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Mathematical modeling of the epidemic diseases

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Abstract

Biology and mathematics are key lessons in our curriculum from elementary to high school. Biology science studies the structure and evolution of living organisms and is directly linked to human life on the planet. Many of us have been wondering, "Why do we learn Math?" And "Where will they serve us?" In this work, by linking the courses of Biology and Mathematics, we highlight the role and value of mathematical science in analyzing and explaining real-world situations and phenomena. The experiment we conducted in three classrooms of our school, enabled us to mathematical modeling of infectious diseases, to draw conclusions, to motivate us to further research mathematical models and simulations that would interpret real-world conditions and contemporary ones. Therefore, it will help in making decisions and preventive measures.

Keywords

Diseases, mathematics, biology, experiment, models

Introduction

People have been confronted with infectious diseases, however, with the contribution of science and technology, they have managed to survive. In the 21st century, scientists, researchers, public health officials and governments continue their efforts to control infectious diseases such as HIV, West Nile virus, various strains of influenza, severe influenza (SARS) and encephalopathy. In the quest for discovering methods and applications that will prevent, protect and cure deadly diseases, mathematical science has made a great contribution. Increasingly, many biological phenomena are thought to involve the processing of information that ultimately involves the use of mathematical sciences. Mathematics has played, a significant role in many important biological breakthroughs, for example the sequencing of the human genome and is of fundamental importance in the rapidly evolving concept of "digital biology".

In our work we carried out an activity that simulates the spread of a disease. Epidemiologists and public health officials want to be able to predict the spread and impact of various diseases. Researchers and scientists conduct computer simulations and then develop mathematical models to predict how infectious diseases will appear and spread. The mathematical tool helps us capture and explain a phenomenon through observations and experiments. Why are mathematical models useful? A mathematical model can be defined as a description of a system using mathematical concepts and language to facilitate proper explanation of a system or to study the effects of different components and to make predictions on patterns of behavior [1]. Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application. Mathematical modeling: a) is indispensable in many applications; b) is successful in many further applications; c) gives precision and direction for problem solution; d) enables a thorough understanding of the system modeled; e) prepares the way for better design or control of a system; f) allows the efficient use of modern computing capabilities. Learning about mathematical modeling is an important step from a theoretical mathematical training to an application-oriented mathematical expertise and makes the student fit for mastering the challenges of our modern technological culture [2].

Theoretical Background

Before conducting the experiment, we studied and extracted our knowledge from the biology textbooks on the scientific concepts involved in the science of epidemiology (bacteria, viruses, immune system, vaccines, etc.), ways to transmit infectious diseases. Pathogenic microorganisms, such as bacteria, viruses, parasites or fungi, cause infectious diseases and can spread, directly or indirectly, from one person to another [3].

Our activity simulates the progress of an illness in a closed population using SI and SIR models.

- SIR Model: A simple mathematical model of an epidemic, expressing the relationship of time with different groups of people
- Susceptible (S): These people have not yet contracted the disease. However, they are also not immune to this and thus may be infected with the disease in the future.
- Infected (I): These are people who are infected with the disease and can transmit the disease to sensitive people.

- Recovered (R): These are people who have been restored from the disease and are immunized, so they cannot be infected again.

Methodology

Our simulation consists of two parts

Part 1: Model SI

An orange cube represents a susceptible individual, a red cube represents an infected individual, and a green cube represents a person who has been previously affected by the disease and has been restored. Each cup draw represents the interaction of two people in your class and the possible transmission of illness or recovery from the disease.

The steps we followed are as follows:

1. Select random 2 cubes from the class cup.

2. Then analyze the interaction of the cubes. If he comes in contact with 2 orange or 2 red cubes, they both need to be returned. If they choose 1 orange and one red cube, then we put the red back and replace the orange cube with a red one.

3. The numbers of susceptible and infected individuals are recorded in the category of the monitoring sheet after each test (matrix 1,2,3)

4. The test is repeated for each student in the class.

For a larger population sample, and therefore more accurately, the experiment was conducted in three classes of our school.

5. We then represent the number of susceptible and infected individuals in a one-year graph, where the time period is the independent variable (x-axis) and each measurement (S and I) is a dependent variable (y-axis) (graphs 1, 2,3).

