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Building an Instrument by Generating a Function

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Abstract

Building a musical instrument requires precise calculations of the positions of its holes to obtain desired notes. To make these calculations, sound waves should be examined. Sound is a longitudinal wave and has five distinguishing aspects: amplitude, frequency, wavelength, period, and speed. A note is characterized by the frequency of the wave. In this experiment, the frequencies of notes and the speed of sound in air is used to calculate the length of a closed-end tube needed to obtain that frequency. Then, these values of lengths are used to create sound and the frequencies are measured with a tuner. The water level is changed until the desired frequency is reached. Then, a graph of length and 1/frequency is drawn to experimentally determine the slope, from which a function of length of tube is written in terms of frequency. With this equation, length required for any note can be calculated.

Keywords

Standing Waves; Frequency; Wavelength; Speed; Node; Antinode; Note

Introduction

Building a musical instrument requires knowledge of waves' properties. When a note is played, different waves on the same medium reflect from the fixed ends and interfere to create a standing wave. (Strings, Standing Waves and Harmonics, 2020) The standing waves oscillate; however, their antinode doesn't change position horizontally. Instruments use standing waves to create pitches.

All the holes and strings in musical instruments are created by calculating frequencies of notes and the corresponding lengths. This can be achieved by using the relationship between the speed of sound, frequency, and wavelength. (Strings, Standing Waves and Harmonics, 2020)

As speed is displacement/time, the speed of a wave can be expressed as wavelength/period. As frequency is the reciprocal of period, substituting the frequency in the equation will result in the commonly used equation: $v=\lambda f$, where v represents velocity, λ represents wavelength and f represents frequency.

When a wave is formed inside a tube, which has a closed end and an open end, an antinode will be formed at the open end and a node will be formed at the closed end (See Figure 1). An antinode is where the amplitude of the wave reaches its maximum value and a node is where the amplitude of the wave is at its minimum value. The wavelength can be found by calculating the distance between two crests (maxima) or two troughs (minima). Therefore, in a closed end tube, if the wave is at its fundamental frequency, one quarter of a wave will form inside the tube, so the wavelength of the wave at its fundamental frequency in the tube can be expressed as $\lambda = 4L$, where L is the length of the tube. (Standing Waves and Resonance| Electronics Textbook, 2020)

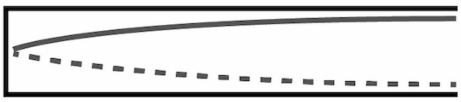

Figure 1: Wave at its fundamental frequency in a closed-end tube (Standing Waves and Resonance| Electronics Textbook, 2020)

However, this equation is only true if the wave is at its fundamental frequency. When an instrument is played, multiple harmonics are created to form a note. These harmonics are the reason why different instruments sound different. The wavelength of these harmonics can be found with the equation $\lambda= 4L/n$, where n is the harmonic number. Since there is always a node at the closed end of a tube and always an antinode at the open end, n can never be an even integer.

Although multiple harmonics are created when a note is played, the first harmonic dominates other harmonics and determines the pitch. (Lenhoff and Robertson, n.d.)

When the speed of sound equation and the wavelength equations are combined the following equation is obtained:

$$f_n = n \times \frac{v}{4L}$$

As n is 1 for the fundamental frequency, the frequencies of the harmonics of a wave can be calculated by multiplying the harmonic number of a wave with its fundamental frequency.

According to this equation, since the speed of a wave only depends on the conditions of the medium it is travelling in, the fundamental frequency of a wave must increase proportionally as the length of the tube decreases. Therefore, while creating a musical instrument, to create a higher pitch, the length of the tube must be decreased. In this experiment, the corresponding lengths of a tube for different notes will be calculated for different pitches by using the speed of sound and the frequencies of the notes. Then, a length vs 1/frequency graph will be drawn to experimentally obtain the speed of sound in air. Then the speed of sound in air will be used to write an integral expression and create a function of length of tube in terms of frequency.

Materials

Three identical tubes, water, meter stick, tuner

Procedure

First, the corresponding frequencies for five notes, C_5, D_5, E_5, G_5, A_5, are found by internet search. This experiment is done according to the A_4=434 Hz equal temperament system. Then, the wavelengths of these frequencies are calculated by dividing the velocity of sound in air, which is 343 m/s, by the frequencies of the notes. Then, the wavelengths are divided by 4 to calculate the length of the tube. As the tubes are identical, the length of the tube, where air can vibrate to create sound waves, can be decreased by increasing the water level. The length of the tube refers to the total length of the tube - the water level. The water behaves like a closed end and the end of the tube behaves like an open end. A disturbance is created by blowing air perpendicularly with a fan at the end of the tube and a tuner is used to determine the frequency of the sound. The length is changed by adding or removing water until the desired frequency is reached. Each length is tested for three tubes and the average of these frequencies is taken to create a frequency over $\frac{1}{L}$graph. The slope of the graph gives the velocity of the sound in air divided by four and the graph is utilized both to predict the corresponding lengths of other frequencies and to compare the theoretical lengths and the experimental lengths.

