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Exponential functions through a real-world context

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Abstract

The exponential function as a mathematical concept plays an important role in the corpus of mathematical knowledge, but unfortunately students have problems grasping it. Paper exposes examples of exponential function application in a real-world context.

Keywords

mathematical modeling; the exponential function; real-world context; functional knowledge

Introduction

Exponential functions in the high school level are, first of all, hard to grasp by students, according to Goldin and Hersowics, (Goldin and Hersovics, 1991). Secondly, this type of function is the base of calculus and differential equations and lots of natural and social phenomena could be explained through the models based on exponential functions. Finally, the great importance of knowledge about exponential functions can be explained by Professor Al Barlett's quote that "The greatest shortcoming of the human race is our inability to understand the exponential function". Barlett also argues that even though the growth is the foundation of our civilization prosperity in the sense of business, economy or technology, this topic is not

adequately represented in the mathematical and physics education. The processes of growth are dominating in our everyday life, and mathematics of growth is mathematics of the exponential function. It is important to understand the exponential growth because that would make us able to evaluate many situations concerning growth (Barlett, 1976).

Application of the mathematical modeling in the teaching process of exponential functions

In this paper we present four examples performed in the classroom which had covered the various mathematical contents related to exponential functions in a real-world context such as domain, function growth, asymptotes, derivatives, and inverse function (Budinski, 2011, Budinski and Subramaniam, 2013, Budinski, 2013a, Budinski 2013b). The content required by Serbian curriculum related to exponential functions are related to the definition of the exponential function, it general properties (such as zero and sign of function), growth of the exponential function, derivative of the exponential function, inverse (logarithmic) function, and graph of the function.

The described examples were performed in the high school (gymnasium) "Petro Kuzmjak" in Serbia in the periods 2013-2015 and 2017-2019. It is important to mention that all lessons were supported with technology, especially with the educational software Geogebra (www.geogebra.org). Technology was recognized as a great remedy in learning about exponential functions. The usage of technology enables students to focus their efforts on the application of mathematics and model determination, rather than focus on computations. Technology can be used in many phases of learning, as working with data, representing the problem graphically or simulating the model. With technology it is possible to skip a routine of calculations which helps students to orientate towards mathematical appliance.

Exponential functions in the paper were presented as process-oriented examples. Examples were close to reality, with avoidance of artificial problems; developed from the point of interest to the students, so it could be analyzed and discussed in detail; rich with information, both relevant and irrelevant, so that the students could investigate and recognize the solution for the problem; unsolvable at the first glance, a little bit complicated, without an obvious path to the solution; broad and connected to other subjects or relevant topics; motivating; opened for further investigation or alternative solutions.

Also, the real situations are chosen to illustrate the cross-disciplinary opportunities, and substantial effort was made to connect lessons with other school subjects, for example, biology, geography or sociology. In this paper we describe four examples with the basic real-world situations of saving money, bacterial growth, local population change, and earthquake magnitude. Some of examples are well known in the teaching practice, such as saving money or bacterial growth, but there are also examples that are up to date with the contemporary world, or of the students' personal interests.

The first example is related to the real-world situation of savings and economy issues. The real-world problem of saving money in the bank can be easily grasped. Process of saving money, could be represented by mathematical model in the form of an exponential function in few steps. The real-world problem is dealing with the invested sum and the amount of money after certain number of years. The first step of the modeling process is to clarify how the amount of money changes according to time. In other words, that the longer the money is in a bank, the greater amount of money would be received. The money amount depends on time. There are two types of mathematical models: (1) simple, which is mathematically a linear function, and (2) compound, which is mathematically an exponential function. Two types of mathematical models were an excellent tool for browsing the features of both, linear and exponential functions. Also, this phase of mathematical modeling could be used to discuss the domain and the co-domain of the function with reflection on the solution of the real-world problem. As seen in Fig_1 we can see investigation of features of linear and exponential functions to the real-world context using software Geogebra. For example, it can be noticed that there is no evident difference of the money amount calculated with linear or exponential model in a short period of time.