Part 2: The SIR model.

1. Randomly select 2 cubes from the class cup.

2. The interaction of the selected cubes is then analyzed. If 2 orange cubes are selected both should be returned. If they choose 1 orange and 1 red cube, we put the red one back and replace the orange cube with another red one. If you select 2 red cubes, put 1 red back and replace the other with 1 green. If you select 2 green cubes, place both of them, back.

3. The numbers of susceptible, infected and rehabilitated individuals will be recorded in the cover sheet category after each test (matrix 4,5,6)

4. After the tests, we determine the prevalence and incidence of the disease for each time.

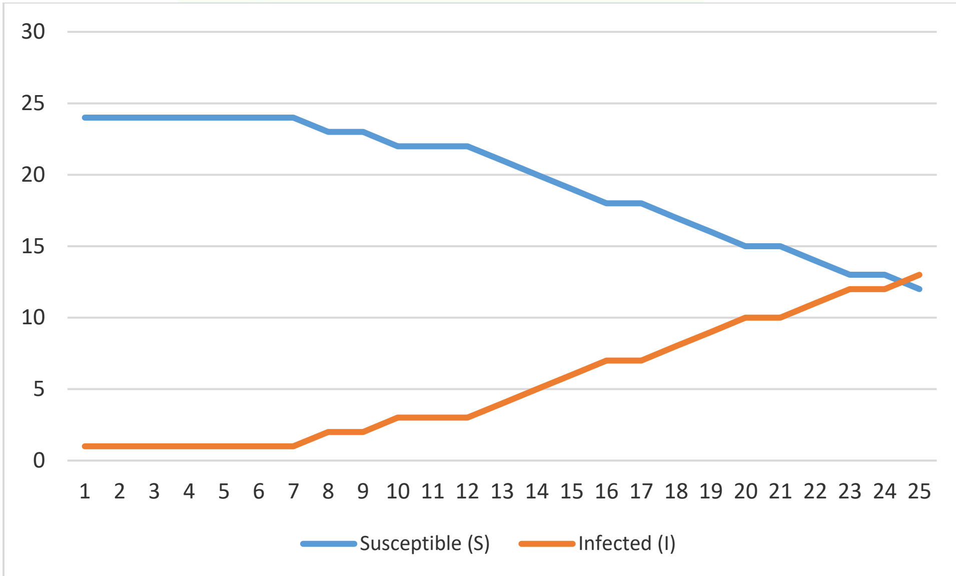
5. We represent the number of susceptible, infected, and recovered individuals in a single timeline, where the time is the independent variable (x-axis) and each count (S, I, and R) is a dependent variable (y-axis) (graphs 4,5,6,7,8,9)

Results

Part I - Model SI

Time periods	Susceptible (S)	Infected (I)
1	24	1
2	24	1
3	24	1
4	24	1
5	24	1
6	24	1
7	24	1
8	23	2
9	23	2
10	22	3
11	22	3
12	22	3
13	21	4
14	20	5
15	19	6
16	18	7
17	18	7
18	17	8
19	16	9
20	15	10
21	15	10
22	14	11
23	13	12
24	13	12
25	12	13

