, the development of metacognitive strategies through open-ended problems contributes to young people's capacity for participation in a diverse society in which conflicts are not only an inescapable part of life, but their resolution, in disciplined ways, is a major source of growing new knowledge and practice.

### References

- Biryukov, P. (2002). Metacognitive Aspects of Solving Combinatorics Problems. *International Journal for Mathematics Teaching and Learning*, 1 – 19.
- Dee, T. (2004). Are there Civic Returns to Education? *Journal of Public Economics* 88, 1697-1720.
- Dunlosky, J. & Bjork, R. (2008). The Integrated Nature of Metamemory and Memory. In J. Dunlosky & R. Bjork, (Eds.), *Handbook of Memory and Metamemory* (pp. 11-28), Psychology Press.
- Ericsson, K. & Simon, H. A. (1980). Verbal reports as data. *Psychological Review*, 87, 215–251.
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231–235). Hillsdale, NJ: Erlbaum.
- Goos, M. & Galbraith P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics*, 30, 229 – 260.
- Hartman, H. J. (2001). Developing students' metacognitive knowledge and strategies. In H. J. Hartman (ed.), *Metacognition in Learning and Instruction: Theory, Research, and Practice*, (pp. 33–68). Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Kapa, E. (2001). A metacognitive support during the process of problem solving in a computerized environment. *Educational Studies in Mathematics*, 47, 317–336.
- Kramarski, B., Mevarech, Z., Arami, M., (2002). The Effects of Metacognitive Instruction on Solving Mathematical Authentic Tasks, *Educational Studies in Mathematics*, 49, 225 – 250.
- Ku, K. & Ho, I. (2010). Metacognitive strategies that enhance critical thinking, *Metacognition and Learning*, vol. 5, 3, 251-267.
- Mevarech, Z., Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal*, 34, 365-394.

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- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nelson, T. O. & Narens, L. (1990). Metamemory: A theoretical framework and new findings. *The Psychology of Learning and Motivation*, 26, 125–141.
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*, Orlando, Florida, Academic Press.
- Schoenfeld, A. H., 1990. Learning to think mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws. *Handbook for Research on Mathematics Teaching and Learning*. Macmillan, New York. pp 1 – 103.
- Schraw, G. & Moshman, D. (1995). Metacognitive Theories. *Educational Psychology Review* 7,4), 351–371.
- Stemhagen, K. & Smith, J. (2008). Dewey, democracy, and mathematics education: Reconceptualizing the last bastion of curricular certainty. *Education and Culture*, 24 (2), 25– 40.

## **Learning, access and power in school mathematics: A systemic investigation into the views of secondary mathematics school teachers**

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**Abstract** In this study, we adopt a systems theory approach to investigate the views that in-service mathematics teachers hold about mathematics (both as a discipline within the system of all disciplines and as a course within the system of school courses) with respect to issues concerning learning, access and power. We present a questionnaire that was constructed according to an inter-systemic, multi-focussed approach and we discuss the first results obtained from a small scale quantitative study (N=19) with mathematicians who work in the secondary education ('Lykeio') with 15-17 years old students in Greece. The conducted analyses suggest that the mathematicians' epistemological views are also evident in their views about mathematics as a school course. Moreover, our multi-focussed approach revealed aspects of their views that otherwise would be conflated.

### **Mathematics in school: systemic inquiries about access and power**

Mathematics educators have investigated issues of access and power, which are linked with the teaching and learning mathematics –within and outside the classroom– from a variety of perspectives, including cultural, and socio-political (Chaviaris, Stathopoulou & Gana, 2011; Powell & Frankenstein, 1997; Wagner & Herbel-Eisenmann, 2011). Furthermore, mathematics educator have acknowledged the fact that issues of social participation, equity and democratic access should be discussed, on the one hand, in terms of the personological characteristics that determine the self and the society (including gender, ability, ethnicity etc; Atweh, Graven, Secada & Valero, 2011) and, on the other hand, in terms of the corpus of the mathematical knowledge that constitutes powerful mathematical ideas (Skovsmose & Valero, 2002).

Mathematics educators have discussed and implemented systemic ideas regarding a variety of topics, including mathematics teaching, learning and curricular reform (Begg, 2003; Bouvier, Boisclair, Gagnon, Kazadi & Samson, 2010; Davis & Simmt, 2003; English, 2008; Kalavasis, Kafoussi, Skoumpourdi & Tatsis, 2010; Wittmann, 2001).

In this study, we address these complex phenomena within the school unit by adopting a *systems theory* perspective (Bertalanfy, 1975) about the interacting and the interrelating parts of implicit and explicit beliefs, which form integrated teaching practices.

In a previous study (Moutsios-Rentzos & Kalavasis, in press), we presented a theoretical and methodological framework, which considered mathematics as elements of two interrelated systems: the system of all disciplines and the school system. The latter was viewed through the lenses of three foci: the official regulations, the current state of school practices and the personal hypothetical decisions. This framework facilitated a qualitative shift of our research focus: *from* investigating views about mathematics *to* investigating views about mathematics within the system of consideration (disciplines or courses).

We draw upon this framework to identify the views of in-service mathematicians about mathematics and school practices in terms of access and power, with the purpose of gaining deeper understanding about the inter-systemic and intra-systemic identified relationships.

### **Setting the scene**

In Greece, in order for someone to enter university (s)he has to sit National Exams. As a result, in the private sector, a preparatory system has been developed (consisting of private tutors, private preparatory schools etc) that complements and co-exists with the central educational system to meet the social need for the children to enter the university department of their choice. The democratic nature of this phenomenon is questionable, since, for example, students with access to greater monetary means may also have access to more ‘successful’ (in the National Exams) preparatory techniques.

Acknowledging the importance of such issues in the teaching and learning mathematics, mathematics educators in Greece attempted to outline the topic, stressing that the existence of a parallel educational universe indicates the insufficiencies of the ‘formal’ educational system (Kalavasis & Meimaris, 1994). Notwithstanding the ethical and political issues that the existence of such a parallel universe raises, it is suggested that these educational worlds may be co-ordinated to achieve the educational and social goals.



Furthermore, the Greek educational system is currently moves towards a de-centralised system, consisting of semi-autonomous school units that set their own pedagogical targets. Consequently, this change highlights the importance of identifying of the in-service mathematicians' views about the aforementioned issues, since their more autonomous decision making becomes crucial in the everyday school practices.

### The questionnaire: structure and procedures

For the purposes of this study, we drew upon the three-part questionnaire structure employed in Moutsios-Rentzos and Kalavasis (in press). The first part includes 16 closed items investigating the mathematicians' views about mathematics within the system of all disciplines, focussing on its links with, for example, *everyday life* (usefulness, importance, problem-solving etc), *success* (including general success in life, social recognition, professional success etc), *personological characteristics* (gender, socioeconomic status, talent etc). The second part focuses on their views about mathematics as a course within the system of school courses, in line with our tri-focussed approach ('official regulations' – 'current state' – 'hypothetical decisions'). Ten topics are investigated through thirteen triplets of closed items (both mathematical understanding and success in exams are investigated for three topics), including: *cheating and reliability of assessment; teaching reinforcements and financial difficulty, gender, talent and absenteeism* etc. In Figure 1, a sample triplet of items is presented. Each item consists of a common part (the topic) and a unique part, in line with each of the three foci. Note that the participants' answers are two-faceted: a 'Yes/No' dichotomy indicating their *agreement or disagreement* and a 4-point Likert scale to identify their *degree of agreement or disagreement*. Finally, the third part of the questionnaire consists of questions about the participants' age, education and working experience.

According to your opinion, the official regulations suggest the teaching of mathematics should incorporate more use of ICT to help more students to gain deeper understanding of mathematics?	Y	1	2	3	4
	N	1	2	3	4
Do you think that in reality in schools ICT is used more in teaching mathematics to help more students to gain deeper understanding of mathematics??	Y	1	2	3	4
	N	1	2	3	4
As a mathematician and assuming you had the power, would you promote the teaching mathematics with ICT more to help more students to gain deeper understanding of mathematics?	Y	1	2	3	4
	N	1	2	3	4

Figure 1: A sample of the triplets of items included in the questionnaire.

In this paper, we discuss only the identified differences in the degree of agreement/disagreement with the items. Considering the size of our sample, we focussed only on descriptive techniques. First, we computed the *intensity* (degree of agreement/ disagreement) for each participant's answer on an item. For example, if someone answered 'Yes' and '3' on an item this resulted in an intensity of '+3'. Accordingly, an answer of 'No' and '2' resulted in '-2'. Subsequently, we computed the *mean of the summative intensity* for each item, as the mean of the algebraic sum of all answers in our sample. The 'mean of the summative intensity' was regarded as a measure of the degree of agreement/disagreement per item for the whole sample, thus providing us with an overview of their views. In our discussion, we considered the items the means of which ranked on the top or low 20%, in order to identify the *strongest views* about the topics of interest. Subsequently, we compared and contrasted these views in terms of the two systems ('disciplines' – 'school courses'), of the tri-focussed approach ('official regulations' – 'current state' – 'hypothetical decisions') and of goal-setting ('mathematical understanding' – 'exam success').

### **Sample and current results**

In Greece, after 9 years of obligatory education, the students may choose to attend Lykeio (16-18 years old). Upon successful completion of Lykeio, the students who want to enter university sit the National Exams. In this study, we focus on mathematicians who teach in Lykeio as their teaching practices are immediately linked with the National Exams.

At this point, we present the results from 19 male mathematicians (N=19). Our study will be completed with the investigation of the female sample and we hope to be able to present the entire study in the CIEAEM's 64<sup>th</sup> conference in July.

Focussing on the system of disciplines, the results of our analysis suggested that the Greek in-service male mathematicians, viewed mathematics to be more useful in everyday life and in everyday problem-solving than other disciplines. Furthermore, more than other disciplines, they consider mathematics to promote logical reasoning. On the other hand, they do not think that excelling in mathematics, more than in other disciplines, is linked with gender, socio-economic status or getting positions of power. Finally, it appears that they view that mathematics does not have a weaker relationship with the natural world than other disciplines.

Considering the systems of 'school courses' and our tri-focussed approach, the analysis revealed that the male mathematicians have *only nega-*

*tive* strong views regarding the *official regulations* and the employment of *ICT* in mathematics teaching, when linked with exam success. Nevertheless, they don't hold strong negative views for the same topic with respect to mathematical understanding. Moreover, they feel that official regulations do not promote the provision of teaching reinforcements within the school especially to *female* students or to *mathematically gifted* students, or to students that are usually *absent* from class due to socio-economic reasons. Importantly, they also hold *strong negative* views about these three topics with respect to what they feel that happens *in reality* in schools.

Furthermore, with respect to the mathematicians' *hypothetical decisions*, the mathematicians would promote, on the one hand, the teaching reinforcement within the school of the students who are usually *absent* from class due to socio-economic reasons, while, on the other hand, they would promote the employment of assessment practices that enable the mathematically *talented students to 'stand-out'*. Considering the promotion of mathematical understanding, they would employ more *ICT* in the teaching of mathematics and they would avoid the teaching of '*exam techniques*'. In order to achieve greater success in exams for students facing *financial difficulties*, they would provide them with teaching reinforcement within the school. Finally, the mathematicians would promote the *teaching reinforcement* of the students *within the school*, in terms of achieving greater mathematical understanding and exam success.

In conclusion, we argue that the inter-systemic, tri-focussed approach adopted in this study helped in more validly identifying the views and practices of the Greek in-service mathematicians.

### References

- Atweh, B., Graven, M., Secada, W., & Valero, P. (Eds.). (2011). *Mapping Equity and Quality in Mathematics Education*. Dordrecht: Springer.
- Begg, A. (2003). Curriculum: Developing a systems theory perspective. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.). *Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia*, Melbourne: Deakin University.
- Bertalanffy, L. V. (1968). *General System Theory: Foundations, Development, Applications*. NY: George Braziller.
- Bouvier, F., Boisclair, C., Gagnon, R., Kazadi, C., & Samson, G. (2010). Interdisciplinarité scolaire: perspectives historiques et état des lieux. *Revue de l'Interdisciplinarité Didactique*, 1(1), 3-14.
- Chaviaris, P., Stathopoulou, C., & Gana, E. (2011). *A socio-political analysis of*

- mathematics teaching in the classroom*. Paper presented at CIEAEM 63, Barcelona, Spain. Retrieved January 29, 2012, from <http://www.mathforlive.net/documents/ChaviarisC.StathopoulouE.Gana.pdf>
- Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, 34(2), 137-167.
- English, L. (2008). Introducing complex systems into the mathematics curriculum. *Teaching Children Mathematics*, 15(1), 38-47.
- Kalavasis, F., Kafoussi, S., Skoumpourdi, C., & Tatsis, K. (2010). Interdisciplinarity and complexity (I-C) in mathematics education: A proposal for their systematic implementation and the role of an international scientific community. *Revue de l'Interdisciplinarité Didactique*, 1(1), 31-40.
- Kalavasis, F., & Meimaris, M. (Eds.). (1994). *Θέματα διδακτικής μαθηματικών II* [Topics in mathematics education II]. Rhodes, Greece: University of the Aegean.
- Moutsios-Rentzos, A., & Kalavasis, F. (in press). Σχολείο, κρίση και συγκριτική τοποθέτηση των μαθημάτων στο σχολικό χωροχρόνο: Μια συστημική προσέγγιση 'εν δυνάμει' εκπαιδευτικών στελεχών για τα μαθηματικά [School, crisis and comparative place of school courses in the school time-space: a systemic perspective of prospective school principals about mathematics].
- Powell, A., & Frankenstein, M. (Eds.). (1997) *Ethnomathematics. Challenging eurocentrism in mathematics education*. Albany: State University of New York Press.
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L.D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century* (pp. 383-407). Mahwah, USA: Lawrence Erlbaum.
- Wagner, D., & Herbel-Eisenmann, B. (2011). *Trying to be democratic in mathematics class*. Paper presented at CIEAEM 63, Barcelona, Spain. Retrieved January 29, 2012, from [http://www.mathforlive.net/documents/Wagner\\_HerbelEisenmann.pdf](http://www.mathforlive.net/documents/Wagner_HerbelEisenmann.pdf)
- Wittmann, E. C. (2001). Developing mathematics education in a systemic process. *Educational Studies in Mathematics*, 48(1), 1-20.

## The interrelationships of mathematics and the school unit as viewed by in-service school principals: a comparative study

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**Abstract** In this study, we adopt a systems theory approach, in order to investigate the views that in-service school principals hold about mathematics, both as a discipline within the system of all disciplines and as a course within the system of courses taught in school. Mathematics as a school course was investigated through an intra-systemic tri-focussed approach: ‘official regulations’ – ‘current state’ – ‘hypothetical decisions’. A small scale (N=46) quantitative comparative study was conducted, including in-service school principals from Brazil (N<sub>B</sub>=16) and Greece (N<sub>G</sub>=30). The results of the conducted analyses suggest differentiations in the epistemological views about mathematics between the Greek principals and their Brazilian counterparts, which are also evident in their views about mathematics as a school course.

### Mathematics through a systems theory perspective

The notion of ‘relationships’ seems to be embedded within the very essence of mathematical thinking, which occurs in the relationship amongst different representations of a notion (Duval 2006). In accordance with this perspective, we draw upon a *systems theory* perspective (Bertalanfy, 1975) to describe the complex phenomena concerning mathematics and the school unit. A *system* is a set of interacting and interrelating parts forming an integrated whole, which has a specific purpose (*ibid*). In mathematics education research, systemic ideas have been discussed in relation with, amongst oth-

ers, mathematics teaching and learning (Davis & Simmt, 2003; English, 2007, 2008; Kalavasis, Kafoussi, Skoumpourdi & Tatsis, 2010; Wittmann, 2001) and curricular reform (Begg, 2003, 2005; Bouvier, Boisclair, Gagnon, Kazadi & Samson, 2010; English 2007, 2008).

In a previous study (Moutsios-Rentzos & Kalavasis, in press), we presented a research framework (theoretical and methodological) aiming to investigate the school principal's views about mathematics and their corresponding managerial practices. In that study, we were interested in identifying, comparing and contrasting, the views that the school principals hold about mathematics when considered as an element of two interconnected systems: a) the system of all disciplines, and b) the school system. Furthermore, regarding the school system, we adopted a *tri-focussed* approach: a) How the *official regulations* should be about mathematics?, b) What is the *current state* of school practices regarding mathematics?, and c) What would be the principal's *hypothetical decisions* regarding mathematics assuming they had the power to put them into practice? This perspective enabled a qualitative shift on the type of questions we seek to answer; e.g. instead of asking 'Is mathematics a useful subject?', we ask 'Is mathematics *more* useful than *other* disciplines?'.