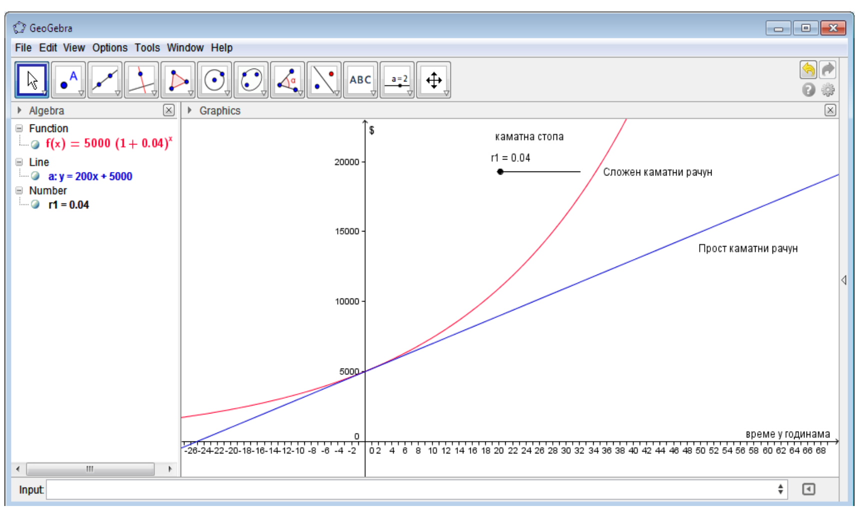

Fig_1 Features of linear (straight line graph) and exponential (curve line graph) function in the real-world context of money saving

The second example is dealing with the exponential function growth and asymptotes. The chosen real-world problem is concerning the problem of population change in local communities. This kind of real-world problem tackle both, mathematical as well as social issues. The real data from the census of our country (or another country) should be used in order to analyze the change in the population and make an assumption for the future. This kind of set up for a problem provide less information for students than they are used to. That could be possibly very encouraging for students and has a potential to increase their motivation to the investigation of the problem. For example, by observing the change in the population over the course of several decades can lead to the prediction of the future population by a mathematical model which would describe the data. The first step is to recognize the change, and label the independent and the dependent variable, which, in this case are the time and population, respectively. In this particular example it could be done by using a Geogebra option of function fitting to obtain a mathematical model. The choices could be exponential and logarithmic fitting, after observing the plotted data. For example, as seen in Fig_2 it can be noticed that the population data graphically corresponded to the graph of the exponential function. That was the reason why to choose the command of exponential fitting to receive the mathematical model in form of the exponential function which could finally represent the change of the observed population. After that, the calculation and prediction the number of inhabitants can be done for the future in ten or a hundred years. In this part, the asymptotes and other features of exponential functions could be revised and connected to the real-world problem. Great attention needs to be paid to the problem of infinity in the mathematical and real context. In this example the stress is put on the substantial use of technology in the modeling process, since the majority of calculation and graphical representation was done by Geogebra.