Matrix 1: Disease Prevention of sheet for 10th grade of class (A Lykeiou) –

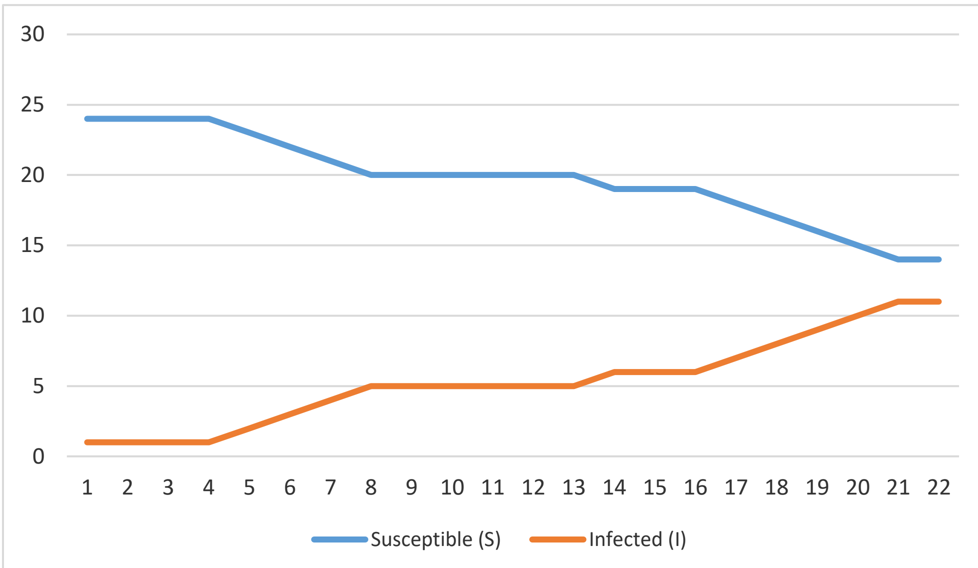


Graph 1: Disease Prevention of 10th grade of class (A Lykeiou)

Time periods	Susceptible (S)	Infected (I)
1	24	1
2	24	1
3	24	1
4	24	1
5	23	2
6	22	3
7	21	4
8	20	5
9	20	5
10	20	5
11	20	5
12	20	5
13	20	5
14	19	6
15	19	6
16	19	6
17	18	7
18	17	8
19	16	9
20	15	10

21	14	11
22	14	11
23	14	11

Matrix 2: Disease Prevention of sheet for 10th grade of class (A Lykeiou) –

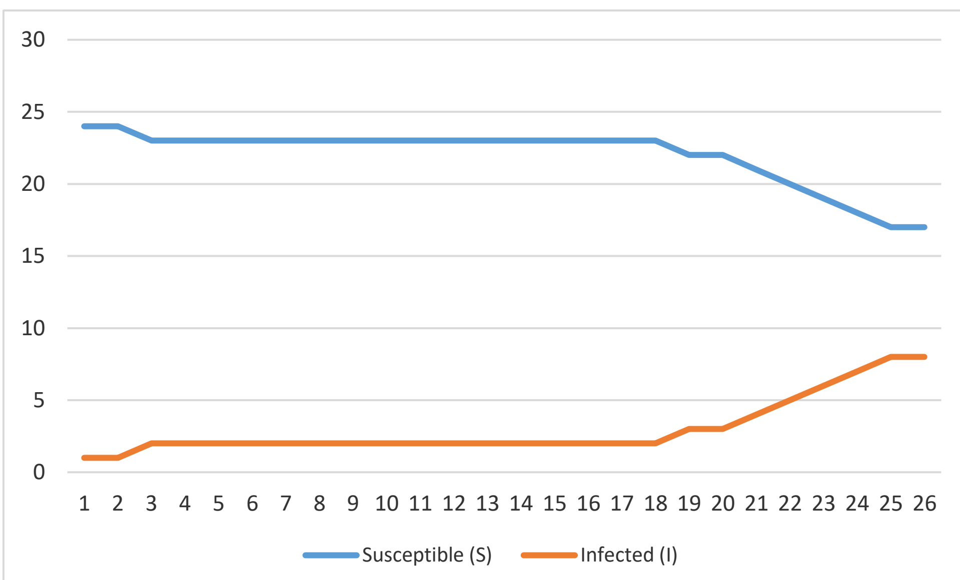


Graph 2: Disease Prevention of 11th grade of class (B Lykeiou)

Time periods	Susceptible (S)	Infected (I)
1	24	1
2	24	1
3	23	2
4	23	2
5	23	2
6	23	2
7	23	2
8	23	2
9	23	2
10	23	2
11	23	2
12	23	2
13	23	2

14	23	2
15	23	2
16	23	2
17	23	2
18	23	2
19	22	3
20	22	3
21	21	4
22	20	5
23	19	6
24	18	7
25	17	8
26	17	8

Matrix 3: Disease Prevention of sheet for 12th grade of class (C Lykeiou)



Graph 3: Disease Prevention of 12th grade of class (C Lykeiou)

Part I - Model SI

Matrix 4 Disease Spread Monitoring for sheet of 10th grade class (A Lykeiou)