In this study, we draw upon this framework to compare and contrast the views and practices concerning mathematics of the school principals of two countries (Brazil and Greece). *The purpose of this study is to investigate whether or not sociocultural, economic and structural differences (the model of the emerging economy society vs. the European model) appear somehow in the identified views and practices of the in-service principals, thus offering deeper insight on the inter-cultural and intra-cultural aspects of mathematics.*

### **Setting the scene**

Recently in the State of São Paulo (Brazil), there has been a move from a relatively decentralised system for Basic Education (6-17 years old) based on the National Curriculum Parameters (NCP) to a centralised system (the Official Curriculum of the State of Sao Paulo), including, for example, the everyday use of "Teacher Notebook" and "Student Notebook" provided by the central secretary of education of this State (SEESP). This movement radically differs from the situation in Greece, where the educational system is currently transformed from an absolutely centralised system towards a decentralised system consisting of semi-autonomous school units that set their own pedagogical targets adapted according to the conditions within which each unit is defined. Thus, it appears that both educational systems

are in a dynamic state, in the process of reaching equilibrium. Moreover, in both countries the value of mathematics is greatly appreciated as shown by the hours of teaching dedicated in the curricula. Nevertheless, both countries appear to perform poorly in the PISA assessment.

Considering the educational and the socioeconomical commonalities and differences of both countries, we argue that a ‘snapshot’ of the school principals’ views about mathematics in such a dynamic state will help in revealing aspects of the phenomenon that are inter-cultural and aspects that linked to the special characteristics of each country.

### The questionnaire

For the purposes of this study, we drew upon the three-part questionnaire employed in Moutsios-Rentzos and Kalavasis (in press). The first part includes twelve items (11 closed and 1 open) investigating the principals’ epistemological views about mathematics within the system of all disciplines, focussing on: *usefulness, importance, reasoning, epistemology and truth*. The second part focuses on the principal’s views about mathematics as a course within the system of school courses, in line with our tri-focussed approach (‘official regulations’ – ‘current state’ – ‘hypothetical decisions’). Thirteen topics are investigated through 13 triplets of closed items. Amongst the topics investigated are *mathematics and school time, mathematics and budget, mathematics and class arrangement, mathematics and didactics, mathematics and evaluation* etc. The last part of the questionnaire consists of questions about the participants’ age, education and working experience. In Figure 1, a sample triplet is presented. Note that each question of the triplet consists of a common part (the topic) and a unique introductory part, in accordance with each of the three foci. Notice also that the participants’ answers are two-faceted: a ‘Yes/No’ dichotomy indicating the participants’ *agreement or disagreement* and a 4-point Likert scale to identify the participants’ *degree of agreement or disagreement*.

According to your opinion, should the official regulations allocate a bigger part of the budget in materials for the teaching of mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4
Do you think that in reality in schools a bigger part of the budget is spent in materials for teaching mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4
As a school principal and assuming you had the power, would you allocate a bigger part of the budget in materials for teaching mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4

Figure 1: A sample of the triplets of items included in the questionnaire.

### Sample and procedures

46 in-service principals participated in the study (N=46). Sixteen school principals working in Basic Education in an economically developed region of a large Brazilian city were included in the study, representing the 40% of the schools of this region. Basic Education in Brazil is divided into “Ensino Fundamental I” (6-10 years old), “Ensino Fundamental II” (11-14 years old) and “Ensino Médio” (15-17 years old). In these classes, the students are taught mathematics 5 hours per week. In Greece, for these year groups, education is divided in Dimotiko (6-12), Gymnasio (13-15) and Lykeio (16-18). In Dimotiko and Gymnasio the students are taught mathematics 4 hours per week, whereas in the first two grades of Lykeio mathematics is taught for 4 to 5 hours per week. In the third grade of Lykeio there is a range from 2 to 7 hours depending on the specialisation chosen by the students. Considering that in Brazil a school principal may work in a school offering more than one level of Basic Education (which is not possible in Greece), we included in the study 30 in-service principals working in a large Greek city in schools of all three levels.

### Results and concluding remarks

The quantitative analysis was conducted with SPSS 17 (SPSS, Inc., Chicago, IL). Though the comparisons were conducted at both levels of measurement (agreement/disagreement *and* degree of agreement/disagreement), in this paper we discuss only the identified differences in the degree of agreement/disagreement with the items. For this purpose, the Mann-Whitney U test was employed (non-parametric equivalent of the independent samples t-test).

The results of our analysis suggested that the Greek in-service principals statistically more than their Brazilian colleagues, viewed mathematics as being more useful than other disciplines ( $U = 141$ ,  $p = 0.017$ ,  $r = -0.35$ ) and that the mathematical results are the least disputed ( $U = 156.5$ ,  $p = 0.037$ ,  $r = -0.30$ ). The analysis of the comparison of the views that the principals hold about mathematics as a course according to our tri-focussed approach (‘official regulations’ – ‘current state’ – ‘hypothetical decisions’) suggested that the only statistically significant difference between the two countries that was evident in all three foci, was the Greek’s view that mathematics are ( $U = 26$ ,  $p < 0.001$ ,  $r = -0.72$ ), should be ( $U = 53.5$ ,  $p < 0.001$ ,  $r = -0.62$ ) and preferably would be taught ( $U = 68.5$ ,  $p < 0.001$ ,  $r = -0.52$ ) in the morning more often than other courses.

When considering the *official regulations* the Brazilian principals feel stronger that the educational system should promote mathematics to be



taught jointly with other courses ( $U = 119.5$ ,  $p = 0.008$ ,  $r = -0.39$ ) and that more training programmes should be offered about the teaching of mathematics than about the teaching of other courses ( $U = 89.5$ ,  $p = 0.001$ ,  $r = -0.49$ ). In contrast, they hold weaker views with respect to linking the school unit success with mathematical success ( $U = 133$ ,  $p = 0.021$ ,  $r = -0.34$ ).

Moreover, the Greek principals feel stronger that, *in reality*, there is more cooperation with the families regarding mathematics than regarding other courses ( $U = 115.5$ ,  $p = 0.006$ ,  $r = -0.40$ ). On the other hand, the Brazilian principals, more than their Greek colleagues, think that *in reality* in schools more class hours are allocated for the teaching of mathematics in relation with other courses ( $U = 132$ ,  $p = 0.045$ ,  $r = -0.29$ ), which ‘justifies’ their *hypothetical decisions* about the same topic, since, if they had the power, they would be less eager than the Greek principals to allocate more school hours for teaching mathematics ( $U = 142.5$ ,  $p = 0.044$ ,  $r = -0.30$ ).

Furthermore, the principal’s *hypothetical decisions* significantly differ in that the Brazilian principals appeared to have stronger views about changing the desk arrangements for the mathematics hour ( $U = 128.5$ ,  $p = 0.017$ ,  $r = -0.35$ ) and about encouraging the teaching of mathematics jointly with other courses ( $U = 74$ ,  $p < 0.001$ ,  $r = -0.56$ ).

Consequently, the Greek principals appear to differ from the Brazilian principals in their perspective about mathematics, as they highlight the usefulness and undisputed truth of mathematics. Importantly, this difference of epistemological perspective is also reflected in their school managerial perspective, including the Greek principals’ stronger view that the official regulation should link school success with mathematical success and the Brazilian principals’ hypothetical decision making about jointly teaching mathematics with other courses and rearranging the desks when teaching mathematics. Finally, the Brazilian principals think that mathematics is taught more class hours than other courses and that they would reduce those hours if they had the power. Consequently, it can be argued that the inter-systemic, tri-focussed perspective adopted in this study helped in more efficiently presenting and in more validly comparing the two populations, thus revealing aspects of their views, of their practices and of the links between them.

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### References

- Begg, A. (2003). Curriculum: Developing a systems theory perspective. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.). *Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia*, Melbourne: Deakin University.
- Begg, A. (2005). Statistics Curriculum and Development: New Ways of Working. In Burrill, G., & Camden, M. (Eds.). *Curricular Development in Statistics Education: International Association for Statistical Education 2004 Roundtable*. Voorburg: ISI.
- Bertalanffy, L. V. (1968). *General System Theory: Foundations, Development, Applications*. NY: George Braziller.
- Bouvier, F., Boisclair, C., Gagnon, R., Kazadi, C., & Samson, G. (2010). Interdisciplinarité scolaire: perspectives historiques et état des lieux. *Revue de l'Interdisciplinarité Didactique*, 1(1), 3-14.
- Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, 34(2), 137-167.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103-131.
- English, L. (2007). Complex systems in the elementary and middle school curriculum: A focus on the modeling. *The Montana Mathematics Enthusiast, Monograph 3*, 139-156.
- English, L. (2008). Introducing complex systems into the mathematics curriculum. *Teaching Children Mathematics*, 15(1), 38-47.
- Kalavasis, F., Kafoussi, S., Skoumpourdi, C., & Tatsis, K. (2010). Interdisciplinarity and complexity (I-C) in mathematics education: A proposal for their systematic implementation and the role of an international scientific community. *Revue de l'Interdisciplinarité Didactique*, 1(1), 31-40.
- Moutsios-Rentzos, A. & Kalavasis, F. (in press). *Σχολείο, κρίση και συγκριτική τοποθέτηση των μαθημάτων στο σχολικό χωροχρόνο: Μια συστημική προσέγγιση 'εν δυνάμει' εκπαιδευτικών στελεχών για τα μαθηματικά* [School, crisis and comparative place of school courses in the school time-space: a systemic approach of perspective school principals about mathematics].
- Wittmann, E. C. (2001). Developing mathematics education in a systemic process. *Educational Studies in Mathematics*, 48(1), 1-20.

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## **Family Math: Doing Mathematics to Increase the Democratic Participation in the Learning Process**

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### **Abstract**

*In this brief paper I draw on a large tradition of research looking on the impact of families and community members on the learning process (focusing on the Mathematics realm). The story of Marisol is presented to the audience for discussion. She is a working class mother from a neighborhood in Barcelona. She devotes her life to support her children to perform successfully in the school. This paper provides some insights to think on how parents and other relative members can participate democratically within their children' learning process of mathematics.*

There is a broad agreement on the idea that family engagement in education has a positive impact on children' performance and achievement (Fan and Chen, 2001; Jeynes, 2003; Smit, Driessen, Sluiter, & Slegers, 2007). Cai and colleagues (2003) found that parents' involvement is a statistically significant predictor. However, not all types of parent engagement have the same encouraging effect. According to scientific research, democratic ways of involvement generates more efficiency than any other type of involvement (INCLUD-ED, 2009; Díez-Palomar, & Kanés, 2012). Students with the most supportive parents demonstrate higher mathematics achievement and more positive attitudes towards mathematics than students with the least supportive parents (Cai, Moyer, & Wang, 1999). In fact, according to the survey piloted by Cai and colleagues almost twenty years ago, parents that are defined as "motivators", "resourceful", "providers" and "monitor" were founded to be better predictors of mathematics achievement than the case of parents acting as "mathematics content advisors" or "mathematics learning counselors."

The impact of the discussion about democracy in the field of

mathematics teaching and learning emerges from number of practices in very different sites and situations (Goñi, et al., 2010). Democracy is usually quoted as a reason for learning mathematics, in order to be able to act critically as a citizen (Skovsmose, & Valero, 2002; Goñi, et al. 2010, Niss, 1994). In the realm of families' engagement on Mathematics, we can refer to the idea of "democracy" in order to look for explanations of the increasing trend all over the World of families looking over the curriculum to find clues in order to help their children with this subject. Previous research has pointed out that many families (and other caregivers) find substantial barriers to their efforts in helping their children to learn mathematics out of the classroom. Changes in the curriculum of mathematics (Jackson, & Epstein, 2006), second language learners (Baker, Street, & Tomlin, 2006), cultural gap (especially for ethnic minorities and migrants) (Abreu, Cline, & Shamsi, 2002), are some of the main constraints reported by the research work in this field. When parents and other relatives or caregivers try to answer their children's' questions about particular mathematical ideas, many of them felt overwhelmed for the situation: current curriculum looks different to them, teachers' strategies to teach mathematics are not the same as they remember from the ones used by their teachers years ago, and children do not appreciate different procedures to solve the activities in the homework, because "it is not what the teacher says."

According to the CIEAEM 64 discussion paper, "globalization determines in a new way the notion of democracy in society, raising questions about the complexity of national political systems and their relationship with global, economic forces, challenging local assumptions about culture, identity, and the importance of specific mathematical skills for a given curriculum." This paper is linked to the research topic 2, which is "*Democracy in mathematics classroom practices*". Drawing on the idea of "democratic access" to mathematical ideas (Skovsmose, & Valero, 2002), in this paper we discuss up to what point is this true. What are the boundaries framing this democratic access? What are the systemic constraints? We explore these questions not from theoretical lenses, but in real situations: workshops of mathematics addressed to families.

This paper emerges from a research project funded by the European Commission: FAMA *Family Math for Adult Learners*. Drawing on a critical communicative methodological approach, we discuss what are the main constraints of adult learners (family members) regarding the opportunity to participate in their children's mathematics learning, in a democratic way.

According to the evidences emerging from research, one of the main barriers hindering families' engagement in mathematics is their lack of knowledge regarding this subject. Many teachers use to invoke this argument in order to do not allow parents or other caregivers to teach mathematics to the children. According to these persons, teaching is only a teachers' responsibility, while families must to focus on making sure that children attend the school. FAMA provides many evidence demonstrating that improving opportunities for family members to learn (refresh) mathematics has a positive impact both in adults' self-confidence and sense of empowering, and children's performance. In cases where the educational community operates by integrating all its members in the same learning project, on an equal basis, results achieved are much better than when initiatives emerge from actions imposed by the "school" authority (Díez-Palomar, Gatt, & Racionero, 2011).

In this paper we analyze the case of Marisol and her daughter. She is a 47 year-old woman. She lives in a working class neighborhood, in Barcelona (Spain). Originally that area was a rural environment. In the 19<sup>th</sup> and 20<sup>th</sup> Centuries, this side of Barcelona experienced a process of heavy industrialization. Peasants also developed one of the main farm markets of the city at that time. People living in this area were either industrial workers or peasants. During the first decade of the 21<sup>st</sup> Century, this neighborhood was a destination for many immigrant people coming especially from Eastern Europe and Mid-Asia (Azerbaijan, Turkmenistan, etc.). Also people from North Africa settled in this area. Marisol is a Catalan" mother (with Spanish origin). She is middle class, and very active within the group of families involved in the school. She is one of the family leaders of her community.

When Marisol was a child, she went to school, and then as she grew up, she took a course in VET. According to her opinion, she is "working class." She and her daughter participated in a workshop of mathematics for families for three years. Marisol was very reluctant against teachers and the school at the beginning. In one of the interviews that we conducted during the research process, she states: "I have no idea. I was coming nicely to talk with him [the teacher] and he told me... I do not now, perhaps he was angry that day, I have no idea, he told me if I was questioning his methodology of work, and I was telling him that I was not questioning anything, I was just trying to share with him my viewpoints drawing on what I was observing in my daughter, who was not understanding Mathematics, and she used to get good grades, and now she was getting bad scores, and... so I would like... to

share... and he was very angry on me and he told me “look, no... I’m going to another meeting”, and he left the discussing like that, and I was very upset, I don’t know.” In fact, her daughter was having serious problems with mathematics. Marisol was concerned about that situation, thus she was one of the first participants in the workshops series for parents.

One of the main difficulties for Marisol regarding mathematics was her lack of memories of some of the key-concepts that her daughter was starting to learn in the school. When she begun attending the workshops, she felt very disoriented. “And I [Marisol] say “honey, when I attended for the first time [the workshop] I was very confused because there were 30 years without using equations” and I said “Ah Jaime [the facilitator] this is too difficult for me! I do not remember it!” Marisol was complaining several times about her lack of memories. But not only memories; Marisol was also concerned because the new curriculum in mathematics. She was aware that there were some differences between what she learnt at the school, and what her daughter (and son) were learning at that time, in the high school. “And of course, because I was using another way [to solve equations] and now she says that she adds one here, and take away another one in the other side [of the equal sign].” I took four or five sessions before Marisol started to felt comfortable with the “new” curriculum used by her daughter. She learnt all these concepts and strategies within the workshop for families. The impact of attending such activity was very positive for her. According to her own opinion, “my daughter loves mathematics a lot; she is very fast and when I taught her how to think [mathematically] she improved her grades. Then I felt blocked and in the second year of ESO [secondary education] with the equations, I felt lost... until Jaime came... and I said: My goodness, you fall from Heaven! Thanks good!!! So she [the daughter] improved again [her scores].” Marisol learnt how to teach equations to her daughter; hence she was able again to support her with her mathematics. Three years later, her daughter was applying to a Polytechnic university in Catalonia. Their story is an example of how democratic actions with families and other caregivers could have positive potential encouraging students to move further and reach better scores in mathematics.