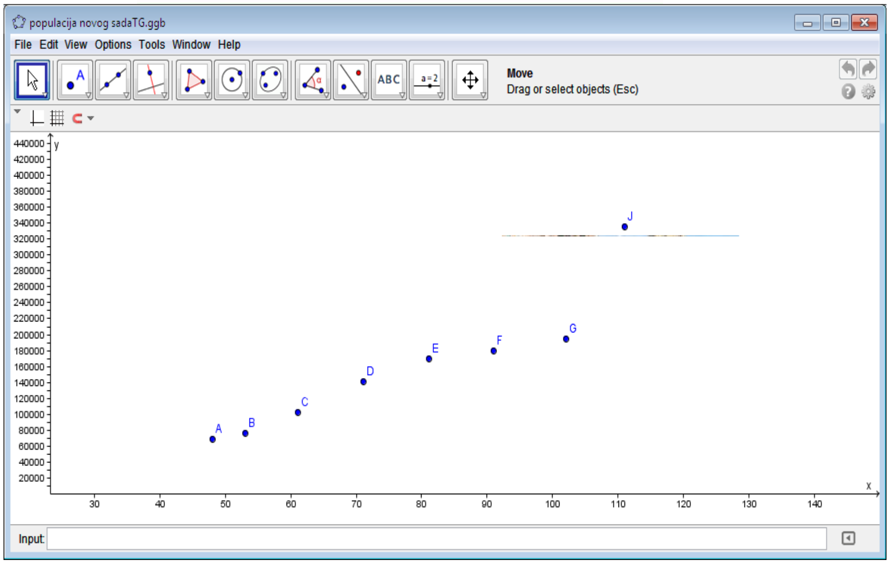

Fig_2. Observation of the population data during the modeling process using Geogebra

The third problem is the real-world problem of bacteria growth, which allows to explore the domain of exponential functions. Trough this problem the mathematics of derivatives, slopes and tangent line could be explored. Even though the real-world situation of bacteria growth is difficult to grasp, or even artificial, when it comes to the interpretation of mathematical results in the real-world context (Castilo-Garsow, 2013), there is a lot of mathematical content that can be conveyed trough. With this example the problem of the domain of the exponential function is illustrated, in a mathematical and real-world context. Bacterial growth is described simply by splitting. That lead to the most often represented model of bacterial growth which is function y=2x,, where y represents the number of bacteria, and x represents the time period, but on the domain of natural numbers. For example, the mentioned function describes the situation where the number of bacteria doubles every minute, with the beginning of zero minute and one bacterium. Its graphical representation is shown in the Fig_3.

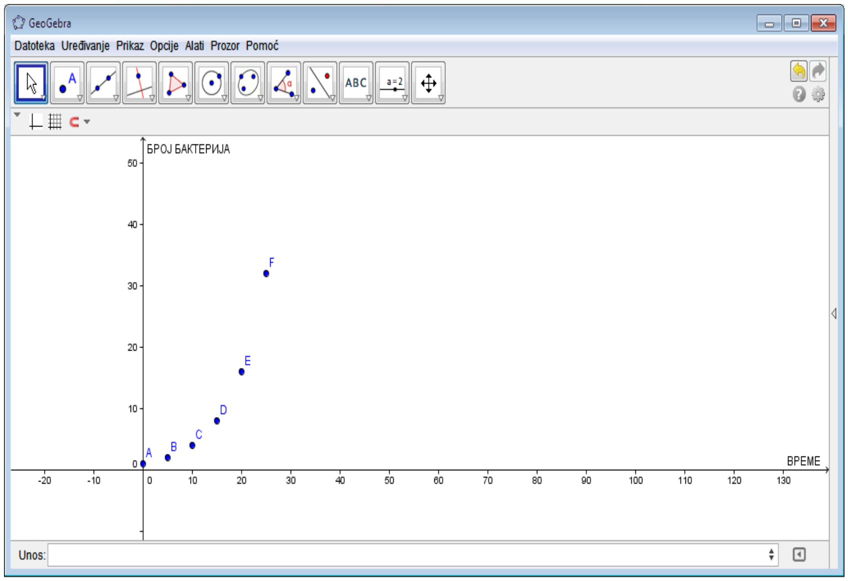

Fig_3. Graphical representation of the function y=2x which describes the bacterial growth in the domain of natural numbers

The problem that could be posed to the students in this phase is how to explain the bacterial growth in the domain of the real number. In order to discuss the obtained model, a slight inaccuracy could be noticed, since for example, at the time of 0.2 minute the model predicts 1.14869 bacteria, which is impossible in real life. It can be explained that the real-world situation of bacterial growth might take into account, for example, the overlapping of the bacterial generation (Castillo-Garsow, 2013). After considering the nature of the real-world situation in more details, it can be noticed that the graph of the function $y=2^x$ on the domain of the nonnegative real numbers could be approximated as in the Fig_4. Only this small step in the modeling process contributed to at least two things: (1) the deeper understanding of the concept of the function and its definition by the domain, and (2) the fact that the modeling process is a complex activity where mathematical and real context do not correspond bijectively, so some compromise has to be made in order to obtain an acceptable solution. After the clarification of that, other mathematical concepts were browsed, especially the first derivative of the exponential function, and its application to the real-world situations.