Time periods	Susceptible (S)	Infected (I)	Restored	Disease Spread/ Infections / Total Impact	New Infections / Total
1	12	13	0	52,0%	0,0%
2	12	12	1	52,0%	0,0%
3	12	12	1	52,0%	0,0%
4	11	13	1	56,0%	4,0%
5	10	14	1	60,0%	8,0%
6	9	15	1	64,0%	12,0%
7	8	16	1	68,0%	16,0%
8	7	17	1	72,0%	20,0%
9	7	17	1	72,0%	20,0%
10	6	18	1	76,0%	24,0%
11	6	17	2	76,0%	24,0%
12	6	17	2	76,0%	24,0%
13	6	16	3	76,0%	24,0%
14	5	17	3	80,0%	28,0%
15	4	18	3	84,0%	32,0%
16	3	18	3	84,0%	32,0%
17	3	17	4	84,0%	32,0%
18	2	18	4	88,0%	32,0%
19	2	17	5	88,0%	36,0%
20	2	16	6	88,0%	36,0%
21	2	15	7	88,0%	36,0%
22	2	14	8	88,0%	36,0%
23	2	13	9	88,0%	36,0%

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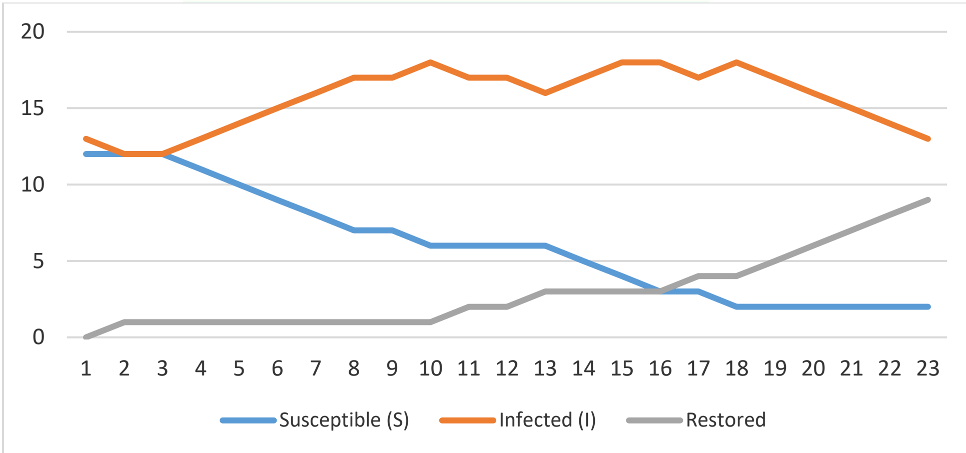
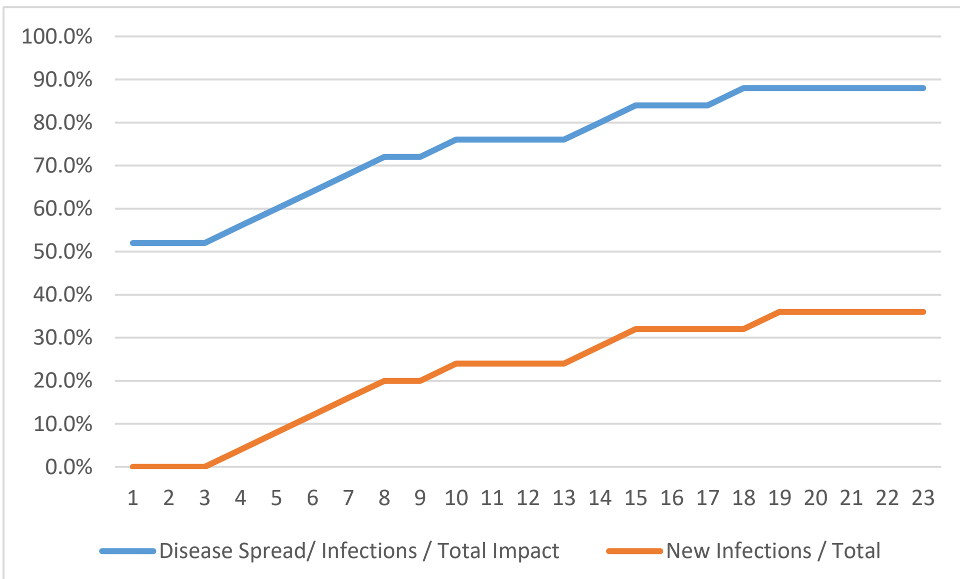