Results and Discussion

As the speed of sound is equal to frequency times wavelength, the wavelength of a note is calculated by dividing the speed of sound in air (which is 343 m/s) by the frequency of the note and the length is calculated by dividing the wavelength by four. Then, the lengths are modified until the desired frequency is reached with the tuner (See Figure 2).

Note Name	Frequency (Hz)	Calculated λ (m)	Calculated Length (m)	Tube Length (m)			
				Trial 1	Trial 2	Trial 3	Average Value
C_5	516.12	0.664	0.166	0.166	0.166	0.166	0.166
D_5	579.32	0.592	0.148	0.152	0.154	0.155	0.154
E_5	650.27	0.524	0.131	0.131	0.134	0.133	0.133
G_5	773.30	0.440	0.110	0.112	0.112	0.112	0.112
A_5	868.00	0.392	0.098	0.093	0.098	0.095	0.095

Figure 2: Table of experimental values of tube length for five pitches

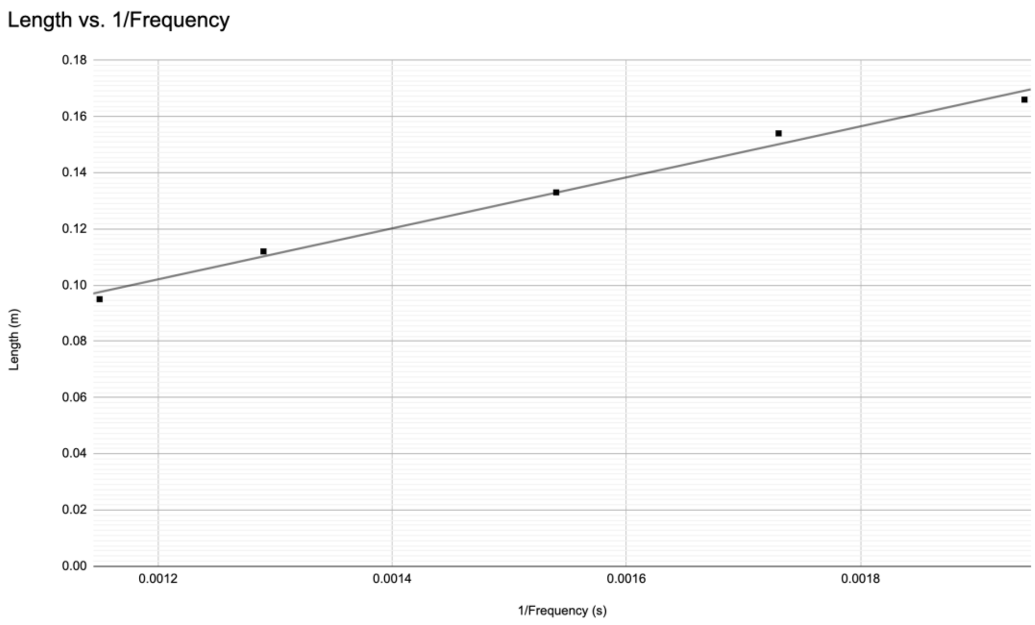

Figure 3: Length vs 1/Frequency graph

The actual values of the lengths are very close to the best fit line (See Figure 3), so the equation can be interpreted to other frequencies without yielding a large percent error.

The slope of the length vs 1/frequency graph gives the velocity of the sound divided by four. As the slope of the graph is 88, the calculated speed of the sound in the medium is 352 m/s. The length of the tube for any frequency can be calculated with the equation:

$$L(f) = 0.098 + 88\left(\frac{1}{f} - 0.00115\right)$$

This equation can be used to calculate other required lengths to create waves with certain frequencies.

Although the real speed of sound in air is 343 m/s, the experimentally determined value is 352 m/s. The percent error is 352-343343= %5. In the experiment, it was assumed that the water created a node as it behaves like a closed end; however, the water may not have behaved exactly like a closed end due to the vibrations in the water. Although the main source of vibrations in the experiment are air molecules, there might also be vibrations due to water molecules and glass, which can change the resultant waves' properties. Therefore, this can be the reason for the experimental error.

Conclusion

Building a music instrument requires knowledge of waves. Since the speed of sound is constant in a certain medium and desired frequencies are known, the wavelengths can be calculated. By using the wavelengths and the properties of waves in closed- end tubes, the corresponding lengths to desired frequencies can be calculated and a function of length can be written in terms of frequency. Since the tube may not behave as assumed, the equation might deviate in real life conditions. Therefore, an experiment should be designed to find the real values of tube length and these lengths should be used to create a length vs 1/frequency graph. The slope of that graph should be constant and represent the velocity of sound in air divided by four. Finally, the slope of that graph can be used to write an integral expression and create a function of length in terms of frequency.

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