### References

- Díez-Palomar, J., Gatt, S., & Racionero, S. (2011). Placing Immigrant and Minority Family and Community Members at the School’s Centre: the role of community participation. *European Journal of Education*, 46(2), 184–196.

- Díez-Palomar, J., Kanés, C. (Eds.) (2012). *Family and community in and out of the classroom: Ways to improve mathematics' achievement*. Bellaterra: UAB Press.
- Fan, X., & Chen, M. (2001). Parental involvement and students' academic achievement: A meta-analysis. *Educational Psychology Review*, 13(1), 1-22.
- Goñi, J., et al. (2010). *Educación matemática y ciudadanía*. Barcelona: Graó.
- INCLUD-ED Consortium. (2009). *Actions for success in schools in Europe*. Brussels: European Commission.
- Jeynes, W. (2003). A meta-analysis. The effects of parental involvement on minority children's academic achievement. *Education and Urban Society*, 35(29), 202-218.
- Niss, M. (1994). Mathematics in society. In B. Biehler, R.W., Scholz, R. Sträber, B. Winkelmann (Eds.) *Didactics of Mathematics as a Scientific Discipline* (pp. 367-378). Dordrecht: Kluwer Academic Publishers.
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (Ed.), *International Research in Mathematics Education* (pp. 383-408). LEA.
- Smit, F., Driessen, G., Sluiter, R. & Slegers, P. (2007). Types of parents and school strategies aimed at the creation of effective partnerships. *International Journal about Parents in Educaiton*, 1(0), 45-52.
- Cai, J., Moyer, J.C., and Wang, N. (1999). Parental roles in students' learning of mathematics: An exploratory study. *Research in Middle Level Education Quarterly*, 22(3), 1-18.
- Jackson, K., and Epstein, Y. (2006). *How parents are framed in reform-oriented elementary mathematics curricular materials*. Paper presented at the 28<sup>th</sup> Annual Meeting of North American Chapter of the International Group for the Psychology of Mathematics Education, Merida, Mexico.
- Cai, J. (2003). Investigating parental roles in students' learning of mathematics from a cross-national perspective. *Mathematics Education Research Journal*, 15(2), 87-106.
- Baker, D., Street, B.V., and Tomlin, A. (2006). Navigating schooled numeracies: Explanations for low achievement in mathematics of UK children from low SES background. *Mathematical Thinking and Learning*, 8(3), 287-307.
- Abreu, G.d., Cline, T., and Shamsi, T. (2002). Exploring ways parents participate in their children's school mathematical learning: case studies in multiethnic primary schools. In G. de Abreu, A.J. Bishop and N.C. Presmeg (Eds.) *Transitions between contexts of mathematical practices* (pp. 123-147). Dordrecht: Kluwer.

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## **Lifelong mathematics education in Greece: an investigation on the who, what, why & how**

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**Abstract** In this study, we discuss mathematics from the perspective of lifelong education. We synthesise theories from adult learning and mathematics education to suggest a theoretical framework that could be employed to research adult mathematics education. A mixed-methods study is suggested to investigate the characteristics of the adult learners' population in Greece and their motivation, as well as the mathematical content and pedagogical practices of the Greek adult mathematics education programmes.

### **Lifelong education and mathematics: a matter of democracy**

Lifelong education, though it depends on and affects all existing educational providers, extends “beyond the formal educational providers and incorporates all agencies, groups and individuals involved in any kind of learning activity” (Tight, 2002, p. 40), since it is assumed that individuals are, or can become, self-directed, realising the value of lifelong education. From the early stages of their life, we (humans) are required to deal with various situations that are (informal) mathematical activities (Clements, 2001). In our adult life we encounter rich experiences and activities that require our informal (at least) mathematics reasoning and learning within a variety of non-school settings, including the home environment, professional practices and cultural activities. Considering at the same time the rapid technological advances, which draw upon predominantly mathematical ideas, it is argued that neither mathematics lifelong education should be limited in school mathematics. Thus, mathematics educators have looked in the adults learning mathematics (Coben, O’Donoghue &



Fitzsimons, 2002), acknowledging that mathematics education may offer to marginalized groups the necessary tools to become critical citizens and to understand the complexities of national political systems and their relationships with global, economic forces. Skovsmose and Valero (2002) discuss the notion of *democratic access*, referring to “the possibility of entering a kind of mathematics education that contributes to the consolidation of democratic social relations” (p. 397). The citizens of the modern society are expected to be able to reason with specific mathematical ideas that are embedded within the fabric of our society, since the “world of the twenty-first century is a world awash in numbers” (Steen 2001, p.1), radically affecting our social understanding. Nevertheless, the nature and content of these ‘powerful ideas’ (Skovsmose & Valero, 2002) is dependent on which aspect (logical, psychological etc) the researchers choose to focus. Skovsmose (1998) coined the notion of *mathemacy* to identify mathematical ideas and competencies that are crucial “for acting in the world structured by mathematics” (p. 200), directly linking mathemacy with democracy.

In this paper, we address these issues (clearly linked with the scope of this conference and in specific with the issue of “Democracy in mathematics curriculum: How does school mathematics contribute to critical thinking and decision-making in the society?”), situated in adult mathematics education, focusing on identifying the current reality in Greece and linking it with existing theories: *What is the profile of the Greek adult learners? Why do they learn mathematics? What is the corpus of mathematical knowledge that they learn? What are the current pedagogical practices in Greek adult mathematics education?*

### **Who, why, what and how in adult mathematics education**

Who is the adult learner? Over 40 years ago, Knowles (1968) proposed “a new label and a new technology” (p. 351) of adult learning to render it distinguishable from pre-adult learning; *andragogy* contrasted *pedagogy*. Knowles’ andragogical model was based on five assumptions (self-directing beings, experience, readiness to learn, problem-centeredness, internally rather than externally motivated; Knowles & Associates, 1984) and despite the fact that it provided the adult educators with a “badge of identity” (Brookfield, 1984, p. 43), it also provoked a lot of controversy and philosophical debate. Cross (1981) argued that adult learners, given the variety of learning experiences are a diverse population, who are either externally or internally motivated to learn. Furthermore, she argued that adult learners possibly undergo physical aspects of aging (i.e. hearing,

vision, energy, and health) that affect their learning and that cognitive, personality, and socioeconomic factors may affect their ability to learn.

In contrast with these models, McClusky's theory of margin underlined both personal and situational characteristics, while Jarvis' learning model distinguishes among learning and non-learning responses (Merriam & Caffarella, 1999). Moreover, Mezirow's (1991) model of transformative learning stipulates experience as a starting point and as its content for reflection, employs a critically reflective stance as a necessary condition for transformation and regards the whole process as change that is enhancing and developmental (cf. Brookfield, 1987).

Why do adult learners engage with the study of mathematics? According to Habermas, scientific domains are constituted through different knowledge-guiding interests and mathematics are based on a technical interest. Nevertheless, viewing mathematics as a language, Barker's (2004) speech act theory of language raises the question: What can be done by means of mathematics? Mathematics is also a source for decision making and action, thus leading to power negotiating within a democratic context (Skovsmose, 1998).

In a case study concerning the Greek adult learners' profile, Sofos (2008) attempted to delineate the profile and the needs of adult students (studying in state institutes in the island of Rhodes) according to adult learning models (including the models of Knowles, Cross, McClusky etc). Sofos adopted a mixed methods approach employing grounded theory techniques to construct a questionnaire administered to 314 adult students (N = 314). He found that the Greek adult students appeared to be mainly self directed and to be motivated by external motives. Their educational choices appeared to be directly linked to their professional expectations and they seemed to pursue proficiency in the area of their specialisation. Moreover, they seemed to be aware of the demands this educational activity it has on them as well as of their abilities to cope with it.

Following these, we consider the question: What is the mathematical content that is appropriate for adult learners and their needs as members of a democratic society? There is a plethora of mathematics, in terms of both its content (numeracy, statistics, probability etc) and the context with which it is linked (school mathematics, everyday mathematics, workplace mathematics etc). Hence, it is crucial to identify the mathematical content of the adult learning curriculum (Coben, O'Donoghue & Fitzsimons, 2002).

The increasing orientation of adult education and lifelong learning in general as important contributors in "the global competition between nations

and companies” (Illeris, 2003, p. 396) has led to the realization that an all inclusive approach to learning should “avoid any separation between learning, personal development, socialization, qualification and the like by regarding all such processes as types of learning when viewed from different angles or positions” (ibid). Illeris (2003) is most interested in the learning process itself, identifying three dimensions always present and active in a learning activity: *cognition* (knowledge and skills), *emotion* (feelings and motivations) and *environment* (society). Cognition and emotion interact in the acquisition of knowledge and skills, while the environment refers to the “external interaction, such as participation, communication, and cooperation” (Illeris, 2004, p. 83), facilitating “the sociality of the learner” (ibid).

Though this model bears some analogy with Knowles’s (1968) andragogical model (for example, cognition could be related with the assumptions of *experience* and adult learners’ *readiness to learn*, and the emotional dimension with the assumption of the prepotency of *internal motivation over the external* one and the individual’s degree of *self-direction*), the andragogical approach seems to lack the environment (societal) dimension. Moreover, Illeris’ model could interpret resistance to or rejection of learning along with transformational learning, because “very special and demanding situations, often with a crisis-like character, can lead to deep and comprehensive transformative learning processes that include simultaneous changes in all three learning dimensions and have to do with the very identity of the learner” (Illeris, 2002, p. 229).

#### **Method and concluding remarks**

In this study, we concentrate on the adults who choose to follow an adult learning programme, amongst those offered by the Greek educational system; namely, Adult Education Centres’ classes and Second Chance Schools’ classes. We intend to gather data from a whole region of Greece (Dodecanese), anticipating a sample of more than 150 people.

We adopt a combination of quantitative and qualitative techniques, depending on the research question. First, with respect to the Greek adult learners’ profile and their reasons why they choose to follow such a programme, we shall employ the questionnaire Sofos (2008; see previous section) developed for these purposes. Second and third, regarding the corpus of mathematical knowledge of adult learning *and* the way that this knowledge is taught in adult education programmes, we shall examine the existing textbooks and curricula regarding mathematics, as well as the

preferred pedagogical practices as suggested by the policy makers. Drawing upon qualitative techniques and an initial coding scheme based on theory and official regulations, we intend to identify patterns and/or trends, which will be compared with empirical evidence, as well as with Greek and European policies, in order to identify ‘discrepancies’, thus gaining further understanding about which mathematical ideas are considered ‘powerful’ enough to be incorporated in the Greek lifelong learning educational system. In conclusion, we posit that this study will provide a comprehensive overview of the current adult educational system in Greece, in terms of the mathematical content and pedagogy, as well as the characteristics of the Greek adult learners’ population. Therefore, we argue that it will be possible to juxtapose this information with existing empirical and theoretical evidence, thus allowing for subsequent pedagogical proposals in terms of both mathematical content and teaching practices.

### References

- Barker, S. (2004). *Renewing Meaning: A Speech-Act Theoretic Approach*. Oxford: Oxford University Press.
- Brookfield, S. (1984). Self Directed Adult Learning: A Critical Paradigm. *Adult Education Quarterly*. 35(2), 59-71.
- Brookfield, S. (1987). *Developing Critical Thinkers*. San Francisco: Jossey-Bass.
- Clements, D.H. (2001). Mathematics in the Preschool. *Teaching Children Mathematics*. 7, 270-275.
- Coben D., O’Donoghue J., & Fitzsimons G. E. (Eds.) (2002). *Perspective on adults learning mathematics: research and practice*. Dordrecht: Kluwer.
- Cross, P. (1981). *Adults as Learners: Increasing Participation and Facilitating Learning*. San Francisco: Jossey-Bass.
- Illeris, K. (2002). *Three Dimensions of learning*. Roskilde, Denmark: Roskilde University Press/Leicester, UK: N IACE
- Illeris, K. (2003). Towards a contemporary and comprehensive theory of learning. *International journal of lifelong education*. 22(4), 396-406.
- Illeris, K. (2004). Transformative learning in the perspective of a comprehensive learning theory. *Journal of Transformative Education*. 2(2). 79-89.
- Knowles, M. (1968). Andragogy not pedagogy. *Adult Leadership*. 16(10), 350-352.

- Knowles, M. & Associates. (1984). *Andragogy in Action: Applying Modern Principles of Adult Learning*. San Francisco: Jossey-Bass
- Merriam, B. S., & Caffarella, R. S. (1999). *Learning in Adulthood: A Comprehensive Guide*. San Francisco: Jossey-Bass Publishers.
- Mezirow, J. (1991). *Transformative Dimensions of Adult Learning*. San Francisco: Jossey-Bass.
- Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberative interaction. *ZDM*. 30(6), p. 195-203.
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L.D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century* (pp. 383-407). Mahwah, USA: Lawrence Erlbaum.
- Sofos, E. (2008). *Adult Education Models and Greek Contemporary Education Reality*. Unpublished Doctoral Dissertation. Rhodes: University of the Aegean.
- Steen, L. A. (2001). Democratic access to powerful mathematical ideas. In L.A. Steen (Ed.), *Mathematics and democracy The Case for Quantitative Literacy* (pp. 1-22). USA: NCED.
- Tight, M. (2002) *Key Concepts in Adult Education and Training*. London: Routledge.

## **WHAT FUTURE MATHEMATICS TEACHERS UNDERSTAND AS DEMOCRATICAL VALUES**

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### **Abstract**

This presentation provides elements to reflect about the characteristics of a pre-service teacher education program for democracy through mathematics training practices. Our first step is to determine prospective teachers' initial ideas. To do this we show the results of an empirical study with two groups of future teachers of mathematics from different cultural contexts (Colombia - Spain). Responses are analyzed and compared for both groups.

### **1. Aims and theoretical framework.**

In this paper, we discuss one of the questions that Professor D'Ambrosio formulated on the possibilities and responsibilities of teacher's mathematical knowledge to build a democratic education through mathematical practices (D'Ambrosio, 2005). We know that teachers have to be sensitive not only about the mathematical content, but also about the development of democratic practices. This is a new professional challenge for which there are not usually well prepared. And in that sense, our specific objective is: to note the initial conceptions of future teachers about building democratic citizenship through mathematics, as a first step in designing a training cycle. We start by analyzing the role that teachers give to some different tasks as part of didactical analysis.

Democracy in mathematics classroom practices designates the possibility of entering a kind of mathematics education that contributes to the consolidation of democratic social relations (Skovsmose & Valero, 2002). We understand that education for democratic citizenship should enable interaction in the class that supports dialogue and negotiation of

meaning through deliberative interaction, based on research attitudes that teachers need to know to incorporate. It's also necessary to include recognizing the capacity building and task analysis to identify social problems of democratic processes that are answered from mathematics. It also involves developing a professional identity that interprets mathematical activity as transformative intercultural inclusive and open for training in scientific autonomy and creativity (Vanegas y Giménez 2010). A few number of researches analyze the development of critical position (from initial training courses as we did. Our study is in line with Adler analysis and confirms the sense of seeking professional content elements that support training for democracy and citizenship.

We interpret the initial conceptions of students as decision-making from external situations (as Hewson y Hewson 1989, did in experimental sciences) and assessments from the complexity of the phenomenon. The actions of teachers in the classroom are as complex as any human actions. They are not only determined by their beliefs and knowledge about mathematics (Skovsmose y Valero, 2002) but are also a way of interpreting substrate professional identity

## **2. Methodology for empirical study**

For the study design a specific structured questionnaire is discussed in two different groups of 24 future teachers in mathematics from 3rd semester in the University District "Francisco José de Caldas" of Bogota and an initial group of 26 students from the University of Barcelona in Spain. In a first phase develops the open questionnaire to these 50 students with the aim of provoking positions indirectly. In a second phase was carried interviews with a group of 5 students in each of the universities. And later deploy with 100 more students in various training groups in Spain.