Graph 4: Disease Spread Monitoring for sheet of 10th grade class (A Lykeiou)

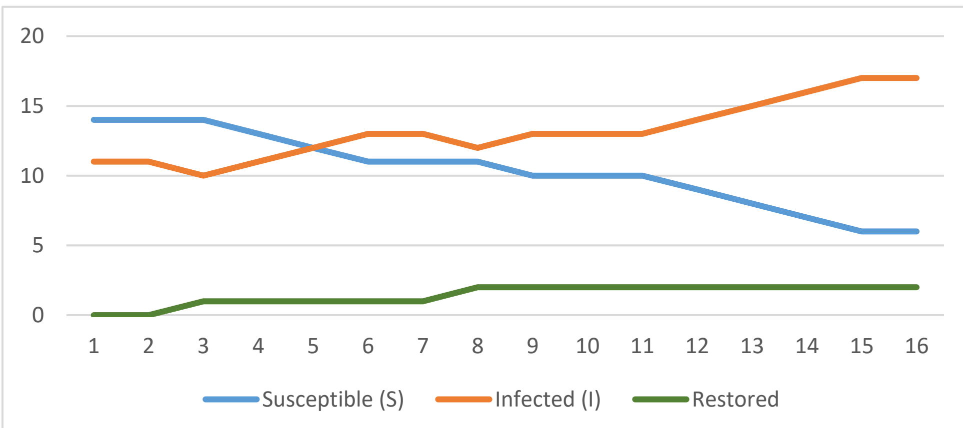


Graph 5: Disease Spread Monitoring Model (Dissemination-Impact) for sheet of 10th grade class (A Lykeiou)

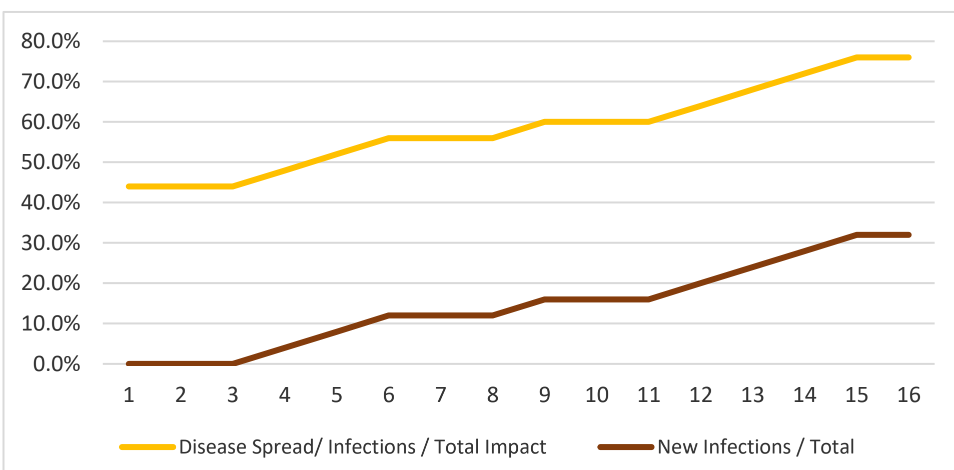
Time periods	Susceptible (S)	Infected (I)	Restored	Disease Spread/ Infections / Total Impact	New Infections / Total
1	14	11	0	44,0%	0,00%
2	14	11	0	44,0%	0,00%
3	14	10	1	44,0%	0,00%
4	13	11	1	48,0%	4,0%
5	12	12	1	52,0%	8,0%
6	11	13	1	56,0%	12,0%

7	11	13	1	56,0%	12,0%
8	11	12	2	56,0%	12,0%
9	10	13	2	60,0%	16,0%
10	10	13	2	60,0%	16,0%
11	10	13	2	60,0%	16,0%
12	9	14	2	64,0%	20,0%
13	8	15	2	68,0%	24,0%
14	7	16	2	72,0%	28,0%
15	6	17	2	76,0%	32,0%
16	6	17	2	76,0%	32,0%

Matrix 5 Disease Spread Monitoring for sheet of 11th grade class (B Lykeiou)