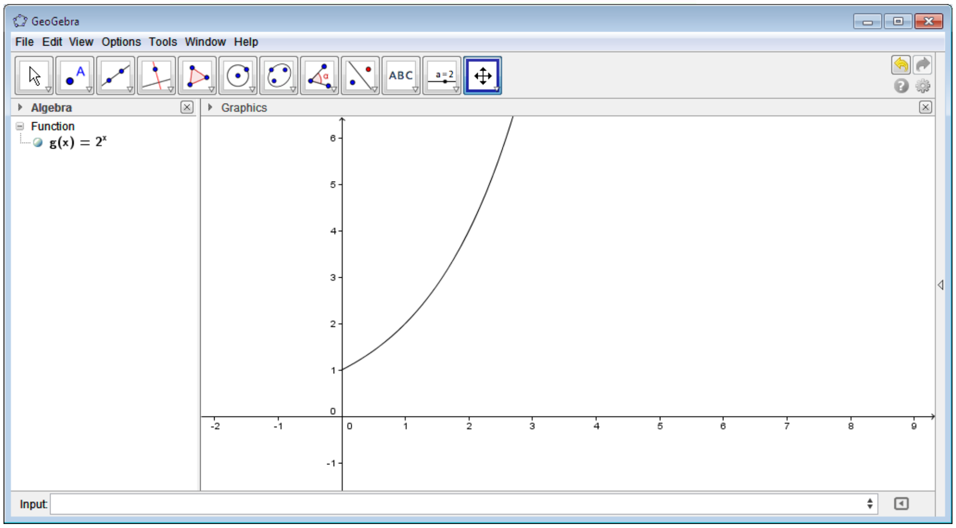

Fig_ 4 Graphical representation of the function y=2^x on the domain of nonnegative real numbers based on the obtained task of bacterial growth

Finally, the fourth problem is bonding the real-world problem of earthquakes and the mathematical problem of inverse function. In this particular example, the connection is made between the exponential function and its inverse function, logarithmic. Firstly, a very powerful real-world situation can be used as an illustration, such as the earthquake in Japan, which according to the news paper, shifted the Earth's axis. In this case, the mathematical model for measuring the magnitude of the earthquake can be offered, which has a form of logarithmic functions. The real-world context of energy released in the earthquake should follow after that. The main task is to compare the energy released in earthquakes that occurred in the past and recent history and mathematical context of inverse function, exponential and logarithmic functions. During the modeling, the notion of inverse function can be explored in detail. As well as the logarithmic scale. With this example, the following goals could be fulfilled: introduction with a ready-made model and critical observation of the received information, for example, from the media. The focus of the lessons is on the problem understanding, the differences between relevant and irrelevant information, and the identification of mathematics underlying the real-world context.

Conclusion

Described examples begin with a complex real-world situation, where in the beginning it was not obvious which type of mathematical procedures should be implemented. The use of technology allows to explore, visualize, and calculate, which lead to the mathematical solutions of the real-world problem. One of the challenges of contemporary mathematical education is to make the knowledge of students applicable and suitable for the demands of the contemporary society. While answering the posed problem, the presented study bridges the educational theoretical research and practice. In the field of instructional perspectives of mathematical modeling, it answered the important questions about classroom instructions occurring in the classroom, which is considered as one of very important problem of the mathematical modeling processes application in education (Cai et al, 2014). The mathematical content such as the exponential function has a great importance in real-world application since the connection to numerous different fields of everyday life and sciences is evident, such as biology, chemistry, technology, physics, statistics, engineering, telecommunication, environment, economy and so on (Ganter and Baker, 2004). Mastering such a topic during high school education should be recognized as a demand of contemporary society.

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