We have begun to introduce future teachers three different activities: (a) the first part of a painting by Paul Klee, with the addition of the dialogue of the students in the construction of mathematical ideas about geometric figures. The second is a problem associated with water scarcity is asked why it is important to assess the savings of this resource. The third is a short approach to Hooke's Law.

Given these tasks, we ask that they are positioned according to power more or less democratic citizenship education. We have asked us to justify your answer. Then we ask: in what way these activities are considered training students as democratic citizens? For the analysis of the positions and beliefs, we have decided to recognize elements of the complex

phenomenon explicit discursive positions on the role of mathematical tasks that are considered to promote democracy.

It's discussed in this presentation a priori characterization of the positions on citizenship and democracy based on two axes: (1) Democratic participation constructive and responsible use of social tools and (2) assuming a critical perspective through a mathematical point of view, Each of them are associated with a set of indicators and sub-categories that emerge from a priori theoretical developments (Vanegas y Giménez, 2010). The proposed work, and other professional development results are discussed in the wider project is not explicit here.

### 3. Discussion.

We recognize the results based on the aforementioned axes for analysis. In both groups, 86% said that, both the task proposed and how to manage the classroom, are important to be considered for introducing citizenship, and just 14% tell us that the task is the most important for introducing citizenship. In the case of UB, people considers group work, dialogue, etc as fundamental aspects, but the behavioral-attitudinal arguments as "*respect the turns of speaking, listening...*" (St 2). In UD people speaks about the use of dialogue, but more in the sense of awareness. At UB, including indicators referred to appears the so-called "treatment of coexistence" and "identification of collective entities as openness" From dialogical approach to constructive and responsible participation we can say that there is no allusions in the UB. At UD, 16% come statements like, "*acculturating the student also proposing to locate sources basins in the country in which they live*" (St 34) pertaining to the usefulness of mathematics in the management of resources. In both groups, teachers identified that the future is clearly positioned in citizenship which are tasks that are associated with sustainability issues.

Other indicators of democratic identity are harder to find. In fact many students are arguments of type "*in the task ... it works to agree and address the controversy*" (St 3). In rare cases, if references to respect as democratic identity with phrases like "*because it is an activity that presents art as a means to observe discuss and share different points of view so that students will come consensus on the ways ...*" (St 7)

What the Columbian students do not discuss in almost all cases the value of dialogical positions in the work of art, such as shared meaning which would stand in a position of questioning social construction. An idiosyncratic example says "*... the different points of view of each student shall be provided in a participatory manner ....*" (St 13) In some cases



referred to social impact to train students in the sense of responsibility that did not appear in any student at UB.

Although some prospective teachers Colombians value their concern for the elements of dialogue themselves as social knowledge building, not directly associated with the social construction of knowledge and promotion of democratic citizenship. Neither associated with a didactic analysis of the situation that requires the teacher has a record (real or experiential) of dialogue to see if it supports this construction of critical knowledge. However if you allude to take collective intentionality group aimed at improvements seen in phrases like "*in the mathematical training the students learn that not all the same, and that these differences are good. This leads to ... learn to work together.*" (St 25). We found some references to consistency and commitment.

#### **4. Conclusions and perspectives.**

From the data results, we found that almost 80% of prospective teachers look professional action in citizenship training, focused on the task at hand with their questions and not so much in developing the elements of it. In the case of Spanish students the arguments are based on the value of dialogue and participation in class as a social construction, as well as educational components, while Colombians are more reflections on the mathematical aspects.

From the results, we also characterized three types of future teachers: committed, ecological, and conformist no space to explain here in detail. From the results analyzed, only found two cases in each group (UD and UB) in which there is evidence of committed teachers. They were characterized because they are almost the only ones showing indicators corresponding to the a priori categories that we considered to be facing more rigorous public relations / mathematics. The study shows 50% of the UD group as ecologist Professor and 31% in UB. An important part of future teachers of UB were considered as shown traditionalist.

We conjecture that can not simply be due to cultural differences, but rather due to the fact that they have already received one year of specific training. The study showed a short amount of people in a critical tendency and only half of the students are situated in ecological and open position. This is consistent with other studies that showed on the social / mathematical, as soon as hard to beat the professors.

It has been shown that both influences and cultural traditions that Colombian students have had specific training in mathematics and its teaching. The results allow us to influence the construction of educational

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knowledge as further work is didactic analysis processes based study to recognize the potential for developing civic competence, because it is not as spontaneously as we think.

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**References.**

- Christensen, O. R., Stentoft, D., & Valero, P. (2008b). Power Distribution in the Network of Mathematics Education Practices. In K. Nolan & E. De Freitas (Eds.), *Opening the Research Text: Critical Insights and Interventions into Mathematics Education* (pp. 131-146). New York: Springer
- D'Ambrosio, U (2007) Peace, social justice and Ethnomathematics. *TMME Monograph1*, p.25 Pontificia Universidade Católica de São Paulo, Brazil..
- Hewson, P. W. & Hewson, M. G. Analysis and use of a task for identifying conceptions teaching science. *Journal of Education for Teaching*. 15. 191-209
- Skovmose, O (1998) Linking Mathematics Education and Democracy: Citizenship, Mathematical Archaeology, Mathemacy and Deliberative Interaction. *ZDM* 98/6:195-203.
- Skovmose, O., & Valero, P. (2002). Democratic access to powerful mathematics in a democratic country. In L. English (Ed.), *Handbook of international research in mathematics education* pp. 383-408. Mahwah, NJ: Lawrence Erlbaum Assoc.
- Vanegas, Y.; Giménez, J. (2010) Aprender a enseñar matemáticas y educar en ciudadanía. *Educación Matemática y Ciudadanía*. Goñi, J. M. y Callejo, M.L. (Coords.). Serie: Didáctica de las matemáticas. Nº 282. Barcelona. Graó.

## **SPACES FOR THE DEMOCRATIC PARTICIPATION IN AND OUT OF THE MATHEMATICS CLASSROOM**

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### **Abstract**

This presentation reflect about two different spaces in which democracy appears in math classrooms. They relate to both different research experiences: within the classroom and parent's involvement.

### **1. Democratic participation and mathematics.**

Skovsmose (1998) distinguished between four different meanings when linking mathematics education and democracy: (1) citizenship (2) mathematical archeology (3) mathemacy and (4) deliberative interaction. In this paper we have a look on two out of these four dimensions of analysis: the classroom as a privileged space for students and teachers to interact and create episodes of "democratic participation", and the workshops of mathematics for families as a case of active and critical citizenship of a group of parents aiming to build bridges between the teachers' teaching practices and the families' efforts to help their children with this subject.

The first teacher experience is analyzed within a design based research project to improve citizenship competences for teacher training, funded by Spanish Ministry of Education (EDU2009-08120). In order to construct an effective relationship in the classroom, the teacher creates a cooperative learning environment, respect the students, and motivate the students to achieve sufficiency in their social relations (Alobiedat, & Saraierh, 2009). Teacher and some students have been interviewed, to hear the voice of all the participants.

The second vignette draws on data from a project titled "Teacher training towards a Mathematics Education of parents in multicultural contexts" (ARIE/2007 -00026), funded by the General Office of Research and Univer-

sities (AGAUR) from Catalonia. This is an exploratory case study focusing on families and Mathematics Education.

**2- Xavier as regulator of math democratic practices.**

Xavier, is a 30 years experienced teacher, who wrote articles and a book in which mathematics classroom is a community of practice. in which students feel comfortable doing mathematics. He told us that students feel valued, participate, and feel respected in the constructive pursuit of mathematical work. He also explained to us that cultural diversity does not end when we can communicate using the same language, nor is it entirely true that mathematics education is universal. Javier understands mathematical activity as one that clarifies "should be rich and collaborative, open and overlapping approaches."

In the analysis of the discourse of Professor X, we recognize that despite its interaction pattern authoritarian-Dialogic he act as regulator of mathematical practice assists students and accompany them in mathematical thinking, in general encouraging a dialogue which takes into account the globality of diverse students. Their answers are conciliatory.

Andrew- It's a part. Really.

Gail- I did something different

Prof – Let's see. Just a moment, Before you explain (to Gail) what you did, let's see how Andrew explained. To see everybody if you understood his explanation “

Now, let's see as an example, what Ramsés tell us about such a classroom experience. He is a 13 years-old student, coming from Morocco, repeating the course, and considered as problematic student. He even didn't want to be taped, perhaps because the interviewer is a woman. His expectations are centered on “studying to be prepared when you are old... to find work”. He says the most important for him in school are the colleagues. When we ask him about math classes, he considers giving help for life, “and also to solve problems. Just talking about math classes he smiles. He asserts in other math classes it was different, and there is immediately an allusion to the teacher, not the subject.

Y (interviewer) - What was different?

St. R- The way of explaining, ... I don't know. When we don't understand, he takes more time for us. ... Thus, maths are more easy.

Int. Y- How do you feel your colleagues think about this classroom?

St R- Well... they like it ..../...

Int. Y- About the way of doing ... what do you learn from it?

St. R- The way of doing ...by the teacher... I don't know how to explain. .  
Int. Y- ¿Do you feel you participate more in this class than other classes?  
St. R- He did a lot. Let's say... in order the people understand more . Therefore, everybody understands and participate more. .../...

### **3.- Construction of dialogic spaces for participation of the community**

Prior scientific literature has demonstrated the positive impact that families have when they participate in their children's learning of mathematics (Racionero, 2011). However, not all types of family engagement produce the same positive impact (Díez-Palomar, & Kanés, 2012). Family engagement means to open the school spaces for democratic participation. The more radical the democracy is, the better the students' achievements are (Elboj, & Niemela, 2010). In order to understand this statement, we need to focus on the role that parents (and other caregivers) play in the interaction with students and teachers.

The workshops of mathematics for families are spaces for parents to participate in the school, and learn from the teachers' approaches and methodologies to mathematics education. In these particular spaces parents (and other members of the community) confront their previous knowledge regarding mathematics, with the approaches teachers use in their classroom practices. In a sense, these spaces become places for participating, exchange and learn together. Next quote illustrates the kind of interactions that use to appear in these spaces.

(Context: We are in a classroom placed in a high school. Parents are working with first grade equations with one unknown. Now the topic "how to solve an equation" is showing up. The facilitator solves the problem using a method, and one mother claim that her daughter uses other way to do it. At this point the facilitator explains the method used by the daughter. She has divided the chalkboard into two columns: on the first one there is the method used by the facilitator –which is the one known by the mother-; on the second one the facilitator wrote the daughter' method –which is the one used by teachers and children at the school-).

Facilitator: How it is going? Good?

Mothers: yes... very good (the mom who asked the question is the one who speaks louder).

Mother: We didn't understand it at home.

Facilitator: eh?

Mother: I didn't understand it like this at home; this that you have explained to us my daughter used to say "mom, we wrote this here", and I

say “where do you put this?” because I know it in the other w... in the old way (a noise in the background is heard, like admitting she is right) and I was not able to understand it because there is no explanation on the text book.

Facilitator: But, now did you get it?

Mother: (Some mothers admitting on the background are heard) Kind of, but what happens is that here is so easy... but to me... (She starts to laugh and makes gestures with her hands to say that sometimes the activities are difficult).

Facilitator: ... well... this is the same... but you have to go to....

Mother: (At the same time) now you’re getting it, because, because...

Facilitator: (At the same time) to everybody.

Mother: she explains that she does it that way, but I don’t know how to explain it....

From this experience, two different approaches to equation solving, are observed.

$2x + 5 = 40 - 3x$ $2x = 40 - 3x - 5$ $2x + 3x = 40 - 5$ $5x = 35$ $x = \frac{35}{5}$ $x = 7$	<p style="text-align: center;"><u>Profe</u></p> $2x + 5 = 40 - 3x$ $2x + 5 - 5 = 40 - 3x - 5$ $2x = 35 - 3x$ $2x + 3x = 35 - 3x + 3x$ $5x = 35$ $x = \frac{35}{5}$ $x = 7$
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Figure 1: Detail of the chalkboard grounded on the field notes.

Figure 1 illustrates these two different approaches to how to solve an equation. In the dialogue, the mother claims that she was not able to understand why her child was taking away the number five. Her comments suggest that she was trying to figure out where the five came from, to understand the whole process. But not additional explanations were present in the textbook, and the mother was feeling blocked in helping her daughter. The workshop of mathematics for families became a space for this mother to share her troubles, talk around them, and look for some clues in order to clarify the teacher’ procedure to solve the equation.

#### 4. Final remarks

The access to powerful mathematical ideas should be a human right (Malloy, 2002). However, it is true that because a number of reasons, including teacher of social perceptions of ability, cultural discontinuity in learning and instruction, tracking, poverty, low expectations, etc. (Tate & Rousseau, 2002), many children reject or suffer rejection by their teachers of mathematics (Malloy, 2002). A democratic practice (within the classroom, or out of the classroom) include (1) problem-solving curriculum, (2) inclusivity and rights, (3) equal participation in decisions, and (4) equal encouragement for success (Pearl, & Knight, 1999 among others). Vignette 1 illustrates a clear example of (3) and (4), in a sense. The teacher is looking for ways to involve students in a creative process of participation to discover and create mathematical meaning. In Vignette 2 Workshops become *dialogic spaces* for parents to share their troubles and learn from them.

Democratic teaching involves in-school practices (more teacher-oriented), and out-of-school practices (community-centered). Vignette 1 illustrates how a particular teacher is taking care of his students in order to promote participation, because he is aware of the importance of such competence in terms of learning mathematics. Vignette 2 points out the crucial role played by parents (and other members of the community) to push students' learning, since learning is not an in-school activity, but a global one involving also out-of-school contexts (Aubert, et al., 2008; Díez-Palomar, Gatt, & Racionero, 2011). Civil (2006) highlights the importance of both parents' context and previous knowledge about mathematics (many times defined as cultural knowledge in the kind of communities that she studies), and changes in terms of curriculum of mathematics (the *reform* of mathematics). Throughout her research trajectory, she illuminated how parents solve their lack of mathematical competence (in the context of the *reform* mathematics) giving value to their own mathematical background. Creating spaces for participation is a crucial fact parents to get their knowledge valorized (and legitimated) by the school (what we can call the "guardians" of the legitimize knowledge, drawing on the ideas of Bernstein and others). Gutierrez (2012) also provides much evidence to sustain this interpretation of parents' background. She uses the metaphor *Nepantla* to talk about a body of mathematical knowledge conformed by personal and cultural experience. The idea of dialogic spaces (Díez-Palomar, & Molina, 2009) provides the room for parents and other members of the community to valorize their background (what others call *funds of knowledge* –Moll et al., 1992; González et al., 2001–). These dialogic spaces move our attention over the

traditional classroom (in terms Appelbaum's terms, 2008) towards a broader notion of space, where voices of all actors (teacher, students and volunteers) are included within the pedagogical practices (Bernstein) through interaction (Vygotsky).

From these experiences, we see democratic access to mathematics as a means for solving (potential) conflicts between parents, teachers and students, which according to the literature in the field, is one of the more prominent barriers making difficult the relation between families and schools (Díez-Palomar, Menéndez, & Civil, 2011).

### References.

- Alobiedat, A., Saraierh, R (2009) The Degree of Democratic Practicing in the Classroom by Teachers European Journal of Social Sciences – Volume 9, Number 1 (2009) Available at [http://www.eurojournals.com/ejss\\_9\\_1\\_10.pdf](http://www.eurojournals.com/ejss_9_1_10.pdf)
- Appelbaum, P. (2008). *Embracing mathematics. On becoming a teacher and changing with mathematics*. New York: Routledge.
- Aubert, A., Flecha, A., Garcia, C., Flecha, R., Racionero, S. (2008). *Aprendizaje dialógico en la sociedad de la información*. Barcelona: Hipatia Editorial.
- Civil, M. (2006). Working towards equity in Mathematics Education: A focus on learners, teachers and parents. In S. Alatorre, JL. Cortina, M. Sáiz, and A. Méndez (Eds.) *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Mérida, México: Universidad Pedagógica Nacional.
- Díez-Palomar, J., Gatt, S., & Racionero, S. (2011). Placing Immigrant and Minority Family and Community Members at the School's Centre: the role of community participation. *European Journal of Education*, 46(2), 184–196.
- Díez-Palomar, J., Kanés, C. (Eds.) (2012). *Family and community in and out of the classroom: Ways to improve mathematics' achievement*. Bellaterra: UAB Press.
- Díez-Palomar, J., Menéndez, JM., Civil, M. (2011). Learning mathematics with adult learners: Drawing from parents' perspective. *Revista Latinoamericana de Investigación en Matemática Educativa – RELIME*, 14, 71-94.
- Díez-Palomar, J., Molina, S. (2009). Family Mathematics Education: Building Dialogic Spaces for Adult Learning Mathematics. In D.