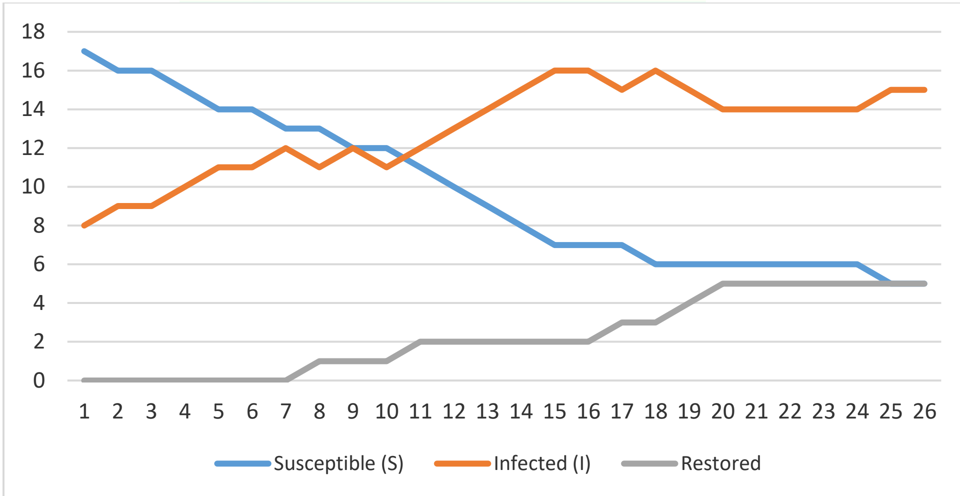
Graph 6: Disease Spread Monitoring for sheet of 11th grade class (B Lykeiou)



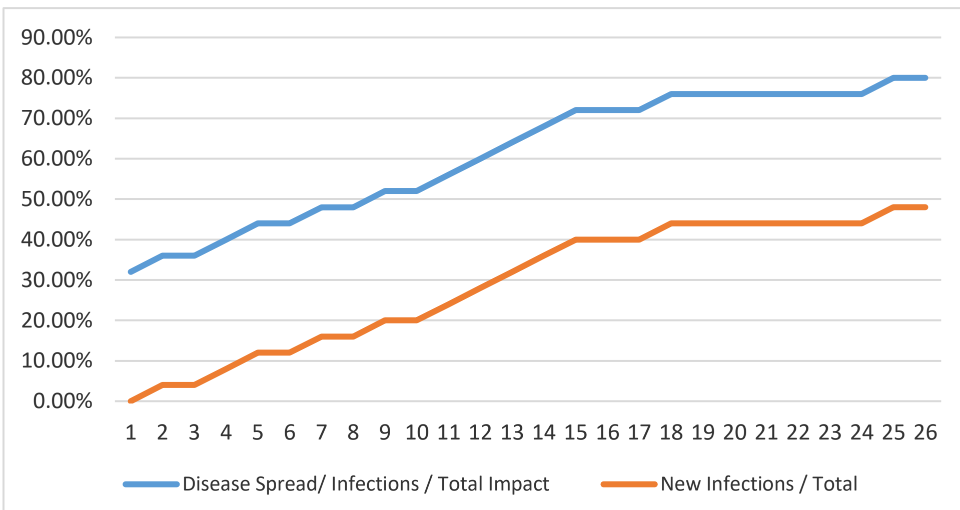
Graph 7: Disease Spread Monitoring Model (Dissemination-Impact) for sheet of 11th grade class (B Lykeiou)

Time periods	Susceptible (S)	Infected (I)	Restored	Disease Spread/ Infections / Total Impact	New Infections / Total
1	17	8	0	32,00%	0%
2	16	9	0	36,00%	4,00%
3	16	9	0	36,00%	4,00%
4	15	10	0	40,00%	8,00%
5	14	11	0	44,00%	12,00%
6	14	11	0	44,00%	12,00%
7	13	12	0	48,00%	16,00%
8	13	11	1	48,00%	16,00%
9	12	12	1	52,00%	20%
10	12	11	1	52,00%	20,00%
11	11	12	2	56,00%	24,00%
12	10	13	2	60,00%	28,00%
13	9	14	2	64,00%	32,00%
14	8	15	2	68,00%	36,00%
15	7	16	2	72,00%	40,00%
16	7	16	2	72,00%	40,00%
17	7	15	3	72,00%	40,00%
18	6	16	3	76,00%	44,00%
19	6	15	4	76,00%	44,00%
20	6	14	5	76,00%	44,00%
21	6	14	5	76,00%	44,00%
22	6	14	5	76,00%	44,00%
23	6	14	5	76,00%	44,00%
24	6	14	5	76,00%	44,00%
25	5	15	5	80,00%	48,00%
26	5	15	5	80,00%	48,00%