- Kaye (Ed.) *Proceedings of the Adults Learning Mathematics Annual Meeting*. London (UK): LLU.
- Elboj, C., & Niemela, R. (2010). Sub-communities of mutual learners in the classroom: the case of interactive groups. *Revista de Psicodidáctica*, 15(2), 177-189.
- Epstein, J. (2011). Preparing Future Teachers to Conduct Programs of Family and Community Involvement for Student Success in School. *Proceedings of the 8<sup>th</sup> ERNAPE conference*, held in Milano (Italy), June 2011. Pages 39-40.
- González, N., Andrade, R., and Civil, M. (2001). Bridging Funds of Distributed Knowledge: Creating zones of practices in Mathematics. *Journal of Education for Students Placed at Risk*, 6(1-2), 115-132.
- Gutierrez, R. (2012). Embracing Nepantla: Rethinking “Knowledge” and its use in mathematics teaching. *REDIMAT. Revista de Investigación en Didáctica de las Matemáticas*, 1(1), 29-56.
- INCLUD-ED Consortium. (2009). *Actions for success in schools in Europe*. Brussels: European Commission.
- Malloy, C. E. (2002). Democratic access to mathematics through democratic education: an introduction. In Lyn D. English (Ed.) *Handbook of International Research in Mathematics Education* (pp. 17-26). Mahwah, NJ.: Lawrence Erlbaum Associates.
- Moll, L., Amanti, C., Neff, D., and Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Qualitative Issues in Educational Research*, 31(2), 132-141.
- Pearl, A., & Knight, T. (1999). *The democratic classroom: Theory to inform practice*. Cresskill, NJ.: Hampton Press.
- Racionero, S. (2011). Egalitarian Dialogue and Instrumental Dimension. Two Principles of Dialogic Learning in the Classroom. *International Psychology, Society and Education*, 2(1), 61-70.
- Skovsmose, O. (1998). Linking Mathematics Education and Democracy: Citizenship, Mathematical Archeology, Mathemacy and Deliberative Interaction. *ZDM. The International Journal of Mathematics Education*, 30(6), 195-203.
- Tate, W., Rousseau, C. (2002). Access and opportunity: the political and social context of mathematics education. In Lyn D. English (Ed.) *Handbook of International Research in Mathematics Education* (pp. 271-300). Mahwah, NJ.: Lawrence Erlbaum Associates.

## WORKSHOPS- ATELIERS

Hellenic Mathematical Society  
International Journal for Mathematics in Education

### **Democracy and Mathematics Circles: Questions, Collaborations, and Social Technologies**

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This workshop is intended to bring together participants in the previous 2011 CIEAEM 63 Mathematics Circles Workshop with new people who are interested in the theme of mathematics education and democracy. After discussing “how to live a math circle” last year, some participants have explored the ideas in the context of their own work. Others are still interested, but have not yet put any of the ideas into practice. The theme for CIEAEM 64 in Rhodes offers a unique opportunity to link the mathematics of mathematics circles with goals of democratic participation (both within places of mathematics teaching and learning, and as citizens in societies that value democratic forms of social organization) and the democratization of mathematics education research (as in the creation of an open network of researchers working in collaborative forms of democratic action).

#### **Le Démocratie et Les Cercles des Mathématiques: Questions, les Collaborations et Technologies Sociales**

Cet atelier est conçu à rassembler des participants de la précédente 2011 CIEAEM 63 Cercles Mathématiques Atelier avec de nouvelles personnes qui sont intéressées par le thème de l'enseignement des mathématiques et de la démocratie. Après avoir discuté de « la façon de vivre un cercle mathématique » l'année dernière, certains participants ont exploré les idées dans le contexte de leur propre travail. D'autres sont encore intéressés, mais n'ont pas encore mis aucune des idées en pratique. Le thème de la CIEAEM 64 à Rhodes offre une occasion unique de relier les mathématiques de cercles de mathématiques avec des objectifs de participation démocratique (tant au sein des lieux de l'enseignement des mathé-

matiques et de l'apprentissage, et en tant que citoyens dans les sociétés que la valeur des formes démocratiques d'organisation sociale) et la démocratisation de la l'enseignement des mathématiques de recherche (comme dans la création d'un réseau ouvert de chercheurs travaillant dans des formes de collaboration de l'action démocratique).

The plan for the workshop follows:

**1. What is a Math Circle?**

This is a review of topics discussed in the previous year, highlighting practical advice about starting a mathematics circle in a community, examples of ways that previous participants have enacted these ideas, and a short introduction to theories of mathematics education teaching and learning that have been applied by mathematics educators so far. The aim of this short introduction is to establish a common vocabulary and welcome new participants to the ongoing discussion.

**2. Connections to the CIEAEM 64 Discussion Paper** (Workshop facilitators will share examples of each of the following; participants will also share examples to expand the list of resources for understanding the connections between math circles and democracy in mathematics education).

- a. How have math circles contributed and decision-making in society?
- b. Which common practices for facilitating and participating in a math circle promise social justice, respect and dignity?
- c. Are there ways that math circles support forms of teacher education and professional development compatible with (a) and (b) above?
- d. How can we establish a network of mathematics educators interested in the potential of math circles in ways that are consistent with democratic values and practices?

**3. Questions that math circle work raises.** Participants in the CIEAEM 63 2011 workshop have identified questions that are generated by initial efforts to enact math circles locally.

- a. Can or should mathematics learning outside of school or in school projects in the community be more joyful or different from mathematics learning inside school? One of the aims of mathematics circles when used in teacher education and ongoing professional development is to help educators to make mathematics learning inside school at least as joyful as mathematics learning outside school; at the same time, mathematics circles outside school might have unique characteristics.

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- b. Are the social interactions inside and outside school different, and will this promote or hinder the mathematical construction of knowledge? Would there be a previous social interaction when creating a math circle outside school? Is this previous social interaction also an aim of the math circle? Can math educators and/or community leaders use the ICT technologies, the ones that many students use for their social interaction, to create a math circle?
  - c. How can a math circle maintain a participant-driven, radically democratic commitment, and not devolve into leaders and followers?
- 4. Social Networking and Math Circles.** A completely new topic for this year is how social networking and web 2.0 technologies can best be exploited in mathematics education.
- a. Web 2.0 has the potential of creating networks of help among teachers and students, and of extending the discussion within the curriculum, enabling a potential for learning inside and outside school.
  - b. The nature of social technologies creates a potential for changing how teachers discuss with students, and especially for solving different mathematical problems.
  - c. There are some technological tools (online forums, chats, wikis, facebook,... ) that promote enhanced social interaction of people creating circles of communication. Those circles can evolve to become math circles. In this evolution we can encounter some difficulties related to the use of the technology, but also with the nature of the relationships between mathematical knowledge of inside and outside school.
  - d. In connection with both difficulties mentioned in (c), there is the question, "Do Information and Communication Technologies genuinely promote a democratic mathematics curriculum?"
- 5. Pursuing a technology-supported democratic research project on math circles and democracy.** This will be an open discussion of how those present will commit to ongoing communication and participation in a social network on-line, sharing ideas, activities, and projects that can be carried out across national and regional borders.
- 6. Continued examination of the goals of a mathematics circle.**
- a. Should or can we agree that the circle members are brought together via mathematics - mathematics facilitates joy, participation, a can-do attitude, and the solution of serious problems? How else might we articulate this?
  - b. Circles take action in their community and in the world, not neces-

sarily studying mathematics; nevertheless, in the process, they use and learn mathematics. How are we planning to engage with this idea?

- c. The question about linkages with a prescribed/proscribed school curriculum is only a form of resistance; it need not prevent this from taking place. Can we, as a group, seriously work in the field of mathematics education without being limited to the most common forms of school-based learning?

**7. Contacts and affiliations will be collected and exchanged, and tentative small group collaborations will be formed before we leave.**

**References**

- Appelbaum, P. (2008). *Embracing Mathematics; On Becoming a Teacher and Changing with Mathematics*. Routledge.
- Clark, W., Logan, K., Luchin, R., Mee, A., & Oliver, M. (2009). Beyond 2.0: mapping the technology landscapes of young learners. *Journal of Computer Assisted Learning* (25), 56-69.
- Gerofsky, S. (2010). The impossibility of 'real-life' word problems (according to Bakhtin, Lacan, Zizek and Baudrillard). *Discourse* 31(1), 61-74.
- Greenhow, C., Robelia, B., & Hughes, J. E. (2009). Learning, Teaching and Scholarship in a Digital Age: Web 2.0 and Classroom Research: What Path Should We Take Know? *Educational Researcher*, 38 (4), 246-259.
- Juan, A., Huertas, M. A., Trenholm, S., & Steegmann, C. (2012). Teaching Mathematics Online: Emergent Technologies and Methodologies (pp. 1-414). doi:10.4018/978-1-60960-875-0

## **Construire un problème de mathématiques dans une optique d'apprentissage actif : l'apprentissage par problèmes (APP)**

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L'Ecole Polytechnique de Louvain a initié depuis 2000 une pédagogie active centrée sur l'apprentissage par problèmes (APP) et par projets dès la première année de baccalauréat. Le but principal du déroulement d'un processus APP est de permettre aux étudiants, via la résolution d'un problème, d'acquérir des connaissances, des compétences, des attitudes, des comportements qui sont recherchés par les enseignants ayant conçu ce problème (Réf. 2, 3, 4, 5).

L'apprentissage par problèmes met l'étudiant en situation de besoin d'apprentissage en lui proposant des défis et en utilisant le groupe (6 étudiants) comme moteur d'apprentissage (Réf. 1). L'idée de faire travailler les étudiants en groupes a pour conséquence une démocratisation du savoir, en particulier en ce qui concerne les mathématiques. L'étudiant en difficulté peut s'appuyer sur le groupe, il n'est pas livré à lui-même.

Malheureusement, beaucoup de collègues hésitent à se lancer dans cette aventure. La difficulté majeure à laquelle ils sont confrontés est la construction d'APPs en mathématiques. C'est ce que nous proposons dans cet atelier : construire un APP en mathématiques et faire la différence entre un bon et un mauvais énoncé.

Nous commencerons par donner des exemples de problèmes de type APP utilisés dans notre institution. Nous en choisirons un et proposerons au groupe de le résoudre. Dans une deuxième phase de l'atelier, la tâche consistera à construire un nouvel APP.

### **Références**

E. Aguirre, C. Jacqmot, E. Milgrom, B. Raucent, A. Soucisse, Ch. Trullemans, C. Vander Borgh: Devenir ingénieur par apprentissage

- actif, *1er colloque de la pédagogie par projet dans l'enseignement supérieur, Brest (France) 27-29 juin 2001*).
- K. Ben-Naoum, V. Wertz: L'apprentissage par problème en mathématiques: une expérience en candidatures ingénieur, *Actes du 19ème colloque de l'AIPU (Association Internationale de Pédagogie Universitaire) «Les méthodes actives dans l'enseignement supérieur», Louvain-la-Neuve, Mai 2002*.
- K. Ben-Naoum, V. Wertz: L'apprentissage par problèmes en mathématiques: Pourquoi? *Actes du 2ème Colloque ENSIETA et ENST Brest: «Questions de pédagogie dans l'enseignement supérieur: réflexions, projets et pratiques» Brest (France), Juin 2003*.
- K. Ben-Naoum, R. Rabut, V. Wertz: Design your problem to-day!, *Proc. 7th International ALE Workshop, Toulouse (France), 2007*.
- A. K. Ben-Naoum, C. Rabut, V. Wertz, L'apprentissage par problèmes dans les matières théoriques (exemple des mathématiques): spécificités et faisabilité, *Actes du 25ème Colloque de l'AIPU (Association Internationale de Pédagogie Universitaire), "Le défi de la qualité dans l'enseignement supérieur: vers un changement de paradigme", Montpellier (France), 2008*.

## **Mathematics and Democracy: teaching electoral systems and procedures**

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### **Abstract**

We can think of democracy as a system of government with four key elements: A political system for choosing and replacing the government through free and fair elections, the active participation of the people, as citizens, in politics and civic life, protection of the human rights of all citizens and a rule of law, in which the laws and procedures apply equally to all citizens.

In this paper we try to emphasize the connection between mathematics and the first key element namely electoral procedures –especially the electoral law. Also through teaching activities and interdisciplinary projects in every day school live, students may realize the second key element. We argue that mathematics is the suitable tool to realize “fair” and “representative” in modern democracy. Through this approach also a number of mathematical skills may develop.

### **Mathematics and governance**

Almost all election laws were designed by mathematicians! But what is the electoral law? It is an algorithm by which the votes of citizens turned into seats in a representative body or used to elect one person. How then this is associated with math?

Today's democracy functions as a multiparty system of selection of representative bodies. Based on the existence of organized parties: political



institutions with members shared common political background and propositions, and representatives: citizens selected by the electorate.

Usually there is more than one electoral unit. The electorate is divided into geographically defined subsets. The constituencies are defined by population and geographical criteria. In fact this is the problem of partitioning a set into subsets that share common characteristics.

The choice of an electoral system allows government to take into consideration historical data, tradition, ethical and social characteristics. The distinction between majority and proportional systems helps to discuss the conditions set by each system, while the definition of algorithmic process helps in the understanding of algorithmic thinking.

Examples: partitioning and defining constituencies in Greece and other countries.

Activity: A school with classes and sections can be divided to electoral units.

### **Representation: processes and decisions**

The selection process of representation must meet some requirements. The first and fundamental political rule is the well known principle of the equity of the vote "one man-one vote". This means that every citizen has an equal right to participate and vote has the same value. That is that the number of representatives elected by a group of people must be proportional to its size.

Furthermore, there is the question whether choice is based primary on parties or on persons. Dependent with the above is the issue is the proximity citizen-representative and the role of the candidate. This discussion highlights the interdisciplinary approach between mathematics and political science.

Because there are alternatives available, the topic is suitable for organizing debates and other learning activities.

Examples: the number of members of MP's per constituency in Greece and elsewhere. Describing proportional representation procedures based on several systems. Calculate the deviation between population and seats. What is fair? Using data from previous elections to calculate several representation indices, gives the opportunity to estimate "fairness".

Activity: Knowing the number of seats (eg, a school board to 15 members) calculate the number of delegates representing a section (section or class) depending on the partition. Calculate the deviation corresponding to each solution and set the "best selection" criterion.

### **Algorithms: transforming rules into practice**

In elections groups of candidates (or parties) are involved. In addition to "fair representation" according constituency population, "fair representation" deals also with the share of political unit (candidate or party). So the electoral law describes how votes are assigned to seats. As in the case of constituencies so in the case of parties, proportionally should occur. According to the electoral law "proportionality" can be derived from the total share or from each individual electoral unit. It can also result in part from the electoral units and partly from the total share of votes.

The distinction in proportional, majoritarian and mixed systems, gives the opportunity, applying these algorithms, to understand the concept and principles of computer programming.

Examples: based on election results calculate for different electoral systems seats shares in Greece and other countries. Compute the deviation (between votes and seats) for several electoral systems in each country. Argue about the outcomes.

Activities: Calculate the number of seats in the 15-member school board according the electoral unit (section, class or school). Running simulation of electoral systems for know election results, map seats and votes using proper software.

### **Electoral systems based on persons**

Select a person from each electoral unit. This is definitely the simplest system, but creates problems of "fair representation". The elected representative represents only the share of the electorate who voted for him. Calculating the percentage of those who are not represented, we can argue about various systems. If misrepresentation gives an index of dissatisfaction, total and partial dissatisfaction could computed.

Example: UK single member constituencies, voter turnout.

Activity: choose one representative from each class (section) and calculate the corresponding measures.

Select many (more than one) representative from each unit. The available alternatives are the ranking the candidates of a list or voting simultaneously for several candidates in the list. In the second alternative voter express his/hers preference for more candidates. That is, each voter has a number of votes (up to a maximum). According to the available

number of votes ( $N$  voters,  $k$  candidates,  $m$  [maximum number of votes]), “representativeness” and “fairness” may be computed

Preferential vote (STV). In this alternative each voter expresses his/hers preference for a candidate but also is able to rank the candidates according to his/hers criteria. Ranking means that he/she expresses how his/her vote should be handled in case it wasn't used for his/hers preferred. This procedure minimizes total waste.

Examples: Familiarize with STV (electoral system of Ireland).

Activity: Using the above systems in the selection of school board. Divide students into two groups. Assign randomly system (open list and STV) to group and compare the outcome.

### **List and mixed systems**

Among all electoral systems the usual party list system gives the opportunity to consider the effect of choosing a party among many based on voter-party proximity, or on voter-party agreement considering issues. An interesting alternative is the so-called mixed systems where each voter has two votes. One is expressed for a party list, while the other is expressed for a candidate among several running for one member constituency. There are several algorithms to compute overall distribution of seats. Among them the one used in Germany and the one used in Russia. In the case of Germany the distribution of seats is based on total share of votes, while in Russia only a fraction of seats is distributed according to the total share and the remaining are distributed according to the share in each constituency.

Examples: for several countries compare electoral systems. Debate for and against.

Activity: use a mixed two-votes system for the school board elections.

### **Defining electoral units (constituencies).**

Districting and re-districting criteria.

Several criteria for districting are introduced. Well-known criteria are those of the population parameters (density, number of inhabitants), parameters that have to do with physical and artificial boundaries (such as mountains, rivers, roads, expressways), parameters that have to do with the shape and form of the constituency. The discussion on the criteria and the search for feasible solutions includes mathematical concepts as continuity and compactness, concepts from graph theory (connectivity, colorings), calculations with combinations and arrangements, description of algorithms. It also offers the possibility of interdisciplinary approach to history and

geography of the region, study of demographical characteristics from official sources (statistical offices), the study of social characteristics (distribution of occupations and wealth), while the comparative observation in order to revise the boundaries enables study of changes in population statistics.

Also, the search process for "fair" and "objective" solution is the subject of investigation which involved elements of law and political science.

Examples: UK (single member constituencies), and discussion about the districting authority and the relevant legislative provisions, Greece (constitutional provisions for the definition of regions, Hungary (counterfactual), discussion on the biased districting (the gerrymandering example).

Activity: partition the municipality into electoral districts for the municipality elections using one member districts solutions and multi-members solutions. Analysis Consider social, demographical and geographical parameters.

### **The analysis of electoral results: statistical correlation and causality**

Analysis of the results is available to discuss issues such as participation and turnout the representativeness of the electoral system, using thematic maps to record changes in space and time.

Moreover, the discussion of political surveys can help in understanding statistical concepts and social research. Using sophisticated statistical methods wider conclusions can be drawn. This topic is connected also to the methodology of project, which is introduced in education to the use of simulation techniques and it is suitable for debating on several themes.

### **References**

- Maths in Political Science. Electoral system design and redrawing administrative and electoral units, Th. Chadjipadelis, (in press)
- Requirements and criteria for the redrawing of local authority map, Th. Chadjipadelis, From prefectural to regional local government, G. Sotirelis, Th. Xiros (ed), Papazisis, 2010:99-141 (in Greek).
- Statistics teacher of the new era: another specialized mathematician or a totally different person? Theodore Chadjipadelis, Efi Papparistodemou, Maria Meletiou-Mavrotheris. ICOTS-8. 2010:  
<http://icots.net/8/talk.php?k=3A2>

- Digital democracy, Th. Chadjipadelis, Informatics in Education. Techniques, applications, teacher training, K. Tsolakidis (ed). University of the Aegean, Rhodes, 2002:71-76. (in Greek)
- Criteria for determining electoral districts (in relation to the electoral system): mathematical approach, Th. Chadjipadelis, Law and Politics, 1983: 4:253-256. (in Greek)
- Political Gerrymandering and the Courts, Bernard Grofman, Algora Publishing, 1990
- Mathematics and democracy: recent advances in voting systems and collective choice, Bruno Simeone, Friedrich Pukelsheim, Springer, 2006
- The International Institute for Democracy and Electoral Assistance, <http://www.idea.int/>
- Census at School, <http://www.censusatschool.ie/>

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## **NUMBERS IN THE FRONT PAGE: MATHEMATICS IN THE NEWS**

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### **Aim**

This workshop aims to engage the participants in a critical discussion on the use of numerical and statistical concepts in the (re)construction of crucial aspects of economic and social reality by newspapers and TV shows, intended to neutralise public reactions and to promote specific political interests of the time.

Such a use of mathematics is observable the last two years in Greece concerning facts related to situations which are causes or results of the experienced economic and social crisis. This use of mathematics poses interesting questions which are challenging critical mathematics educators, who are facing the following dilemma: they must teach the mathematical meanings of the numerical and statistical concepts and at the same time they have to select some of their referential meanings and reject others, which at the time are widely promoted in public life.

### **Theoretical background**

#### ***The mathematical reading of the world***

As has elsewhere analyzed (Chassapis, 1977), any mathematical construct, i.e. any mathematical entity defined in the context of a mathematical theory, acquires its specific mathematical meaning from the propositions of the particular theory in which it is embedded. Mathematical constructs, however, are used to describe or are “applied” to real world situations. Any such mathematical description of any real world situation may be considered as a micro-theory of that particular aspect of the real

world, in which case any implicated mathematical statement may be interpreted as a statement about that aspect of the real world.

The mathematical construct of addition on integers, for instance, is defined and thus it acquires its meaning within a theory of numbers and, on the other hand, may be used and considered as a reasonable micro-theory of simple financial transactions describing income or credit by positive integers and expense or debt by negative integers. In such a case any mathematical statement implied by that particular application of addition to a particular situation of financial transactions may be interpreted as a statement about that particular aspect of the social reality.

A description of a real world situation in terms of a mathematical construct is set up on the basis of a mapping between that construct and the real world situation. However, every such mapping is mediated by non-mathematical concepts that describe the real world situation and by the associated linguistic expressions that signify these non-mathematical concepts. These non-mathematical concepts in association to their linguistic expressions assign another non-mathematical meaning and simultaneously specify a reference of that mathematical construct within the particular description of the particular real world situation. This meaning may be considered as constituting the referential meaning of the mathematical construct which establishes its affinity to the real world situation.

The mathematical construct of addition, for example, may be used to describe a class of real world situations described by concepts of change, combination or comparison. Particular instances of these concepts (e.g., the growth of a quantity as an instance of change, the union of many quantities as an instance of combination or the difference of two quantities as an instance of comparison) in conjunction to their signifying linguistic expressions attribute to the addition a new meaning, differentiated from its mathematical one, specifying at the same time particular references of addition within its particular applications to particular real world situations. Any one of such meanings constitutes a referential meaning of the mathematical construct of addition on numbers.

### ***Mathematics and politics***

There can be no doubt that *“there is a constitutive interrelationship between quantification and democratic government. Democratic power is calculated power, and numbers are intrinsic to the forms of justification that give legitimacy to political power in democracies. Democratic power is calculating power, and numbers are integral to the technologies that seek to*

*give effect to democracy as a particular set of mechanisms of rule. Democratic power requires citizens who calculate about power, and numeracy and a numericized space of public discourse are essential for making up self-controlling democratic citizens”* (Rose, 1991: 675).

On the other hand, it may be claimed that the referential meanings assigned to mathematical constructs being properly manipulated do not merely inscribe a preexisting real world situation but constitutes it. Techniques of inscription in numerical formats and accumulation of facts about aspects of the “national economy”, the “public debt”, the “tax incomes” or the “labor salaries” render visible particular domains with a certain internal homogeneity and external boundaries. The collection, manipulation and presentation of numerical data participate in each case in the fabrication of a “locus” within which thought and action can occur. Numbers delineate “fictive spaces” for the operation of governments, and establish a “plane of reality”, marked out by a grid of norms, on which governments can operate according to the case (Miller & O’Leary, 1987; Rose, 1988; Miller & Rose, 1990).

At the same time, every such fabrication of real world situations ostensibly endorsed by the objectivity and neutrality of mathematics and enhanced by the publicity power of media prevails or actually is imposed as the unique representation of reality and finally as the reality itself (Skovsmose, 2010).

The relation between mathematics and politics has been addressed from different perspectives according to their focus. Foucault (1979), for instance, has analyzed the relation between government and knowledge by considering “governmentality” or the mentalities of government that characterize all contemporary modes of exercise of political power in the Western democracies. In this perspective Skovsmose (1998, 2010) has further exemplified the role of mathematics in the constitution of political, calculative, practices. The link between numerical information and a politics of calculated administration of a population is another perspective emphasized, for example, by Pasquino (1978). Latour (1987) has analyzed how in modern societies events and processes are inscribed in standardized forms which can be accumulated in a central locale where they can be aggregated, compared, compiled and calculated about and how through the development of complex relays of inscription and accumulation of “data” new conduits of power are created between those who wish to exercise power and those over whom power is to be exercised.



***Crisis in Greece as an example***

The last two years in Greece almost every public political debate and governmental decision concerning the experienced economic and social crisis, and as a consequence most of the TV news bulletins and newspaper front pages, are flooding with numbers and numerical indices interpreted and commented according to the case. Therefore, the data provided and the challenges posed by such a situation may be considered as a characteristic example of the use of numbers in political manipulations. Considering that,

as Alonso and Starr (1987:3) point out, acts of social quantification are “politicized” not in the sense that the numbers they use are somehow corrupt - although they may be – but because “political judgments are implicit in the choice of what to measure, how to measure it, how often to measure it and how to present and interpret the results”.

Such a situation raises fundamental issues concerning politics and mathematics and it has to be problematized from the viewpoint of critical mathematics education.

**Organisation of the workshop**

The contributors will introduce the topic presenting a discussion framework on the aforementioned background and they will present typical cases of selected newspapers’ front pages as well as TV news outlines which contain numerical data for supporting political arguments or governmental decisions of the time.

On this basis, questions will arise aiming to stimulate a critical discussion on the use of mathematics for promoting political interests and the role of critical mathematics educators in the demystification of mathematics and its use in everyday life situations, without at the same time undermining the value of its learning (Skovsmose, 1998).

**References**

- Alonso, W. & Starr, P. (eds.) (1987), *The politics of numbers*, New York: Russell Sage Foundation.
- Chassapis, D. (1997), The social ideologies of school mathematics applications: A case study of elementary school textbooks. *For the Learning of Mathematics*, 17(3), 24-26.
- Foucault, M. (1979) On governmentality, *Ideology and Consciousness*, 6 (1), 5-22
- Latour, B. (1987), *Science in Action*, Milton Keynes: Open University Press.

- Miller, P. and O'Leary, T. (1987), Accounting and the construction of the governable person, *Accounting Organizations and Society*, 12 (3), 235-265.
- Miller, P. & Rose, N. (1990), Governing economic life, *Economy and Society*, 19, 1-31.
- Pasquino, P. (1978) Theatrum Politicum. The genealogy of capital - Police and the state of prosperity, *Ideology and Consciousness* 4, 41-54.
- Rose, N., (1988) Calculable minds and manageable individuals, *History of the Human Sciences*, 1,179-200.
- Skovsmose, O. (1998), Linking Mathematics Education and Democracy: Citizenship, Mathematics Archeology, Mathemacy and Deliberative Interaction. *Zentralblatt für Didaktik der Mathematik*, 30 (4), 195-203.
- Skovsmose, O. (2010), Can Facts be Fabricated through Mathematics? *Philosophy of Mathematics Education Journal*, 25, Available at <http://people.exeter.ac.uk/PErnest/pome25/index.html>

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## **Geometrical-mechanical artefacts for managing tangent concept**

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### **ABSTRACT**

This workshop deals with the didactical use of geometrical-mechanical artefacts for managing tangent concept in vygotskian perspective and adopting Rabardel's theory on instrumental approach.

From this point of view, during the workshop activity, we present a pathway for the constructive use of the tangent to reflect with CIEAEM participants on suitable didactical practice for student of 12th grades and university level. Specifically the workshop deepens the *kinematical* properties of the tangent through a laboratorial didactic activity as prototype of the democratic approach to the problem solving. In particular we propose some "new" geometrical-mechanical artefacts presented according to the vygotskian perspective to mediate the meanings related to machines embodied knowledge. From this point of view the proposed artefacts constitute the link between the subtended mathematical content and the laboratorial activity defined on a democratic practice in the classroom.

#### **1. ARTEFACTS IN EDUCATION: A POSSIBLE THEORETICAL FRAMEWORK FOR A DEMOCRATIC DIDACTICAL PRACTICE IN CLASSROOM**

Recent international studies on laboratorial didactical activities are producing interesting results about the students participation at the construction of mathematical meanings through the adoption of problems, instruments and teacher-student iterations. These researches are set on different theoretical frameworks and inquiry methodologies, and in particular we are interested in extending Bartolini Bussi's works (Bartolini Bussi & Mariotti, 2008) on mathematical machines to concretely manage the tangent concept. This paper discusses a laboratorial activity based on the introduction of

“new” artefacts<sup>1</sup> according to Rabardel's framework (Rabardel, 1995) whose purpose is not exclusively technical/practical but mainly theoretical, oriented to develop mathematical meanings, processes and in general arguing, conjecturing and demonstrating attitudes.

The mathematical meanings mediated by laboratorial activities are defined starting from the mathematical knowledge considered as a historical-cultural object (Norman, 1993) so strongly linked with the curricula and the teachers' decisions about the skills and knowledge to be taught. The national and international Math Education researches evince that, even if motivating, intriguing and pedagogically useful, it is not sufficient to propose an instrument and to suggest its use for a problem to a student or to a group of them to mediate the underlying mathematical meanings, but generally these laboratorial activities need the support of teachers paying attention at the didactic strategies centered on the use of artefacts and guiding the evolution of signs and *system of signs* (in the sense of Arzarello, Robutti, 2009) toward what is recognizable as mathematics through the focus on students cognitive and meta-cognitive level. About the didactic concrete use of artefacts, Vygotskij showed how practically the artefacts are used to accomplish purposes other ways unreachable, on the contrary the mental activities are supported and developed through signs produced in the internalization processes, defined by Vygotskij as *psychological tools*. The first ones are outside oriented, the others inside. Since many years the fundamental role of the artefacts in the internalization process is widely recognized and, differently from other psychological approaches which clearly distinguish the technological artefacts and the concrete signs, the vygotskian perspective poses the existence of an analogy between them. There have been really few works which treat the tangent concept from this point of view.

According to this, our paper introduces a workshop laboratorial activity subdivided in steps and oriented to a deeper new reading of how the mathematical knowledge of *kinematical* tangent's properties can be used in a concrete way according to a constructivist framework based on a democratic approach to the problem solving and, in general, to the related teaching/learning phases.

With this aim, planning the workshop laboratorial activity, we are focusing both on *values* and *norms* that are inherent in the democratic laboratorial activities both on *students behaviours* while using the artefacts. These as-

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<sup>1</sup> The artifacts are new for its application in didactic, but are historically based on *tractional motion* (Bos 1988; Tournès 2007, 2009).

pects could give us an optimal combination between the students cognitive resources "observed" in classes and potentials and limitations of the democracy approach as an ideal form of social organization of the laboratorial activity, linked with our working hypotheses about the artefact's use and the related milieu (Brousseau, 1997).

## **2. Didactical problems of the tangent content**

Students very often learn just mechanically to "calculate" the tangent in analytic or geometrical contexts, without having an unitary and conscious vision at a meta-cognitive level.

At a geometrical level, if the teacher explicitly asks to trace the tangent to a curve in a point and to study the properties in the evolution of the point on the curve, in some cases even the high-school students are satisfied just by a perceptive evidence ignoring the need of a rigorous "demonstration" linking the traced line with the relations of the curve. At the analytic level usually there is a strong break between the students formal activity in the analytic register and the conversion (Duval, 1993) in the geometric one. Our artifacts try to solve this cognitive gap. Still more complex there appears the situation about the physic interpretation of the tangent concept which usually require a lot of work of the teachers who, moving on different epistemological bases (a vision of the mathematical Knowledge as a unitary theoretical Science), are distant from the often fragmentary vision of the learners. During the workshop the CIEAEM participants will be invited to discuss about it in small groups and all together. From this point of view we propose specific didactic pathways and strategies which make possible to connect the two dynamically evolving worlds of teacher/learners, and which are able to mutually re-interpret themselves through the signs and the signs systems produced by both of them.