Matrix 6: Disease Spread Monitoring for sheet of 12th grade class (C Lykeiou)



Graph 8: Disease Spread Monitoring for sheet of 12^{th} grade class (C Lykeiou)



Graph 9: Disease Spread Monitoring Model (Dissemination-Impact) for sheet of 12^{th} grade class (C Lykeiou)

For SI graphs: The graphs of infected appear to be symmetrical. This is because for each infected person, the population of susceptible is also reduced by one. Consequently, infected people grow at exactly the same rate as susceptible ones. What the difference in the charts for each class shows is that the rate at which an epidemic is spreading is not always the same but varies according to environmental conditions. According to the conventions of the experiment, the only thing that changes from one class to another is the interactions between the cubes, since each student "pulls" different combinations each time, so they become random. Therefore, we could mention that different contacts make a disease spread. Typically, in the diagram of the 10th grade the lines intersect, that is, there is a period of time during which the populations of the susceptible and the infected are equated. Afterwards, the infected continue to grow, thus outgrowing the susceptible. If the experiment did not stop, the lines would continue to run symmetrically until the senses were reset and the infected ones retained the maximum value. In the diagram of the lines are close enough, but not intersecting, while in the diagram of the 11th grade, at the end of the experiment, there seems to be a significant distance between them throughout the experiment. This means that in the experiment in 11th grade class, the disease spreads at a slightly slower rate than in the 10th one, while the 3rd experiment shows the slowest rate of spread by all classes.

For SIR Charts: The lines susceptible and infected are no longer the same as in the previous chart. Although they look symmetrical, they are not exactly as a new population group, the restored, is added, which directly affects the sensitive-infected interaction. The impact of the introduction of the new population is evident in the diagram of 10th grade the infected-susceptible lines intersect again due to the reduction of the infected with the restoration of the health of some. In this experiment, the values of the sensitive ones are very close to the value of 0. The infected, of course, cannot obtain the maximum value, as the group of the restored is inserted. So, if the experiment were to continue, eventually the number of susceptible and infected people would be reduced to zero, and the value of the restored ones would reach and maintain the maximum value. In the diagram of the 11th grade, the restored ones do not affect the course of the other two lines as strongly as only two infected ones recover. However, we certainly cannot talk about symmetrical lines of susceptible-infected. Finally, in the diagram of the 12th grade, the populations of the susceptible and the infected are intersected twice, while the susceptible and the restored are identified and remain stable for a period.

As for the lines of propagation and impact, their segments appear to be completely parallel to each other. This is because they both increase by one for each new infection per period but have a constant difference in the original number of infected. In addition, although these two increase in a similar way to the line of infected, they are not parallel to this, because the infected are sometimes reduced due to the restoration of health of some, so the chart is affected accordingly.

Discussions

Categorizing the cubes into three colors that symbolize susceptible, infected and restored individuals realistically simulates reality. In addition, the random selection of the cubes that come into contact simulates quite well the likelihood of two people coming into contact, and if the conditions are met, one of them becomes infected.

There are many conventions to make the experiment easy, so this simulation is obviously wrong in some places. For example, it is not possible to contact only 2 people in each time period. In addition, there is no fixed incubation period, and the duration of the disease is identical to that of infectivity. Realistic and specific timescales for both disease, infectivity and incubation of individuals could be set.

Conclusions

Infectious diseases are an extremely critical issue in our daily lives. In our work we presented their dissemination through an experiment implemented in our school. By adding mathematical equations to the model, epidemiologists can predict how many people will be susceptible, infected, or restored at any given time. It is extremely important to be aware of the conditions in other countries, especially regarding our hygiene, but before traveling too far it is essential to consult your doctor about possible preventive measures or vaccines.

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