## **3. PLAN OF THE WORKSHOP**

The workshop will be organized in the following steps:

1- orientation to the topic through the presentation of a simple artefact to evince the tangent to a curve used at a didactical level with 12th degree students: the *Tangentograph* introduced in (Di Paola & Milici, 2012).

2- whole group discussion focused on the role of the wheel in the managing of the tangent to avoid the "lateral movement" in the contact point.

3- small groups interaction focused on a more complex artefact and a suitable fixed form. We will present a square root artefact. The CIEAEM participants will explore the artefact and build some related use schemes (Bartolini Bussi & Mariotti, 2008). At the end the groups will share their re-

ports.

4- to promote a meta-reflection about the previous laboratorial activity we invite the small groups to theoretically and practically work on the decomposition and reassembling of a different artefact to trace the exponential curve (Perks, 1706) and explore the differences with the previous artefact.

The steps discussed above evince the explicit way to observe the mathematics classroom as a micro society in which democratic relationships among their students can be established during their direct and indirect communication and argumentation on the mathematical content using the artifact.

#### 4. MATHEMATICAL ASPECTS ABOUT THE SQUARE ROOT ARTEFACT

The choice of the square root function was made because of the nature of the function, that, even if simple, evinces many significant aspects that can be highlighted in the geometrical/mechanical interpretation. The workshop will particularly focus the attendants on a concrete “new” reading of the mathematical concepts of direction, tangent (geometrical and analytical approach with the derivative), continuity, real function asymptotic behaviours, differential equations.

Even if the function  $f(x) = \sqrt{x}$  is algebraic, we'll not interpret it as the converse of  $x^2$ . Specifically the machine will solve the differential equation  $f'(x) = \frac{1}{2f(x)}$  with the boundary condition  $f(1) = 1$ , whose unique solution in  $[0, +\infty[$  is the square root.<sup>2</sup>

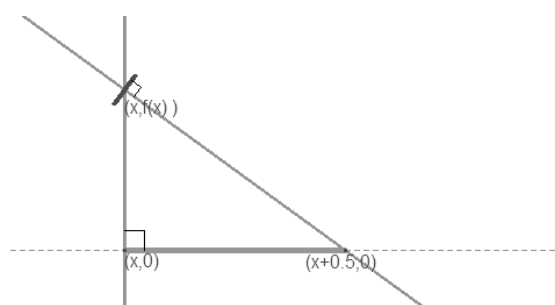


Figure 1. Diagram of the *square root* realized in *Cabri Géomètre II Plus*  
It follows a table about the possible translation between analytical and

A sketch of the  $f(x) = \sqrt{x}$  artefact: to work it has to be shifted horizontally along the “basis cathetus” (the segment  $[(x, 0), (x+1/2, 0)]$  will be later identified as “hypotenuse”).

The red segment in  $(x, f(x))$  represents a wheel, implementing the condition that the tangent in  $(x, f(x))$  has to be perpendicular to the hypotenuse.

<sup>2</sup> This definition is solved by the square root just for the real values, it doesn't apply in the complex field.

geometric/mechanical semiotic registers<sup>3</sup> (Duval, 1993) and the way we dynamically analyze the artefact and its mechanical components (constraints, rods,...).

Analytical Register	Geometrical/Mechanical Register
Domain: $\mathbf{R}^+$	It is possible to observe a <u>great difference with the analytical register</u> . So the artefact, statically observed, <i>doesn't permit to evaluate the domain</i> . That because the abscissa values are used in a dynamic way. On the other side it is possible to realize how the artefact blocks when $f(x)=0$ .
$f(x) \geq 0$	Knowing that when $f(x)=0$ the artefact blocks and $f(1)=1$ , then the function is always be not negative.
$f'(x) > 0$	The tangent has to be perpendicular to the <i>hypotenuse</i> , so the derivative is positive when $f$ is not negative (in the whole domain).
$\lim_{x \rightarrow +\infty} f(x) = +\infty$ $\lim_{x \rightarrow +\infty} f'(x) = 0$	Being increasing, $f$ can't oscillate. By <i>Reductio ad Absurdum</i> suppose that $f$ converges, so $f'$ tends to 0. Mechanically it implies the <i>hypotenuse</i> tends to be parallel to the ordinates (even if physically it can never happen), and that occurs just is $f$ tends to infinity, thus the proposition is false. Once observed the divergence, for the absurd reasoning, the tangent tends to be parallel to the abscissa.

Table 1. Translation between analytical and geometric/mechanical semiotic registers

## CONCLUSIONS

From a social point of view, Vygotskian perspective in semiotic mediation in general, and about the proximal development zone in particular, can be seen as a prototype of the democratic approach to the problem solving: the aim is common to everybody, and everyone has to share its own experience, knowledge, ideas and criticism to obtain a commonly accepted solution. Like in an ideal society, each attendant would actively take part to the discussion, maybe as a creative or as a critic, but everyone has to understand the problem and the possible approaches to reach a conscious participation. Mathematically, we think that the workshop could be significant for two reasons:

<sup>3</sup> In this paper the geometrical/mechanical is still confused with the analytic one, because these artefacts are not only able to evince some properties (such in dynamic geometry) but also to prove them in a specific register. Actually, this register isn't autonomous. We are working on this aspect defining a suitable theory to better highlight the primitive concepts for the construction and the functioning of the machine. In addition, to describe the table in a simple way, the geometrical and mechanical registers, even if different, are summarized in the same column.

- 1- to reflect in a concrete way on the mathematical contents related to the *kinematical* properties of the tangent through a laboratorial didactic activity;
- 2- to discuss with the CIEAEM participants a *good practice* to the didactical use of these geometrical-mechanical artefacts, reflecting about a possible experimental pathway aimed to shift similar laboratorial activities with 12th grade students and specific course at university level.

### REFERENCES

- Arzarello, F. & Robutti, O. (2009), Embodiment e multimedialità, nell'apprendimento della matematica, *L'insegnamento della matematica e delle scienze integrate*, vol. 32, n. 3, 243-268.
- Bartolini Bussi M. G. & Mariotti M. A. (2008), Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective, in L. English (ed.), *Handbook of International Research in Mathematics Education (second edition)*, Routledge.
- Bos, H. J. M. (1988), *Tractional motion and the legitimation of transcendental curves*, Centaurus, 31, 9-62.
- Brousseau, G. (1997), *Theory of Didactical situations in mathematics. 1970-1990*, English edition by M. Cooper, N. Balacheff, R. Sutherland and V. Warfield (Kluwer Academic Publishers).
- Di Paola, B. & Milici, P. (2012), Geometrical-mechanical artefacts for mediating tangent meaning: the Tangentograph, Manuscript submitted for publication.
- Duval R. (1993), Registres de représentations sémiotiques et fonctionnement cognitive de la pensée. *Annales de Didactique et de Sciences Cognitives*. 5, 37-65.
- Milici, P. (in press), Tractional Motion Machines extend GPAC-generable functions, *International Journal of Unconventional Computing*.
- Milici, P. and Dawson, R. (in press). The Equiangular Compass, *The Mathematical Intelligencer*.
- Norman, D.A. (1993), *Things that make us smart*, Addison-Wesley Pub. Com (traduz ital. *Le cose che ci fanno intelligenti*, Feltrinelli, 1995)
- Perks, J. (1706), The construction and properties of a new quadratrix to the hyperbola, *Philosophical Transactions of the Royal Society of London*, 25, 2253-2262.
- Rabardel, P. (1995), *Les hommes et les technologies - Approche cognitive des instruments contemporains*. A. Colin, Paris.



Tournès, D. (2007), La construction tractionnelle des équations différentielles dans la première moitié du XVIIIe siècle, in *Histoires de géométries. Textes du séminaire de l'année 2007*, Dominique Flament (éd.), Paris: Fondation Maison des Sciences de l'Homme, 14p.

Tournès, D. (2009), *La construction tractionnelle des équations différentielles*. Paris: Blanchard.

## **Factors in creating a democratic game play**

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### **Résumé**

L'atelier a comme but de collaborer avec les participants sur des sujets qui concernent la signification d'un jeu démocratique, la création d'un jeu démocratique pour la classe de mathématiques, la construction d'une communication démocratique par le jeu dans la classe de mathématiques ainsi que les valeurs que les élèves sont censés apprendre par un jeu démocratique.

### **Abstract**

*The workshop aims to collaborate with the participants on issues concerning the meanings of a democratic game play, the creation of a democratic game play in the mathematics classroom, the construction of a democratic communication through game play in the mathematics classroom, as well as the values that students are expected to learn from a democratic game play.*

Democratic education, and in particular democratic access in mathematics education, according to many researchers (Skovsmose, 2005; Valero, 1999), is very important for social justice and equity in our world. A fundamental arena in which democratic access is played out is the classroom (Skovsmose & Valero, 2002). In democratic classrooms four qualities are included (Malloy, 2002: 21-22): 'problem-solving curriculum', 'inclusivity and rights', 'equal participation in decisions' and 'equal encouragement for success'. A necessary condition for these four qualities is the existence of democratic relationships between teacher and students as well as between students. The main part of the democratic relationships should be the construction of a communicative environment in the classroom. According to Valero (1999: 22) communication "allows the establishment of a shared language for which all participants are responsible and in whose creation

and modification all participants have a role to play". Alro' and Skovsmose' (1996) communicative model, in a democratic concern, consists of five elements: getting in contact with, discovering, identifying, thinking aloud, challenging, reformulating, negotiating and evaluating.

A context which encourages the communication is game play (DeVries, 1980; Hart & Mas-Colell, 1997). Game play could be a kind of learning environment which creates the appropriate conditions where all students who participate in, can access mathematical ideas (Olson, 2007). Games usually include a relevant set of social practices that could contribute to the emergence of democratic practices. For example, playing games such as board games is a kind of mathematics classroom practice (Skoumpourdi, 2010) which could enact particular values such as justice, respect and dignity, as well as particular ethical principles such as patience and fair play (Berlinger, 2005).

The workshop aims to collaborate with the participants on issues concerning democratic game play in the mathematics classroom. During the workshop we will have the opportunity to explore:

- the meanings of the democratic game play,
- the factors that create a democratic game play in the mathematics classroom,
- the forms of a democratic communication through game play in the mathematics classroom,
- the values that students are expected to learn from a democratic game play.

Other questions that will concern us are the following:

- How can we create the appropriate playing environment in which all the students could participate equally?
- Can democratic relationships among players be established during their play?
- What are the forms of a democratic discourse during the game play?
- To ensure a democratic game, which should be the role:
  - of the choice of the game?
  - of the rules?
  - of the players?
  - of the teacher?
  - of the process of playing?

The structure of the workshop will be the following:

1. An introduction about the place of game play within the mathematics classroom will be presented.
2. The participants would separate in groups in order to explore designed board games (such in the photos 1, 2, 3, 4).
3. A short description of the games will be provided.
4. Each group, by playing the game, will record their thoughts on the above issues.
5. Each group positions will be presented.
6. A whole group discussion will be made about the factors in creating a democratic game play.

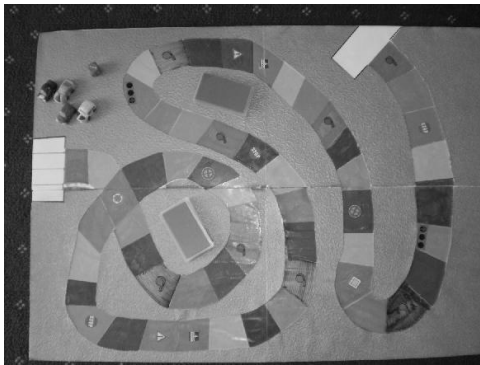


Photo 1

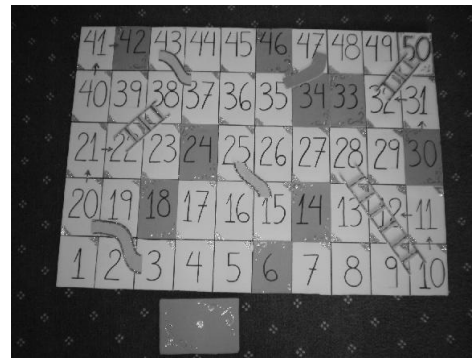


Photo 2



Photo 3

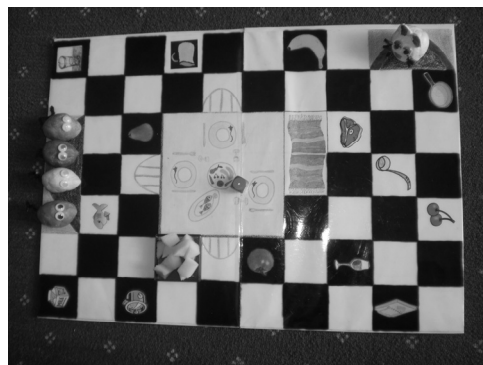


Photo 4

**References**

- Alro, H. & Skovsmose, O. (1996) On the right track. *For the Learning of Mathematics*, 16(1), 2-8.
- Berlinger, Y. (2005). Ethics in Gaming 1.0. *The Games Journal / A magazine about Boardgames*  
<http://www.thegamesjournal.com/articles/Ethics1.shtml> accessed on 20/2/2008.
- DeVries, R. (1980). Good group games: What are they? In C. Kamii & R. DeVries, (Eds.), *Group games in early education. Implications of Piaget's theory* (pp. 3-9). National Association for the Education of Young Children Washington, D.C.
- Hart, S. & Mas-Colell, A. (1997). *Cooperation: Game-theoretic approaches*. NATO Advanced Studies Institute, Series F, Vol 155. Springer – Verlag Berlin Heidelberg.
- Malloy, C. (2002). Democratic access to mathematics through democratic education: an introduction. In L. English (ed.) *Handbook of International Research in Mathematics Education* (pp. 17-26). Lawrence Erlbaum Associates, Publishers, Mahwah, New Jersey, London, National Council of Teachers of Mathematics.
- Olson, J.C. (2007). Developing Students' Mathematical Reasoning through Games. *Teaching Children Mathematics* 13(9): 464-471.
- Skoumpourdi, C. (2010). Play as context for approaching early childhood mathematics: Designing board games. *Journal of Contemporary Education*, 162, 139-156. (GR)
- Skovsmose, O. (2005). Foregrounds and politics of learning obstacles. *For the Learning of Mathematics*, 25(1), 4-10.
- Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (ed.) *Handbook of International Research in Mathematics Education* (pp. 383-408). Lawrence Erlbaum Associates, Publishers, Mahwah, New Jersey, London, National Council of Teachers of Mathematics.
- Valero, P. (1999). Deliberative Mathematics Education for Social Democratization in Latin America. *ZDM*, 31(1), 20-26.

# FORUM OF IDEAS- FORUM D'IDEEES

Hellenic Mathematical Society  
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## Graphical perception: a case study at the university level

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In recent years the education research community has shown a great interest in the problems that can arise when doing didactical activities based on students competences about *graphical representations* in mathematics classroom and scientific environment (Shaughnessy, Garfield, & Greer, 1996). Nowadays these kind of competences are more significant in our highly technological society and these are often key aspects to live as a conscious citizen. The comprehension and the reading graphs reported in newspapers, the comprehension and the reading climate temperature's graphs could be significant examples.

In this paper we discuss a experimental study on the critical factors influencing graph's reading and comprehension of students on the first year mathematics program for Pharmacist at the University of Palermo (Italy).

The answers to a specially designed questionnaire<sup>1</sup> are analyzed on the basis of an a-priori analysis made using a general scheme of reference on the epistemology of mathematics and in particular on the competence used to read and correctly infer the data shown on a graph.

The study was performed by using quantitative data analysis methods, i.e. factorial analysis of the correspondences and implicative analysis (using the C.H.I.C. Software). A qualitative analysis of key-words and terms reported by students during interviews was also used to examine some aspects that emerged from the quantitative analysis. The study's conclusions are consistent with previous research conducted by G.R.I.M. and discussed

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<sup>1</sup> Inspired by INVASI (National Institute for the Evaluation of the Operating System of Education and Training) administered in Italy to Italian students of 3rd grades of Upper Secondary School in 2007.

in literature, but the use of quantitative data analysis allowed us to classify the involved students into three 'profiles' related to different epistemological approaches to knowledge construction (called 'behaviorist', 'cognitivist' and 'constructivist') contextualized to the graphical perception and organized in different cognitive levels: 'Elementary', 'Intermediate' and 'Overall' (Cursio, 1981; 1987).

In our research bringing together different perspectives concerning the processing and the use of graphs, we report a general framework to identify critical factors that appear to influence graph comprehension and we discuss some our results about students behaviors in the phases of *translation*, *interpretation*, and *extrapolation/interpolation* (Friel e alli, 2001).

As the first step of our theoretical/experimental research, we refer to the definition of *graph* proposed by Fry (1984) and the related definition of *graph comprehension* suggested by Friel e alli (2001).

The main research questions involved in this study are the following:

- *Do a university student (attending a first year program of mathematics for Pharmacist) be able to correctly interpret a mathematical graph refereed to a real situations?*
- *What are the critical factors influencing their graph comprehension.*

The results that we will show in the C.I.E.A.E.M. poster will indicate a considerable difficulty of the students on logical thinking, proportional reasoning and graphing ability that confirm literature results discussed on the law competence that students reach at the end of Upper Secondary school levels (Wavering , 1989).

The major part of the students is able to focus their attention only on some particular elementary information presented in a graph. Few students can be considered able to reach competences of intermediate level, characterized by interpolating and finding relationships in the data as shown on a graph. A minority part of the student extrapolate data from graph and analyze the relationships implicit in a graph (i.e., generating, predicting).

#### **Essential references:**

Curcio, F. R. (1981). *The effect of prior knowledge, reading and mathematics achievement, and sex on comprehending mathematical relationships expressed in graphs* (Final Report). Brooklyn, NY: St. Francis College. (ERIC Document Reproduction Service No. ED 210 185)

- Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education*, 18, 382–393.
- Friel, S.N., Curcio, F.R., Bright, G.W. (2001). Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications, *Journal for Research in Mathematics Education*, 32, 124–158
- Shaughnessy, J. M., Garfield, J., & Greer, B. (1996). Data handling. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 205–237). Boston, MA: Kluwer
- Wavering, M. J. (1989). Logical reasoning necessary to make line graphs. *Journal of Research in Science Teaching*, 26, 373–379.



## **CONSIDERATIONS ABOUT PROOF IN TEACHER DEVELOPMENT PROGRAMMES**

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**ABSTRACT:** The aim of this article is to present the conceptions of a group of 14 Brazilians school teachers regarding the role of proving in Mathematics teachers education courses in the sphere of a general learning and, more particularly, taking into account the fact that this subject might constitute a future object of study. Thus, we are guided by the following question that emerges from the research study: "What are the implications for in-service teachers courses of carrying out a piece of work with proof in secondary-school<sup>1</sup> classes?"

Keywords: Proof and Proving, Continued Education, Mathematics teachers education.

### **1. CHARACTERISING THE RESEARCH PROBLEM**

It is well-known that there is a good deal of international research about the introduction of argumentation and proof in secondary school. However, this subject has not been debated much in Brazil, in particular with regard to related areas in teacher education courses. Research among the community of Brazilian Mathematics Educators in this area is comparatively rare.

Some educational specialists argue that we should not undervalue the role of argumentation and proofs in students' learning and support the idea of including this subject in the school curricula.

This attitude in favor of proving in school curricula can doubtless be

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<sup>1</sup> From 11 to 18 years.

explained by the attention that has been given to it in recent years, by several researchers. Schoenfeld (1994), for example, argues that demonstrations are not something that can be taken away from Mathematics as it occurs in a lot of teaching programmes. In his view, proving is an essential feature of practice and communication in Mathematics.

In Brazil, the National Curricular for secondary-school recommend, albeit in a rather half-hearted way, that from secondary-school education onwards, there should be work involving argumentation and proof.

Following a survey that was undertaken with teachers from in-service teachers courses in the public network in 2001, we were able to conjecture that the initial and in-service programmes for Mathematics teachers had not attached importance to teaching skills in proof. This conjecture is strengthened by the results of The National Examination of Courses: the performance of the students in the last year of their initial teacher course was unsatisfactory in dissertational questions, especially those with instructions requiring the student to "prove", "demonstrate" and "justify", even so the students from the universities who, overall, achieved the best results.

As an example, we can refer to the performance of prospective teachers in one dissertational question proposed in 2001, which involved tasks that are usually carried out for students 13 or 14 years old. One question aimed to assess if the future teachers knew how to prove the Bhaskara Formula, used to solve a quadratic equation. The results obtained were surprising: on a scale of 0 to 100, Brazilian average was 8.6.

In the light of this situation, we sought to examine the opinions of some teachers, with strong Mathematics knowledge and whose teaching practice includes some sort of work involving argumentation and proof, about the need to introduce this content into the curricula of secondary-school and how far they had access to them. We also investigated the implications of this innovation for the curricula that governed the initial teacher education.

## **2. THE STUDY**

For the purposes of this study, we carried out 14 interviews with Mathematics teachers from secondary-schools which had control over the subject-matter used at this school level. The teachers undertook to provide evidence of the opportunities to include argumentation and proof in the secondary-school Mathematics curricula and the work that had to be carried out in the initial teacher education so that they would have greater skills in planning and controlling learning situations in this area. During the

interviews, the teachers were also asked to examine the proofs prepared by the students, using the same models employed by Healy and Hoyles (1998) and Dreyfus (2000) which respectively dealt with the opinions of students and teachers about proofs in Mathematics. This study also relied on the work of Knuth (2002).

Reading the testimony of the interviewees allowed us to identify what we call the 'units of meaning', or rather, the particular comments which were most significant in enabling us to provide aspects of the research questions for discussion. In other words, these units were extracts from the remarks of the interviewees and were of great potential significance in the view of the researcher.

### **3. SUMMARY OF POINTS OF AGREEMENT**

The conclusions of our investigation with regard to the role of proofs in initial and in-service teacher education can be outlined as follows:

The notions and beliefs that teachers have about their work with proofs at secondary-school act as obstacles to implementing innovatory ideas.

These teachers do not believe that teaching proof can be a potential means of enhancing doing Mathematics in classroom because this work would be restricted to a few students. In their view, the work with proof – when there is a lot of it - should only be restricted to semi-formal proofs. They explained their opinion by referring to their memories of experiences with demonstrations (almost always unsuccessful), when they were students in secondary-school or in initial teacher course.

However, the analysis that they conducted of work carried out by students (which was shown to them during the interviews), revealed to us that there was a state of tension: their remarks swung between accepting and classifying an "empirical proof" as excellent and creative, and rejecting it on the grounds that it was not really a mathematical proof, or rather, it was not a rigorous proof. This was the case with all the teachers: when analyzing the work carried out by another student, the teacher shifted his position from praising the "experimental qualities" to, the next moment, again rejecting this kind of diligence. This tension was noticeable in the testimony too.

The inclusion of proof in initial and in-service teacher education should be undertaken both in the list of substantive items of knowledge and in the list of pedagogical and curricular types of knowledge.

Our interviewees set out various explanations to show the importance of undertaking rigorous proofs in initial teacher courses subjects. They were needed to learn more Mathematics and were essential when doing or

communicating Mathematics, as well as being an essential component of the culture of this area of knowledge. The participants in this research study thought that in the initial teacher course, a student should learn how to demonstrate, even if she was not going to carry out proofs in the classroom in the future. This was because the teacher has to acquire knowledge beyond what she is going to teach – what is sometimes described as “a supplementary stock”.

The interviewees believed that a Mathematics teacher, who knows how to demonstrate theorems and formulae regarding the subject-matter they are going to teach, can share leadership with their peers.

There was also a consensus about the way that future teachers should study proofs in the initial teacher course; they should experience situations analogous to those they were going to share with their students.

This summary of goals, objectives and methods shows that in initial teacher courses in Mathematics, the proof should be regarded as:

an instrument, which is involved in various subjects in the course (to test the validity, explain, refute, outline theories) and as an important subject in forging links between mathematical topics (historical problems, the connections between different subject areas) or else from the perspective of understanding and looking at concepts and procedures in greater depth;

a subject that will become a constituent part of teaching or more precisely, a part of its pedagogical and curricular perspective (specific aims, examples and counter-examples, analogies, representations, problem situations that need to be given validity, the results of research from a didactic perspective, didactic series).

#### **4. FINAL WORD**

In our interviewees’ testimony about the inclusion of proofs in initial and in-service teacher courses, some explanations were made, regarding a type of knowledge that is classified as “substantive knowledge” (Shulman, 1986). In our view, knowing proofs from the perspective of substantive knowledge means that the teacher must possess a sufficient amount of knowledge to allow him to have intellectual autonomy over the subject. This autonomy means, for example, not only knowing the demonstrations of the theorems and formulae, which will be employed in the future but also having the ability to select and organizes these theorems and knowing their respective applications. It implies knowing how to distinguish between what is of major or secondary importance. It requires, above all, that one knows how to set up problems from the demonstrations in a way that can combine

them with the subject that is being undertaken. To achieve this, it must act as a mediator between the historically produced knowledge and the kind of knowledge which will be adapted by the students.

In other words, they will be able to extend the range the proofs that they explain (Hanna, 1990). In the view of Hanna, not every demonstration has the power to explain and he warns that abandoning demonstrations that validate in favor of those that explain, will not make the curriculum less satisfactory in reflecting acceptable practical mathematics.

Thus, our research shows that demonstrations in initial and in-service teacher courses should be given greater scope than has been given, nowadays. This greater prominence can be achieved if the courses do not make use of proofs just to learn more Mathematics or with the aim of developing mathematics reasoning skills, but could be applied from a didactic, curricular and historical perspective.

Finally, it should be stressed that our discussion about what constitutes the teachers knowledge of proofs is at an extremely important even decisive, moment of history, with regard to initial training courses, though obviously not in a decisive phase. We know that training a teacher to undertake a professional activity is a process that entails several stages – both forwards and backwards – and that in the last analysis, it is always – or nearly always - incomplete.

#### REFERENCES

- Dreyfus, T. (2000) Some views on proofs by teachers and mathematicians. In: Gagatsis, A. (Ed.) Proceedings of the 2nd Mediterranean Conference on Mathematics Education, Cyprus: The University of Cyprus, v. 1, p. 11-25.
- Hanna G. (1990) Some pedagogical aspects of proof. *Interchange*, n. 21(1), pp. 6-13.
- Healy, L., & Hoyles, C. (1998) Justifying and Proving in School Mathematics Technical Report, University of London, Institute of Education.
- Knuth, E. (2002) Teachers' conceptions of proof in the context of Secondary School of Mathematics. *Journal of Mathematics Teachers Education*, 5(1), pp. 61-88.
- Schoenfeld, A. H. (1994) What do we know about Mathematics curricula? *Journal of Mathematical Behavior*, n. 13(1), pp. 55-80.
- Shulman, L. S. (1986) Those who understand: Knowledge growth in teaching. *Education Researcher*, vol. 15, n.2. pp. 4-14.

## How Underachievers become Experts in Maths: the Bètacoach Model

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### **Learning math by explaining**

Students' talk will increase their thinking, especially when performing the *key activities* as formulated by Dekker and Elshout-Mohr (1998): to show one's (thinking) work, to explain one's (thinking) work, to justify one's (thinking) work, to reconstruct one's (thinking) work. Providing explanations highly correlates with learning achievements (Webb, 2009). Some students, however, have so little self-confidence in maths, that they never fully grab the opportunity to learn from explaining (Pijls, 2007).

In primary education, peer tutoring is a commonly used to improve the tutor's reading and math abilities. Vosse (1998) found peer tutoring to be very effective for the tutor's learning, especially when they were pedagogically trained. In the present project, students with low self-confidence and average to low results in maths became *peer tutor* (Bètacoach – in Dutch 'bèta' means 'science') of younger students in the maths lesson.

### **Design: the Bètacoach peer tutoring model**

We started with 13 tutors (from 2 9th grade classes) and their 2 math teachers. These tutors were each responsible for 5 younger students (7th

grade), once a week during the math lesson of these students.

*1. Selection of the coaches*

Students with low confidence in maths, mostly with low to average results. The teachers had to convince these students that they would be able to perform this 'job' and the confidence of the teacher was crucial.

*2. Preparation of the lessons*

The Bètacoaches were trained pedagogically as teachers-in-spe. They learned to ask younger students to show their work and not immediately to tell the answer to a question.

*3. During the math lesson*

The teacher started up the lesson and when the students worked on their mathematical tasks, the Bètacoaches were active. Each weekly lesson, the Bètacoaches worked with the same group of students.

*4. Evaluating the coach experience*

Besides the feedback from their teacher, during and after the lesson, the Bètacoaches got written feedback from their younger students.

## **Results**

After six months working as a Bètacoach, students gained:

- self-confidence in maths
- beter learning results
- learning strategies

*'I now stay questioning until I fully understand the problem'*

*'I became another person in my own math lessons.'*

The younger students gained from the lessons too. They mentioned they could better concentrate, dared to ask more questions, gained better learning results.

And the teacher... gained more information about alle students' homework and progress, developed more focus in the lesson by providing instruction to the coaches and was better able to have some distance and provide help where really needed.

## **Discussion**

Main question for many schools is how to scedule these lessons. In next years we wish to gain insight in how to overcome these difficulties.

## How Underachievers become Experts in Maths: the Bètacoach Model

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!

## PROGRAM OF THE CONFERENCE

	Monday 23/07	Tuesday 24/07	Wednesday 25/07	Thursday 26/07	Friday 27/07
9:00-10:30		Plenary: Koeno Gravemeijer "Aiming for 21 <sup>st</sup> Century Skills"	Plenary: Corneille Kazadi "LA DÉMOCRATIE EN ÉDUCATION Les mathématiques : le terreau propice pour la démocratie"	Plenary: Anna Chronaki "Fragile Experiments with Mathematics and Technology: zooming into teaching and learning enactments"	Special Session: Theodore Chadjipadelis "Misconceptions and misunderstandings electoral statistics. Can statistics education improve democracy?"
10:30-11:00		Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00-11:30		WG 1X4	WG 3X4	WG 4X4	Panel
11:30-12:00		Lunch	Lunch	Lunch	Final Report
12:00-12:30		Meet Plenary speakers 1 & 2	Forum of Ideas	Meet Plenary speakers 3 & 4	Snack
12:30-13:00		Workshop 1X3		Workshop 2X3	
13:00-13:30		Coffee Break		Coffee Break	
13:30-14:00		WG 2X4		WG 5X4	
14:00-14:30					
14:30-15:00					
15:00-15:30					
15:30-16:00					
16:00-16:30					
16:30-17:00					
17:00-17:30	Registration (meeting of animators 17:00)				
17:30-18:00	Opening ceremony				
18:00-18:30	Plenary: Ole Skovsmose				
18:30-19:00	"Mathematics Education and Democracy: An on-going challenge"				
19:00-20:30	Welcome cocktail				
21:00					

## PROGRAMME DE LA CONFERENCE

	Lundi 23/07	Mardi 24/07	Mercredi 25/07	Jeudi 26/07	Vendredi 27/07
9:00-10:30		Séance Plénière: Koeno Gravemeller "Visée vers les apfitudes du 21ère Siècle"	Séance Plénière: Cornelle Kazadi "LA DÉMOCRATIE EN ÉDUCATION Les mathématiques : terreau propice pour la démocratie"	Séance Plénière: Anna Chronaki "Expérimentations fragiles avec les mathématiques et la technologie: focalisation sur les textes d'enseignement et d'apprentissage"	Séance Spéciale: Theodore Chadjipadelis "Idées fausses et malentendus des statistiques électorales. L'éducation à la statistique peut-elle améliorer la démocratie?"
10:30-11:00		Pause Café	Pause Café	Pause Café	Pause Café
11:00-11:30		WG 1X4	WG 3X4	WG 4X4	Table Ronde
11:30-12:00		Déjeuner	Déjeuner	Déjeuner	Rapport Final
12:00-12:30		Rencontre Conférenciers de la séance plénière 1 & 2	Forum d'idées	Rencontre Conférenciers de la séance plénière 3 & 4	Hors-d'œuvre
12:30-13:00		Atelier 1X3		Atelier 2X3	
13:00-13:30		Pause Café		Pause Café	
13:30-14:00		WG 2X4		WG 5X4	
14:00-14:30					
14:30-15:00					
15:00-15:30					
15:30-16:00					
16:00-16:30					
16:30-17:00					
17:00-17:30	Inscription (Rencontre avec les animateurs 17:00)				
17:30-18:00	Cérémonie d'ouverture				
18:00-18:30	Séance Plénière: Ole Skovsmose				
18:30-19:00	"Education en Mathématiques et Démocratie: Un défi continu "				
19:00-20:30	Bienvenue cocktail				
21:00					

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